

79047
J. J. J. J.

I think this Book fit to be Printed.

*June the 4th.
1690.*

JOHN HOSKINS. V. P. R. S.

Nobilissimo Clarissimoq; Viro
Christiani Huygens
Christiano Hugonio Zulichemio Const. F.
Inter Principes Mathematicos Philosophosq;
Reverendo

Hoc Observantia Testimonium Mittit
Sui Nominis ac Eruditionis
Cultor Devotissimus
Gulielmus Molyneux.

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DIOPTRICA NOVA.

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O F
D I O P T R I C K S,

In Two PARTS.

Wherein the
Various Effects and Appearances
O F

Spherick Glasses,

BOTH
Convex and Concave, Single and Combined,
I N
TELESCOPES and MICROSCOPES,

Together with
Their USEFULNESS in many Concerns of Humane Life,
ARE EXPLAINED.

By WILLIAM MOLYNEUX of *Dublin* Esq;
Fellow of the ROYAL SOCIETY.

Ex Visibilibus Invisibilia.

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To the ILLUSTRIOUS
The Royal Society.

THE Design of the ensuing Treatise being the Promotion of a Part of Physico-Mathematical Knowledge in the English Nation; I know not to whom I can more properly present it, than to that noble Body of English Philosophers, whose Foundation by the Royal Charter of King Charles II. is to this very purpose. How far, and how successfully you have hitherto prosecuted the end of this excellent Institution, 'tis needless for me to declare: since the literate World is already so abundantly stored with your learned Labours, and useful Discoveries; whereof I could here recount a List of many Hundreds published by several Members of the Society. But I design not a Panegyrick, but an humble Address for your Favour and Countenance to my present Endeavours: And this I hope for, with the more Assurance, having already seen your favourable Acceptance of many Offerings of this kind, and your ready Incouragement of all such Philosophical Inquiries, as tend to the use of Life, or Advancement of Arts and Sciences.

And on this Occasion I cannot omit expressing my Sence of that excellent Method of Experimental Philosophy, which now, by your Example and Incouragement, does so universally prevail, and is so highly advanced all over Europe, and other Parts of the World.

'Tis wonderful to consider, how the Schools were formerly overrun with a senseless kind of Jargon, which they call'd Philosophy;

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DEDICATION

and which men studied with the greatest Labour and Assiduity, that they might attain the name of Wise and Learned. This certainly was the greatest Cheat was ever imposed on the mind of Man: But why say I, imposed? Men drew it on themselves, and run their own Heads into the Noose: and when they had intangled themselves in a thousand ridiculous Disputes about empty Questions, they vainly thought they had attained the Perfection of Philosophers; whilst they had no Ideas in their Minds answerable to those Noises they made with their Tongues; but took more pains to deceive both themselves and others, than is requisite for the Propagation of true Knowledge. And indeed we may well imagine, that, had the former Ages of the World been at half that Labour and Study for the Advancement of real Knowledge, which they spent in promoting verbose Stuff; Mankind by this time might have been by many Degrees more wise, and consequently more happy even in this Life; for Wisdom only makes men so.

But in this last Age the generous Undertakings of the Philosophick Societies of Europe (to whom your Institution has shewn the way, and been an illustrious Example) have dissipated these dark Mists, and have abdicated this kind of empty Stuff; which had crept into even Natural Disquisitions; and like a Leprosie had quite over-run the whole Body of Philosophy, deforming its Beauty, and ruining its Strength. Men are not satisfied now with noisy Words, and nothing else; but require more solid Foundations of Knowledge, and believe no farther than they can find good Proofs.

This great Change, which Philosophy or the Prosecution of Knowledge in general has received of late years, is manifest in all its Parts; but in none more than in natural Enquiries. To these you have given a clearly new Turn, wholly different from the Methods, by which they were formerly prosecuted in the Schools. And how advantageously the Change has been made, will be evident to any that considers the one and t'other Method.

TO THE ROYAL SOCIETY.

The Commentators on Aristotle, (who was certainly himself a most diligent and profound Investigator of Nature) have rendred Physicks an heap of froathy Disputes, managing the whole Knowledge of Body and Motion, of Animals, Plants, and Minerals; of Celestial, Aerial and Terrestrial Bodies; by Hypothetical Conjectures, confirm'd by plausible Arguments of Wit and Rhetorick, ordered in a Syllogistical form; and answering Objections in like manner: But never studied to prove their Opinions by Experiments. By which Method they were as ignorant of the Properties and Affections of Natural Bodies, as if they were not at all the Subject of their Disquisitions. And yet these were the great Dictators of Physicks for many Ages in our Colleges and Schooles; and no one was accounted worthy the Name of a Philosopher, that would not on their Authority Jurare in verba. He that could Dispute and distinguish, about Sympathy, Antipathy, occult Qualities, Antiperistesis, and a Thousand such other fantastick Terms, was reputed a great Proficient, and deeply vers'd in Natures Secrets: tho all the while perhaps he knew not any one of the admirable Phenomena of the Magnet, or was not at all versed in the History of any one Branch in Nature. They'd rumble out indeed the Definition and Divisions of Comets; but knew nothing of the Laws of their Motions; or other Affections. They'd tell you the Tides depend on the Influence of the Moon; and when you proceed farther, and ask, what is this Influence? They'll yet give you a Word for it, and say, 'tis an occult Quality: If you inquire, what an occult Quality is? They'r at a Stand, and having no farther hard Word here to fly to, are forced to confess 'tis a Quality they know nothing of. Had they not better at first have plainly confest, they know not the Cause of the Tides? no surely; For tho this had been more becoming modest Philosophers, it would not so well captivate the Vulgar, and gain to themselves the Repute of deep Knowledge.

Consult
Magirus,
Eustarbi-
us, Zanar-
dus, Col-
Compla-
tensis
Com. &c.
the com-
mon Na-
tural Phi-
losophy
Books read
in our Col-
leges.

Yet this verbose Philosophy is that, which for many Generations prevail'd in the World: This it is, which is injoin'd to be read and studied in our Colleges and Academies, by the Statutes and Charters thereof.

DEDICATION

thereof: which in this Particular, to the apparent Hindrance of the Advancement of real and useful Knowledge, do yet remain unaltered in our Universities: wherein the first years of young Students may be employ'd with much more Advantage by prosecuting other Methods. And, My thinks, it were now full time (after so happy a Reformation of our Errors in Religion, and purging our Seminaries of Learning from the Fopperies and Superstition of a false Worship) to begin a Reformation of our Human Literature, by establishing more useful Methods of Education, especially for the Employment of our more tender years.

But tho this weighty Undertaking has hitherto been deferr'd in them (the Reason whereof I leave to the Consideration of the learned and reverend Heads of our Universities) yet the strong Wits of many in this last Age have broken all these Fetters; And have happily advanced the true Method of prosecuting Knowledge upon solid Foundations.

This is manifest in every Branch of Learning. Logick has put on a Countenance clearly different from what it appeared in formerly: How unlike is its shape in the *Ars Cogitandi*, *Recherches de la Verite*, &c. from what it appears in *Smigletius*, and the Commentators on Aristotle? But to none do we owe for a greater Advancement in this Part of Philosophy, than to the incomparable Mr. Locke, Who, in his Essay concerning Humane Understanding, has rectified more received Mistakes, and delivered more profound Truths, established on Experience and Observation, for the Direction of Man's mind in the Prosecution of Knowledge, (which I think may be properly term'd Logick) than are to be met with in all the Volumes of the Antients. He has clearly overthrown all those Metaphysical Whymfies, which infected mens Brains with a Spice of Madness, whereby they feign'd a Knowledge where they had none, by making a noise with Sounds, without clear and distinct Significations.

Natural

TO THE ROYAL SOCIETY.

Natural Philosophy is now prosecuted by Observation, Experiment, and History thereof. And indeed if we consider it rightly, there is really no other sort of Natural Philosophy, but this only. For by Natural Philosophy, or Physicks, do we mean any thing else, but the Knowledge of the Properties and Affections of Natural Bodies? And is this to be obtain'd any otherwise, than by Experiment and Observation? Can any Man Dispute me into the Knowledge of the Magnet's Attraction, Direction and Variation; or of the Phenomena of the Mercurial Baroscope without Tryal and Experiment? Can any Arguments prove that a little Sulphur, Nitre, and Charcole should produce such a quick and strong Blast as Gunpowder, before they be actually put together and tryed? Men might have disputed to all Eternity, before their Gibberish could discover the Use of that ordinary despicable Substance, Iron-Ore: To which, for ought I see, (as a most ingenious Author has observ'd) we are beholding for all the Politure and Plenty; all the Learning, State, and Magnificence of the World, beyond the Rudeness, Wants, and Ignorance of the ancient savage Americans: Whose Natural Endowments and Provisions equal those of the most flourishing and polite Nations; But they wanted the Advantagious Uses of this contemptible Mineral. So that he, who first discovered the Use of that one poor Mine; or Tubalcain, that first taught the way of working in Iron, may be deservedly celebrated as the Father of Arts, and Author of most of the Conveniences of Human Life.

I know some will say, that by Natural Philosophy is meant not only the Knowledge of the Properties and Uses of Natural Bodies; but also the Assigning the true Reasons or Causes of these Properties. But in this Particular we are to proceed with great Caution. I know the Mind of man is of that inquisitive, prying Nature; that upon any Appearance offer'd to the Senses, it immediately falls to the search after the Cause producing this Effect. But indeed in Natural Disquisitions, 'tis generally (I may say almost alwayes) to no purpose. We may make plausible Conjectures, and some sort of feasible Guesses; but others perhaps may make others, and these also

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equally

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equally probable. But these deserve not the Name of Natural Philosophy; they serve only for Chat and Diversion. For the omnipotent Contriver of the Universe has order'd Natures Operations to be performed by such fine Springs, secret Motions, and inexplicable Ways; that Man in this Life may well despair of attaining the intimate Knowledge thereof; and must therefore content himself with the Contemplation of plain matter of Fact, in which he cannot be deceived. But yet, that we may not wholly suppress this inquisitive Humour, but may only keep it within just Bounds: It will be granted, That whatever immediate Cause can be assigned to an Effect; and it can be proved so to be by some convincing Experiments; and these be often repeated, and diligently examin'd, and found to agree and conspire together; we may be allow'd to found thereon an Hypothesis or Supposal of this Cause, but no more. We must not positively establish it as the undoubted, adequate Cause; for this we may miss after our most diligent Inquiry. However the Experiments we use and demonstrate to sense, for the establishing our Hypothesis, shall be allowed as unquestionable Verities; and shall be embraced as so many Steps of Advancement in the Knowledge of Nature.

But of the Uncertainty of Assigning Natural Causes I shall give but one Instance, and that perhaps as strong as any we shall meet with in Philosophy. We are apt to think, that the Cause of the Suspension of the Mercury in the Torricellian Experiment is undoubtedly the Gravitation of the Air: And we prove it by a most convincing Experiment; for putting the Baroscope into the Pneumatick Engine, exhaust the Air, and the Mercury immediately subsides. But when we consider it a little farther, we shall find, That hereby we have obtain'd little more certain Knowledge, than plainly the matter of Fact of this latter Experiment, and not the adequate Cause of the first Experiment enquired after. For we can only conclude from hence, that the Equipoise of Liquors is the Cause of the Mercury's Suspension: But what is the Cause of this Equipoise of Liquors, or the Cause of the Gravitation of any Liquors, or any Bodies? That is, What is the Cause of Gravity in general is clearly unknown to us;
and

TO THE ROYAL SOCIETY.

and consequently the ultimate Cause of the Mercury's Suspension is not hereby discovered. 'Tis true indeed, by this Experiment we have most probably arriv'd at the Knowledge of one Link more in the Chain of Natural Causes; but this is not conclusive; this puts not an end to the Enquiry: For so if one looking at a Pendulum Clock should enquire, Why the Pendulum does not cease by degrees from vibrating? And he were answered, That it is kept in motion by the next immediate Wheel that beats on the Pallats: And this were offer'd to be proved by Experiment; for stop the motion of this Wheel, and the Pendulum soon ceases: Would he not presently be satisfied, and go away secure, that he had discovered the Cause of the Continuation of the Pendulum's Motion? And yet certainly he has mistaken one single Link for the whole Chain. For if he proceeded farther; and had enquired, What moved this Wheel? He would find, the next Wheel, and so onwards to the Weight or Spring. But here he's at a Loss; for what moves them, is absolutely unknown.

As to most other Reasons in Natural Philosophy, that usually pass as satisfactory, and are received as Accounts of Nature's Proceedings: We shall generally find them little more, than farther Illustrations of the Matter enquired after in some different Words. Thus if it be asked, What is the Reason the Sun casts a shadow from some Bodies, viz. those we call opaque, and none from others, viz. Transparent Bodies? 'Tis answer'd, Because by the opaque Body the Rays of the Sun are stop't in their Progress, and hindred from enlightning that Part of the Ground or Floor that is behind the Body: In the Transparent Body the Rays pass freely. This would be taken by several as deep Reasoning; and yet is really no more than Tautology; as if we should say, the Sun casts a Shadow, Because he casts a Shadow. But it gives no Account of the Opacity of one Body, and Transparency of another, which truly the Question requires. If we ask, How Fire burns? 'Tis answered, by exciting a violent Motion in the Parts of the Combustible Matter: Which indeed is no more than the same thing in different Words. But how Motion is excited or communicated by one Body to another, is absolutely inexplicable. Yet these

DEDICATION

Kind of Verbal Reasons do generally pass in Talk, and serve to amuse as well as the best.

Since therefore we cannot expect to arrive at the intimate Knowledge of Natures Operations: Let us apply our selves to know as much of Her, as we may be certain of. And this only is in Matters of Experiment and Tryal; wherein by the infallible Guidance of our Sences, we cannot be deceived. For tho, Experimentum periculosum, be an ancient Aphorism; yet we must consider of what Experiments it was pronounced, viz. of those in Medicine and Diseases. It agrees not to all, when diligently enquired into, and often repeated. But when we meet with any Experiment that is thus fallacious, we are not to rely on it; yet of this we may be sure, and lay it down as a discover'd Certainty, that sometimes it hits, sometimes it misses; and in this Truth we cannot be deceived, seeing we have so often found it.

But I have lanch'd out thus far, before I was aware: I must recover my self, and beg Pardon both for this Digression, and for telling you Things which you very well know already, at the same time when I offer you a Petition. But my Desire of propagating this useful Method of Philosophy will excuse my Fault, and at the same time will recommend me the more to your Favour, who are the great Patrons thereof, and in Account whereof your Name is deservedly celebrated all over Europe.

Permit me therefore to lay this Offering at your Feet, it being the Explication of one of the most noble Instruments of Experimental Philosophy. Not that I think thereby to add any think considerable to the vast Treasure of Curiosities you already possess: But that I may have an Opportunity of declaring to the World, how much I am

Your Devoted

Humble Servant,

WILLIAM MOLYNEUX.

April 17.
1690.

ADMONITION
TO THE
READER.

BEFORE the Reader proceed to the following Sheets, I desire he may take notice;

First, That he is not to expect *Geometrical Strictness* in several Particulars of this Doctrine. I say, in *several Particulars*; For many there are, which will bear the most *precise Exactness*: *Kepler, Cavallerius, Herigon, Dechaes, Honoratus Faber, Gregorius, Barrow*, and other Authors have taken this Liberty; as being more desirous of shewing in gross the Properties of Glasses and their Effects in Telescopes, than of affecting a *Nicety*, which would be of little Use in Practice. Thus we shall find in what follows, that many Lines are supposed *equal*, which strictly taken are really not so; but yet are so very little different, that for all use, and ease of Demonstration, they may be taken as *equal*. Thus also we suppose very small *Angles* and their *Sines* to be *proportional*; which *precisely* is not so, but is so to the smallest and most insensible Difference. Thus likewise, we sometimes consider not the *Glasses Thickness*, but suppose it of the *least Thickness imaginable*, or of no Thickness at all; which yet is false, but does hardly prejudice any Demonstration. For *Dioptricks* being a part of *mixt Mathematicks*, conversant about *material Lines* (or Rays of Light) and the refractive Power of a *corporeal Glass*, cannot be delivered with that *Ἀνεξήλικα Geometrika* requisite in *abstracted Mathematicks*.

Secondly, The Reader will find some *Corollaries* and *Scholiums* hereafter delivered, which of themselves more properly might have constituted Propositions. But these did not occur so readily in the course of the Work, And therefore I chose rather to add them as *Corollaries* than

A D M O N I T I O N

than change the number of Propositions and Citations. Of this kind are *Corol. ad Prop. 8. Corol. ad Prop. 15. &c.*

Thirdly, I have presumed to intitle this Work *Dioptrica Nova*, as being indeed almost *wholly new*. Very little being borrowed from others, but what is requisite to shew the former Methods of Authors in demonstrating their Propositions, and to keep up the Consecution and Series of the Propositions of this Book. Besides, I can say, that the *Geometrical Method* of calculating a Rays Progress, which in many particulars is so amply delivered hereafter, is *wholly new*, and never before publish'd. And for the first Intimation thereof, I must acknowledg my self obliged to my worthy Friend Mr. *Hamsted. Astron. Reg.* who had it from some unpublished Papers of Mr. *Gascoignes*.

Lastly, I declare, That if in any thing hereafter delivered, I have made any Mistakes, or not so clearly expressed my self upon Intimation thereof, I shall be most ready to *retract*. And therefore if in any thing I have *slip'd*, or made a *false Step*, I desire the ingenious and candid Reader either to inform me thereof, and upon Conviction, I shall *submit*; or else that he would freely pardon the Error; *Humanum est*.

Tho I had begun, and made some Progress in this Work in *Latin*; yet I thought it convenient (at least for the present) to alter my Design: There being nothing in *this Part of Mathematicks* ever yet publish'd in *English*. And I am sure there are many ingenious Heads, great Geometers, and Masters in Mathematicks, who are not so well skill'd in *Latin*.

If Forreigners may think this Work deserves their Perusal; in time perhaps I may satisfy them.

And because I have studied to be as plain as possibly I could, I have chosen rather to be *prolix* in many Particulars, than leave any Ambiguity to the most ordinary Mathematician. And therefore in demonstrating, I often repeat several Steps tho the least altered; that all things may ly'e as plain as possible. And on this Account I hope, the curious and profound Geometers will pardon me; I know they are used to what is *concise* and *closely* put together, much expressed in a little: But this Method suits not every Reader: And I have chosen to accommodate mine to the plainest Capacities.

Lastly, in this practical kind of Mathematicks, I desire the Reader to have frequent recourse to Experiments, for these illustrate the Theory and make many things clear, which otherwise will pass obscurely.

To the READER.

I use the common Notes of Algebra, with two or three new ones introduced by *Branker* or *Dr. Pell*, such as

\div Signifies *Divided by*, $\div \bar{2}$ signifies *Divided by 2*.

$\bar{2}$ With a Line over it signifies the absolute Number 2.

* Is the Note of Multiplication or drawing into.

In the Demonstrations the larger and lesser Margins or Columns are of the same use as is expressed in *Dr. Pell's Algebra*. To which I refer the Reader.

April 17.
1690.

WILL. MOLYNEUX.

DIOPTRICKS.

THE READER

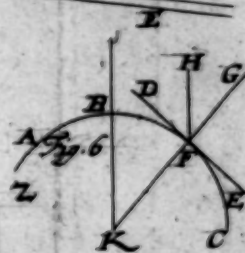
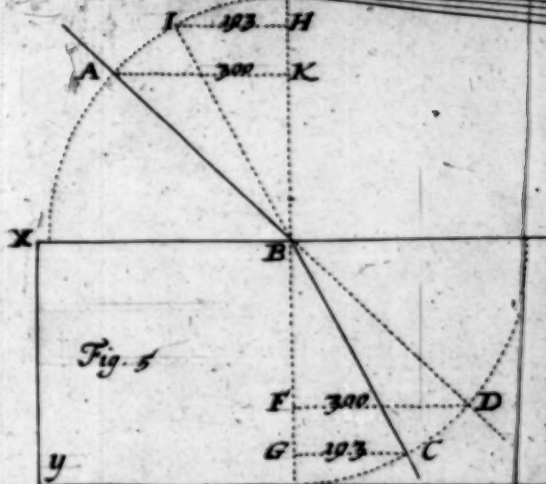
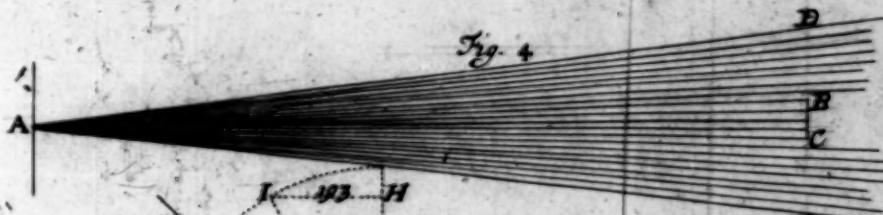
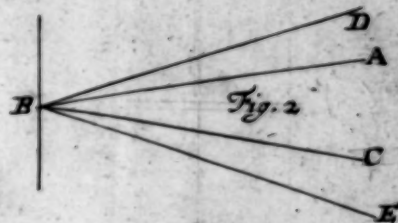
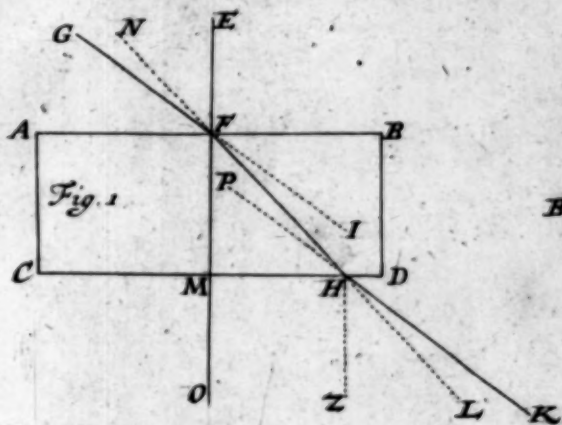
I have the honor to inform you that the new edition of the book is now ready for sale.

The book is now in the hands of the printer and will be ready for sale in a few days.

I am, Sir, very respectfully,
Your obedient servant,
W. L. M. L. R. R. R.

W. L. M. L. R. R. R.

DIOPTRICS



Tab. 1. pag. 1

DIOPTRICKS.

Definitions.

I. **T** *Ab. 1. Fig. 1.* ABCD is a Body of Glass, ^{Tab. 1.}
 EFM is perpendicular to AB: suppose GF ^{Fig. 1.}
 a Ray of Light falling inclined on the Glass
 AD, the Point F is called the *Point of Incidence*.

II. The Angle EFG comprehended between the Perpendicular and the Ray, is the *Angle of Incidence* (with Barrow, &c.) tho by many Dioptrick Writers 'tis called the *Angle of Inclination*; and its Complement AFG, is usually by them called the *Angle of Incidence*. But I shall use the Terms *Inclination* and *Incidence* promiscuously, always designing thereby the *Angle comprehended between the Perpendicular EF and the Ray GF*.

Experiments.

I. GF is a Ray of Light falling inclined on the Glass ABCD; this Ray coming out of a *Rare Medium*, as Air, into a *Dense Medium*, as Glass, does not proceed on its direct Course in a streight Line towards I; but at the Point of Incidence F 'tis bent or refracted towards the Perpendicular FM, and becomes the refracted Ray FH.

II. At H is its Point of *Incidence* again, from Glass a *Dense Medium* to Air a *Rare*, and in this Passage, 'tis refracted from the Perpendicular HZ; so that instead of proceeding directly

B

strait

strait in FHL, 'tis refracted from the Perpendicular HZ, and becomes HK: So that if the Surface AB be parallel to the Surface DC, the Ray becomes again as if it had not been refracted; for now HK runs parallel to GF. The natural Reason of this Refraction is variously assigned by divers, and is properly of a Physical consideration; and what is offered therein, is little more than Hypothetical Conjecture. I shall not therefore mix Guessees with Demonstration: The Matter of Fact is manifest from Ten Thousand repeated Experiments, and this is sufficient to my purpose. But yet there is such an ingenious Hypothesis concerning this Matter, published in the *Acta Eruditorum Lipsie*, Anno 1682. Mens. Junii, pag. 185. by the Learned and Ingenious G. G. Leibnitzius, that I cannot omit inserting it in the Second Part of this Work, Chap. 1.

III. The Ray that falls perpendicular (as suppose EF a Ray) passes unrefracted, but all inclined Rays are refracted.

Definitions.

Tab. 1. Fig. 1. III. The Angle IFH, and KHL comprehended by the Ray directly prolonged, and the refracted Ray is the *Angle of Refraction*.

IV. The Angle HFM or ZHK comprised between the refracted Ray and Perpendicular is the *refracted Angle*.

V. *Diverging Rays* are those that spread and separate the farther from each other, as they flow farther from the Object.

Tab. 1. Fig. 2. Tab. 1. Fig. 2. B is a radiating Point, AB, DB, CB, EB, are diverging Rays.

Tab. 1. Fig. 3. VI. *Converging Rays* are those that approach nigher each other, till they cross, and then become *Diverging*. Tab. 1. Fig. 3. A, B, are two Radiating Points in the Object AB, the Rays CA, CB, do *Converge*, till they cross in C, and then they become *Diverging* DC, EC.

VII. *Parallel*

VII. *Parallel Rays* are those that flowing from one and the same Point of a *remote* Object pass at the same distance, as to sense: But this is not to be strictly taken; for then the Rays flowing from one and the same Point of an Object, cannot be *parallel*, for they always *diverge*: Yet when an Object is at such a great distance, and that parcel of Rays which is considered, is so small, that their Divergence is little or nothing considerable, these Rays are said to be *parallel*.
 Tab. 1. Fig. 4. A is a Point in an Object sending forth its diverging Rays AD, AB, AC, AE; let BC be the breadth of the Pupil or breadth of an Optick Glass: Here if the Point A be so remote, and BC be so small, that the Parcel of Rays BAC, do insensibly run parallel, then these are said to be *parallel Rays*. Tab. 1. Fig. 4.

VIII. Those radiating Points or Objects are said to be *remote*, whose distance from the Eye or Glass is so great, that the breadth of the Pupil or Glass, in respect thereto, is inconsiderable.

IX. Those radiating Points or Objects are said to be *nigh*, when there is a sensible proportion between the Pupils or Glasses breadth, and the distance; so that the Rays flowing from any single Point thereof do not run parallel to each other, but diverge considerably in respect of the Pupils or Glasses breadth.

Experiments.

V. Tab. 1. Fig. 5. ZXY is a Body of Glass, HBG is perpendicular to ZX, AB is a Ray falling on this Glass, and the Angle of Inclination or Incidence is $HBA = DBG$. Kepler tells us, that under 30 deg. of Inclination from Air to Glass, the Angle of Refraction DBC is $\frac{2}{3}$ of the Inclination, therefore the refracted Angle CBG is $\frac{1}{3}$ of the Inclination. Wherefore we lay down the following Proportions,

as confirmed by *Kepler's* Experiments, and usually retain'd by most Optick Writers.

- \angle Inclination DBG : refracted \angle CBG :: 3 : 2
- \angle Inclination DBG : \angle Refraction DBC :: 3 : 1
- \angle Refraction DBC : refracted \angle CBG :: 1 : 2

V. But suppose the Ray CB to proceed from Glass to Air ; at B 'tis refracted from the Perpendicular BH, and becomes BA ; here the Inclination is CBG = HBI, and then

- \angle Incidence HBI : refracted \angle HBA :: 2 : 3
- \angle Inclination HBI : \angle Refraction IBA :: 2 : 1
- \angle Refraction IBA : refracted \angle HBA :: 1 : 3

These Propositions in the 4th. and 5th. Experiments we shall retain in the following Demonstrations, for the Ease and Plainness thereof. But in Calculation we shall observe the Proportion that follows in the 6th. Experiment.

VI. But the most Learned and Ingenious Mr. *Isaac Newton* of Cambridge discovered by most accurate Experiments, that these Proportions of *Kepler* were not sufficiently exact : For *Des Cartes* first found, that Refractions were not to be measured by the Proportion of Angles, but by the Proportion of Sines. See his *Dioptricks* Cap. 2. Sec. 7. And therefore Mr. *Newton* apply'd himself to discover the Proportion of the Sines, and found, That from Air to Glass, the Sine AK or DF of the Angle of Incidence ABK or DBG : is to the Sine IH or CG of the refracted Angle CBG or IBH :: As 300 to 193. (or near, as 14. to 9.) And on the contrary, that from Glass to Air, the Sine of the Incidence : Is to the Sine of the refracted Angle :: as 193 : to 300, or as 9 to 14. But the same Mr. *Newton* in his *Dissertations* concerning Colours and Light, publish'd in the *Philosophical Transactions*, has at large demonstrated, that the Rays of Light are not all *Homogeneous*, or of the same sort, but of different Forms and Figures, so that

that some are more refracted than others, tho they have the same or equal Inclinations on the Glafs: And therefore there can be no constant Proportion settled between the Sines of the Incidence and of the refracted Angles. But the Proportion that comes nearest Truth, for the middle and strong Rays of Light, is nearly as 300 to 193, or 14 to 9.

VII. As the Sine of the Angle of any one Inclination: To the Sine of its refracted Angle:: So is the Sine of any other Inclination: To the Sine of its refracted Angle. Therefore by Experiment, finding the Proportion of the Sines of any one Inclination, and of its correspondent refracted Angle, this Proportion will hold in any other Inclination or Incidence. See *Des Cartes Dioptr. Cap. 2. Sec. 7. Mersenni Optic. Lib. 7. Prop. 12. Decbales Dioptr. Lib. 1. Prop. 1, 2, &c. See also Cap. 1. Sec. 4. Part 2.*

Wherefore the Incidence or Inclination of a Ray being given, 'tis easie finding the refracted Angle; for the Proportion is, as 300: To 193 :: (or as 14 to 9) So the Sine of the Incidence: To the Sine of the refracted Angle, from Air to Glafs.

Or Mechanically thus, A B K being the given Angle of Incidence, by a Sector make the Sine thereof K A or D F 300 parts (or 14) and take 193 (or 9) of the same parts, and make therewith C G or H I the Sine of the refracted Angle; then draw B C, this is the progress of the Ray A B after it enters the Glafs.

VIII. *The Progress of Light through different Mediums is reciprocal:* That is, *Tab. 1. Fig. 1.* Suppose the luminous Point *G* in Air, to send from it the Ray *G E* upon the Glafs *A D*; this Ray by the Glafs is refracted into *F H* within the Glafs. Let us now imagin the Point *H* within the Glafs, to be a luminous Point, sending out from it self the Ray *H F*: This Ray upon its Emerision from Glafs into Air, instead of proceeding

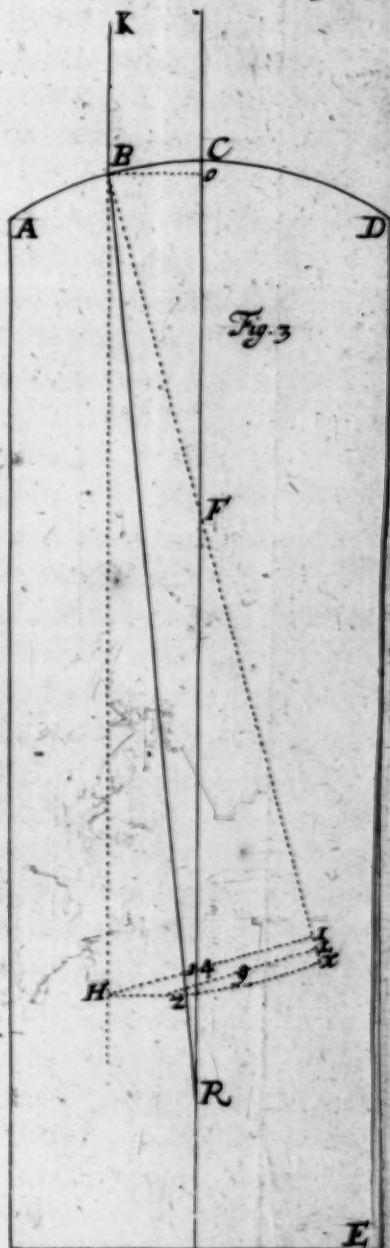
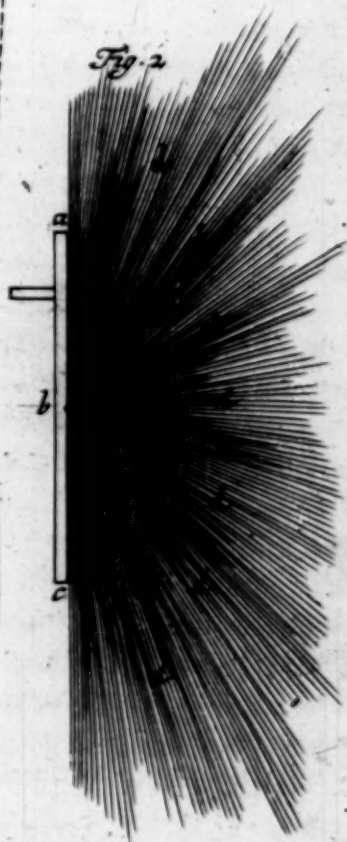
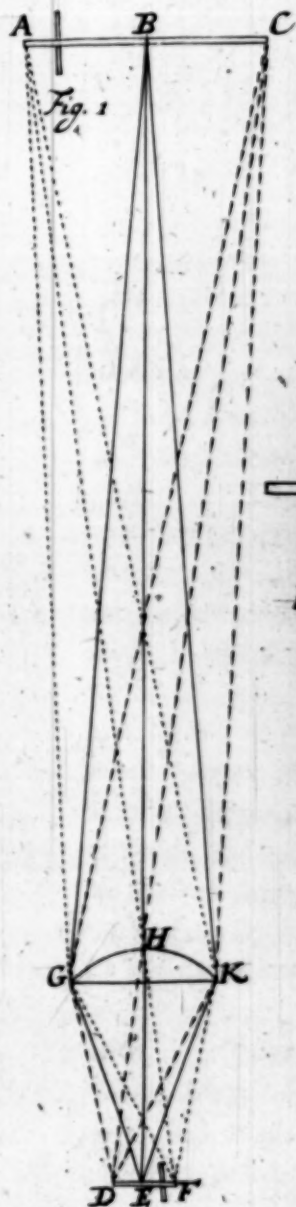
ceeding directly to N, is refracted into FG. And so likewise supposing the Luminous Point G, to send its Ray GF, which after a double Refraction on each Surface of the Glas proceeds first in FH, and afterwards in HK; if we conceive K a Luminous Point, sending its Ray KH on the Glas; this Ray after a double Refraction on the two Surfaces of the Glas shall be refracted first into HF and then into FG; taking the same reciprocal Progress in both Cases.

And this is not so properly proved from Experiment (tho that also does abundantly and most exactly confirm this Truth) but 'tis manifest and self-evident from the very Nature of the thing. For whatever physical Reason there is, which causes the Ray GF proceeding from Air into the Glas AD to be refracted from its direct Course FI into FH; the same reason there must needs be to cause the Ray HF, proceeding from the Glas AD into the open Air, to be refracted from its direct Course FN into FG. For doubtless the Refraction proceeds from the Disparity of the Mediums (but how or in what manner 'tis needless to enquire in this place). Now the difference of the Mediums on their two determining Surfaces, or as the Point of Incidence F, continues the same, let the Ray proceed from which of them soever. For there is as great a reciprocal Difference between Glas and Air, as between Air and Glas; and consequently the Refraction must needs be reciprocal. *Vid. Barrow, Lect. Opt. 3. Sect. 3.*

Definitions.

X. Whatever Line is perpendicular as to the Tangent of any Curve Line at the point of Contract, is said to be perpendicular to the same Curve. *Tab. 1. Fig. 6.* DE being a Tangent to the Arch of a Circle ABC, and FG being perpendicular to DE at the point of Contract F, the Line FG is also perpendicular to the Curve ABC.

XI. A



XI. A Line that is inclined to the Tangent of any Curve, is inclined to the Curve; and the Angle of Inclination to the Curve, is the same with the Angle of Inclination to the Tangent. FH is inclined to the Curve ABC, by the Angle HFG, according to which Angle the same Line HF is inclined to the Tangent DE.

XII. K being the Centre of the Circle ABC, draw BK parallel to HF, the Arch BF measures the Inclination of the Line HF to the Curve ABC, or to the Tangent DE: For the Angle BKF is equal to the Angle HFG. (Pr. 29. 1. Eucl.

XIII. The Rays that proceed from the middle Point of an Object, when the Glas is exposed directly before it, are said to fall *directly* on the Glas; but the Rays that flow from the Collateral Points of an Object, are said to fall *obliquely* on the Glas. Tab. 2. Fig. 1. ABC is a distant Object, sending parallel Rays from each of its Points on the Glas GK, exposed directly before it. Here the Rays flowing from the Point B, are said to fall *directly* on the Glas, as BG, BH, BK. But the Rays flowing from the Point A, or any other Collateral Point, are said to fall *obliquely* on the Glas, as AG, AH, AK. And here 'tis to be noted, that the Point B is said to be *directly* exposed to the Glas, when the Axis of the Glas EHB directly produced, meets this Point B: But all other Points in the Object are Collateral Points; for tho they may be *directly* exposed to some other Points in the Glas, yet *direct* and *collateral*, are here to be understood only in respect of the Glas's Axis or Vertex. Or in short, that Point in an Object is said to be *direct*, the Axis of whose Cone of Rays is coincident or the same with the Axis of the Glas; that Point in an Object is said to be *oblique* or *collateral*, the Axis of whose Cone of Rays falls *obliquely* to the Axis of the Glas. Vid. Def. 17. 18.

XIV. Suppose the Glass GK exposed in the Hole of a Shutter in a dark Room, and that thereby the Object ABC were represented on a white Paper in FED (how this is done we shall see hereafter.) From each Point of this Object there falls a parcel of Rays on the Surface of the whole Glass; thus from the Point B, there falls BG, BH, BK. This parcel of Rays is called *A Cone of Rays*, having its Vertex in the Point of the Object B, and its Base on the Glass.

XV. To this Cone of Rays there is another Correspondent on t^other side the Glass GEK, which has its Base likewise on the Glass, and its *Vertex* in a correspondent Point E of the Representation.

XVI. Both these Cones together (that is BGEK) are called *A Pencil of Rays*.

XVII. Each Pencil of Rays has one Ray amongst the rest, which is called its *Axis*. The *Axis* of the direct Pencil BGEK is the Ray BHE, which falls perpendicularly on the Glass, and passes through it unrefracted. But the *Axis* of any of the oblique Pencils, as AGFK is some certain Ray, as AHF, which, tho' it fall obliquely, and consequently is refracted by the Glass, yet being refracted *from* the perpendicular at its Egress from the Glass, as much as it was refracted *towards* the Perpendicular, at its Ingress into the Glass; it may be taken as not refracted at all; but that part of this *Axis* which is before the Glass (*viz.* AH between the Object and Glass) and that part of it which is behind the Glass (*viz.* HF between the Representation and the Glass) do run parallel to each other, or rather in one Right Line; that part of it only which is within the Glass, being bent out of its strait Course. And that in each oblique Pencil of Rays (for of the direct Pencil 'tis manifest) there is such a certain Ray thus affected, shall appear more fully hereafter. And here we are to distinguish between the *Axis of a Pencil of Rays*, and the *Axis of the Glass*. See Def. 18. The

The Pencils of Rays from various Points are expressed differently in the Schemes (*viz.* by Lines or small Points) for better Illustration, and easier distinguishing their Progress.

XVIII, *Tab. 2. Fig. 3.* A E is a Body of Glass, whose Surface A C D is the Segment of a Sphere, the Centre whereof is F. K B a Ray falling on the Glass; through the Centre F draw C F R parallel to K B, C F R is the *Axis*, and the Point C the *Pole* or *Vertex* of the Glass: Or more generally thus, The *Axis* of a Glass is that *Right Line*, which, being produced both ways, falls on the *Centres* of the *Two Spherick Surfaces* (if the Glass be a double Convex, or double Concave) or which being produced both ways, falls on the *Centre* of the *Spherick Surface*, and perpendicular to the plain Surface, (if the Glass be a Plano-Convex, or Plano-Concave.) So that the *Axis* of a *Convex-Glass* passes perpendicular to the *thickest* part of the *compleat Glass*; and of a *Concave* to the *thinnest* part of the *compleat Glass*. Why *Compleat* is added; *vid.* Second Part, Chap. 4. Sect. 4. The *Poles* of a Glass are those two Points in the Glass through which the *Axis* passes.

'Tis needless to define what is meant by a *Convex* or *Concave-Glass*, &c. for every one knows this already.

XIX. The Ray K B falling on the Convexity A C D, parallel to the *Axis* C R, and being refracted to B R; so that now it runs no more parallel, but crosses the *Axis* in R: the Point R is called the *Point of Concourse*, or of *Convergency*, or the *Focus*.

1. Supposition.

That from every Physical Point in an Object Rays of Light are diffused in direct Lines through the Hemisphere round it. *Tab. 2. Fig. 3.* Thus the Point *b* in the Object *abc* projects from it the Rays *bd*, *bd*, *bd*, wherever they are not hindered by some Opaque Body intervening. And these Rays are
C strait

strait Lines, whilst they pass through a Diaphanous Uniform Medium. But here we are to conceive a radiating or luminous Point, not in a strict Mathematical Sense, but in a Physical, considering it as the least part imaginable in an Object.

2. Supposition.

That the Axis of the Eye, Glass, or Glasses, exposed before an Object be Perpendicular to the Plane of the Object; or that the Eye, Glass, or Glasses exposed before an Object, are supposed directly exposed thereto.

3. Supposition, or Admonition.

Because in many Authors of Opticks, much Perplexity and Error arises from their confused mentioning of Rays, and applying to them the Epithets of Diverging, and Converging, and Crossing; without expressing clearly whether they mean Rays from one and the same Point in an Object, or from Different Points. Therefore we are to consider in the ensuing Doctrin, whether the Name Rays do relate to those from one and the same Point, or from different Points. And that this may be the more easie, I do generally express the Rays from the same Point to be such, that is, I call them Rays from the same Point.

PROP. I. PROBL.

Tab. 2. Fig. 3. AE. is a Body of Glass, whose Surface ABCD is the Segment of a Sphere, the Centre whereof is F. KB is a Ray of Light parallel to the Axis CFR. There being given CF the Radius of the Convexity, and BO the Distance

Distance of the Ray from the Axis; 'tis required to find the Point R, wherein this Ray, after one sole Refraction at the Point of Incidence B, crosses the Axis CR.

1. Produce KB at pleasure to H, and draw BFX at pleasure.

2. In the Right Angled Triangle BOF, we have given BO and BF, whence we may find the Angle BFO = HBF, which is the Incidence.

3. Then say, as 300 : to 193 :: (or as 14 to 9) so the Sine HB of the Incidence HBF, to the Sine ZL of the refracted Angle RBF, then HBF = RBF = HBR = BRF, which is the Angle of Refraction.

4. In the Triangle RBF, all the Angles and one of the Sides BF are known, whence we may find FR: Which was required.

Example.

FB = FC = 2 Inches, or 20000 Parts, an Inch being 10000. BO = 1 Inch, or 5000 such Parts. The Angle of Incidence or Inclination BFO = HBF will be found $14^{\circ} 29'$. And the refracted Angle FBR shall be $9^{\circ} 15'$. Wherefore BRF = HBR = $5^{\circ} 14'$. Whence the Line FR is found 35341. Then RF + FC = 35341 + 20000 = 55341 = CR.

Corollary.

From this Calculation 'tis manifest, That in this Case the Point of Concourse R is distant from the Pole of the Glass C, almost a Diameter and half of the Convexity: For a Diameter and half is 60000, and CR is 55341, which wants only about $\frac{1}{4}$ of 60000.

Scholium 1. *Point of Refraction B, crosses the Axis C R.*

This Proposition is the 34th. of *Keplers Dioptricks*, and in him and other Opticians, 'tis usually thus expressed and demonstrated.

The Rays that are parallel to the Axis, and fall so on the Convex Surface of a Glass, and at their Ingress are refracted, and so pass forward in Glass only, are united with the Axis at the distance of almost a Diameter and half of the Convexity, supposing the Glass to be a Portion of no more than about 40 degrees of its Convexity, or (which is the same thing) supposing the Inclination of the most extreme Ray to be but 15 degrees.

This Limitation of 15 degrees Inclination is added, because above 15 degrees Sines and Angles are not proportional, but under 15 degrees they are nearly proportional, a double Angle having a double Sine, a treble Angle a treble Sine, &c.

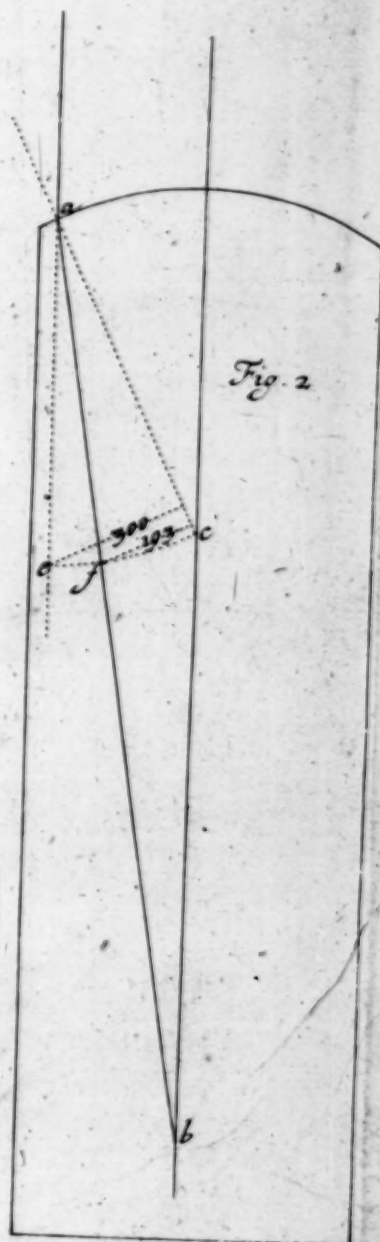
Tab. 1. Fig. 3.

Wherefore they thus demonstrate this Proposition. *Tab. 1. Fig. 3.* $BFC = HBF$ is the Inclination, $HBR = BRF$ is the Angle of Refraction, RCF the refracted Angle. But by *Experiment 4.* RCF the refracted Angle : is to BRF the Angle of Refraction :: as 2 to 1. Therefore the Sines of these Angles, being under 15 degrees, shall be as 2 : to 1. But in plain Triangles the Sines of the Angles and the Sides subtending these Angles are proportional ; therefore the Side RF is double FB or FC . Wherefore RC is thrice FC , or equal to three Semidiameters : Which was to be demonstrated.

Scholium 2.

But a neater way of Expressing and Demonstrating this Proposition is thus.

When



When a Ray falls on a Spherick Glass, and afterwards it being but once refracted, crosses the Axis. It shall be, as the Sine of the Angle of Refraction : to the Sine of the Angle of Inclination :: so the Radius of the Glass's Sphere : to the refracted Ray.

Tab. 3. Fig. 1. Let bet be the Convex Surface of the Glass Tab. 3. Fig. 1.
 ADE , k the Centre of the Convexity,
 bfc the Angle of Inclination, bo the Sine of the Inclination,
 br the refracted Ray meeting the Axis in r , $bbr = brc =$ to
the Angle of Refraction; draw fe parallel to br , then efc
is equal to brc , and ec is the Sine of the Angle of Refraction.
And by reason of the similar Triangles bro , efc , it shall be
 $ec : bo :: ef : br$. Which was to be demonstrated.

Wherefore supposing that from Air to Glass, the Sine of the Angle of Inclination be to the Sine of the refracted Angle as 300 : to 193 : then the Sine of the Inclination shall be to the Sine of the Angle of Refraction :: as 300 : to 300 - 193 = 107. Because the Angle of Refraction is equal to the difference between the Angle of Inclination and the refracted Angle. And Sines and Angles being (in these small Angles) proportional. It shall be, As 107 : 300 :: so Radius of the Convexity : to the Concourse of the refracted Ray with the Axis.

It shall also be, As 107 : 193 :: so the Radius of the Convexity : to the Concourse of the refracted Ray beyond the Centre of the Sphere.

For Tab. 3. Fig. 2. $ae : ac :: 300 : 193$ Tab. 3. Fig. 2.
Dividing the First $ae - ac : ac :: 300 - 193 : 193$
That is $ec : ac :: 107 : 193$
Moreover, $ec = fbc =$ Angle of Refraction.
From 3 and 4 $fbc : fac :: 107 : 193$
But in the $\triangle abc$ $fbc : fac :: Rad. = ac : cb$
From 5 and 6 $107 : 193 :: ac : cb =$ To the Concourse of the Ray beyond the Centre of the Sphere. Which was to be demonstrated.

PROP. II. PROBL.

Tab. 4. Fig. 1.

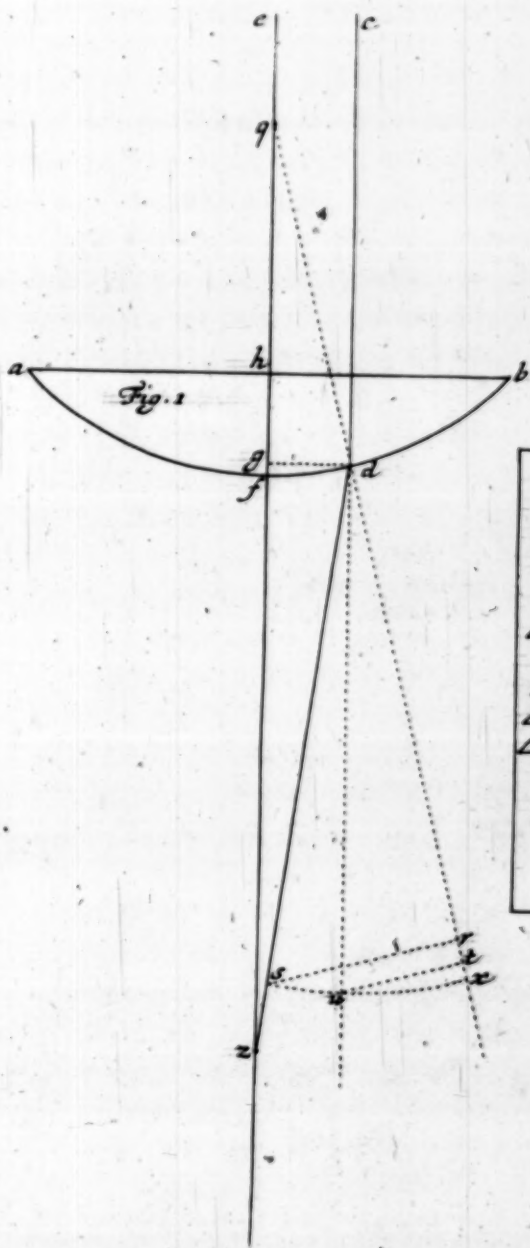
A Plano-Convex Glass afb Tab. 4. Fig. 1. being exposed to a distant Object with its Convex side towards it, and receiving the Ray cd parallel to the Axis cz , 'tis required to determine the Point of Concourse z , from these Data, fh the Glasses thickness, qf the Radius of the Convexity, and dg the distance of the Ray cd from the Axis cz .

The same Problem is proposed from the same Data, for the plane Side towards the Object.

But first for the Convex-Side to the Object: The Mechanical Construction is thus, Tab. 4. Fig. 1. Produce the Ray cd through the Glas at pleasure to m . And from d the Point of Incidence draw the Line dq to the Centre of the Convexity; this is the Perpendicular, and the Angle qdm the Incidence or Inclination. On d as a Centre at any distance strike the Arch moq , in which make mn the Sine of the Angle of Incidence, to op the Sine of the refracted Angle, as 14 to 9 (or op 9 such parts as mn is 14, which is easily done by the Sector) the Line dko shall be the Line in which the Ray would proceed; if the Medium of Glas were continued so far. But because in this Line at k the Ray emerges from the plain Surface of the Glas a dense Medium into Air a rare Medium, from k raise kx perpendicular to the plain Surface ab , and on k , as a Centre at any distance strike the Arch xus ; then the Angle xku is the Incidence of the Ray passing from Glas to Air.

Making therefore tu the Sine of this Angle, to rs the Sine of the second refracted Angle from the Perpendicular as 9 to 14, the Line drawn from k through the Point s , and produced to the Axis at z , shews the Progress of the Ray in the Air after its Emerision from Glas: Wherefore the Point z is determined in the Axis.

Trigon.



Data	
$qf = qd =$	20000
$gd =$	5000
$hf =$	5858
unde	
$\angle dqg = udx =$	$14^{\circ} 28' 40''$
$qg =$	19365
$qf - qg = gf =$	635
$\angle sdx =$	$22^{\circ} 52' 10''$
$\angle sdx - udx = sdu = dzg =$	$8^{\circ} 23' 30''$
$gz =$	33894 +
$gz - gf = fz =$	33259

Trigonometrical Calculation.

But this Point z , or the Measure of the Line fz may be found more accurately by Calculation thus. In the Triangle qdg Right Angled at g are given qd and dg , to find the Angle of Incidence dqg , and the Side qg . Then $qf - bf = qb$, and $qg - qb = bg = di$.

First then, because the Ray passes at d from a rare Medium into a dense, it shall be refracted towards the Perpendicular dq . Wherefore the Analogy is, As 14 : to 9 :: the Sine of the Angle of Incidence $dqf (=mdq)$: to the Sine of the refracted Angle odq (that is kdl). But $mdq (=idl) - kdl = idk = xku$, dm and kx being parallel :) And this is the Angle of Incidence as the Ray passes from Glass to Air.

Secondly, In the little Right Angled Triangle idk , the Side di , and the Angle idk are known; whence the Side ik may be found, then $ib (=dg) - ik = kb$.

Thirdly, Because at k the Ray emerges from a Dense into a Rare Medium, it shall be refracted from the Perpendicular kx ; and the Analogy shall be, as 9 : to 14 :: the Sine of the Angle of Incidence xkn : to the Sine of the Second refracted Angle $xks (=kzb, kx$, and bz being Parallel)

Fourthly, In the Right Angled Triangle $k bz$, the Side kb , and all the Angles are known, whence the Side bz , and consequently $fz (=fb + bz)$ the Distance of the Point of Concurrence from the Pole of the Glass f is discovered.

When the plain Side is turned to the Object, the Method is the same, only shorter; for Tab. 5. Fig. 1. in the Right Angled Triangle qdg , we have qd and dg to find $dqg = udx$, the Incidence, and the Side qg : Then $qf - qg = gf$. Next, As 9 : to 14 :: (or as 193 : to 300 ::) so the Sine of udx : to the

Tab. 4. Fig. 1. the Sine of sdx . But $sdx - udx = sdu = dzg$. Then in the Right Angled Triangle dzg we have dg , and the Angles to find gz . And $gz - gf = fz$. Which was to be found.

Scholium.

In the little Tables inserted in Tab. 4. and Tab. 5. I have given Examples at large of the Calculation of the Progress of the Ray cd , which I presume is pretty truly wrought: But if perhaps any Errors do intervene, they are easily corrected.

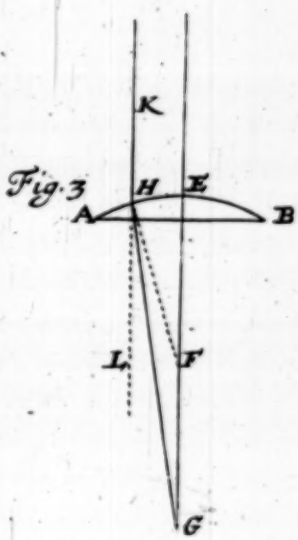
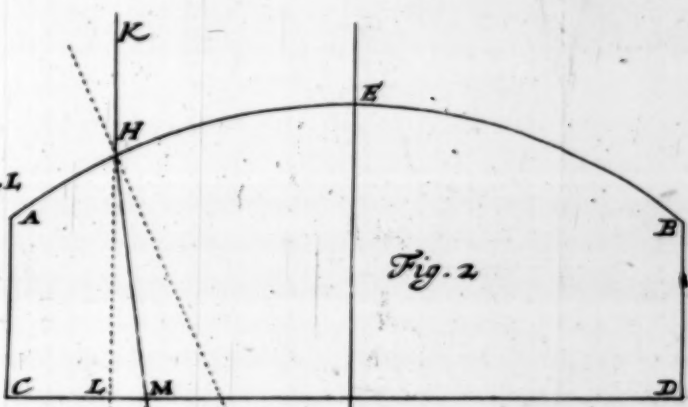
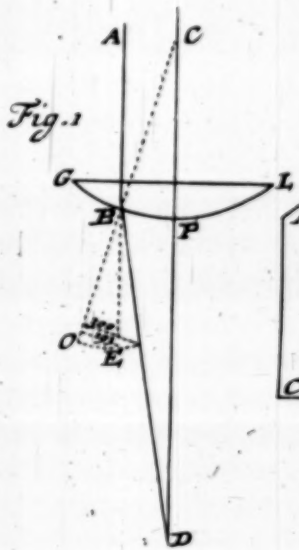
By these Tables we may perceive, that the Point of Concourse z is distant from the Pole of the Glass f about a Diameter of the Convexity, but not fully so much.

Tab. 6. Fig. 1. 'Tis also to be noted in Tab. 4. Fig. 1. That the Point of Concourse z , shall be less or more removed from the Pole f , according as we give the Glass less or more thickness. For 'tis manifest, that the farther we let the Ray run in the Medium of Glass, after it enters it at d , before it emerges into Air at k , the farther the Point of Concourse z shall be protracted from f . For we may imagin the Medium of Glass continued below the Diameter of the Convexity, and then 'tis evident that the Point of Concourse z shall be much more below it. But it can be no more below it than a Diameter and half. By the First Proposition hereof.

This our Second Proposition is usually found in Dioptricks thus expressed and demonstrated.

Parallel Rays falling on a Plano-Convex Glass, are united with the Axis about the distance of a Diameter of the Convexity from the Pole of the Glass, if the Segment be but 30 Degrees.

Demon-



Demonstration.

First for the plain Side towards the Object. *Tab. 6. Fig. 1.* *Tab. 6. Fig. 1.*
 AB is a Ray of Light falling on the Glas GL parallel to the Axis CD. C is the Centre of the Convexity GPL. Joyn CB; the Angle of Incidence or Inclination is BCD, the Angle of Refraction is EBD or BDC.

Wherefore in the Triangle BCD, the Angle BCD being double the Angle BDC (For by *Kepler's* Experiments, which are follow'd by most Authors, from Glas to Air, the Angle of Incidence : is to the Angle of Refraction :: as 2 : to 1.) the Side BD shall be double the Side BC; the Angles and Sides or Sines under 15 Degrees being proportional; but DP is almost equal to BD (for the Thickness and Breadth of the Glas, in Optick Glasses especially in Glasses of Long Foci, are inconsiderable), therefore DP is double CP. Which was to be Demonstrated.

Secondly, for the Convex-side towards the Object. *Tab. 6. Fig. 2.* *Tab. 6. Fig. 2.*
 ABCD is a Plano-Convex Glas (I have here expressed it of this thickness, to shew the Angles and Progress of the Ray more plainly, tho really this great Thickness hinders the Exactness of the Demonstration; for the Glas is to be supposed of the most inconsiderable Thickness). F is the Centre of the Convexity. KH is a Ray falling thereon, parallel to the Axis EG. This Ray, (after it has suffered a Double Refraction, one at its Ingrefs from Air into Glas at H towards the Perpendicular HF, by which it becomes the Refracted Ray HM, and another Refraction at its Egrefs from Glas to Air at M from the Perpendicular PM, by which, instead of going forward towards R, it becomes the Refracted Ray MG) at G crosses the Axis, so that GE is the Diameter of the Convexity AEB.

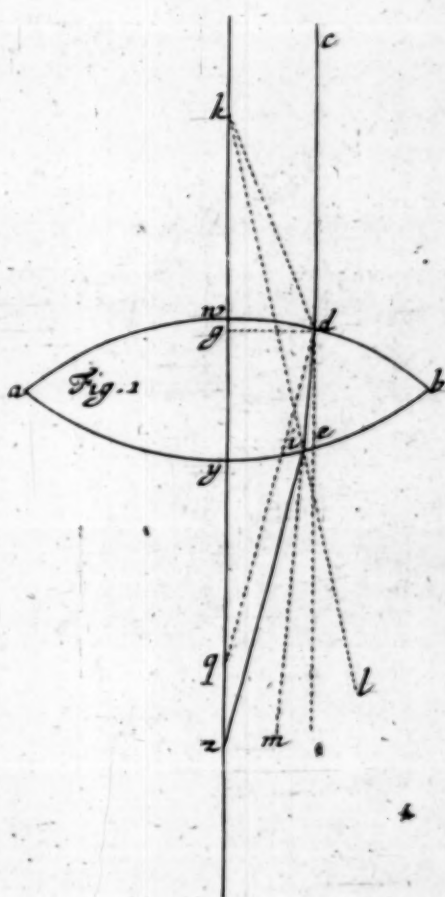
D For

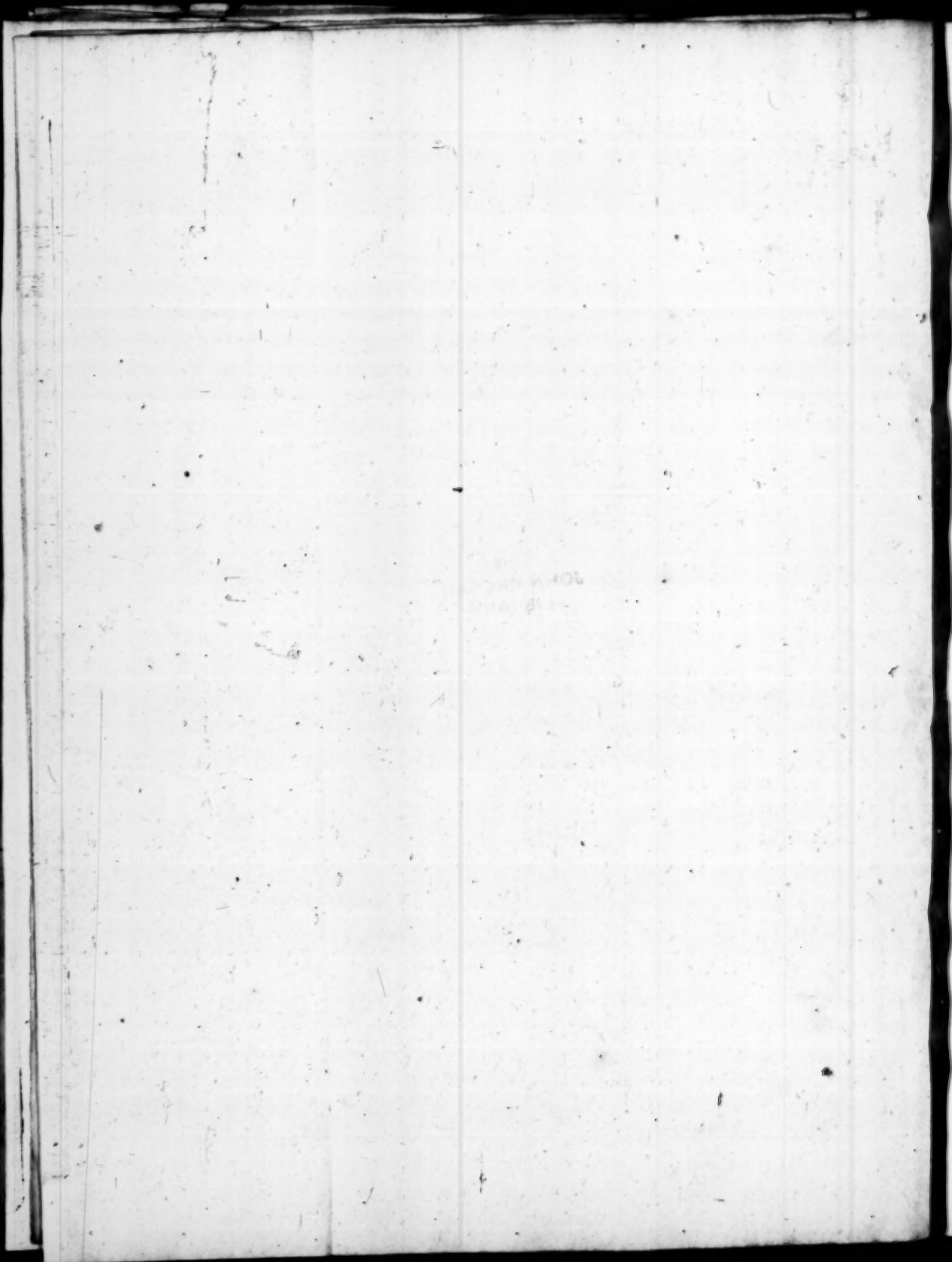
For the Incidence or Inclination of the Ray KH is LHF , of which Angle there is taken off $\frac{1}{2}$ LHM , for the Angle of Refraction from Air to Glass is $\frac{1}{2}$ of the Incidence (by *Exper. 4.*) If therefore the Ray had proceeded onwards in Glass towards R , it had crossed the Axis at a Diameter and half (by the foregoing Proposition), but at its egress into Air at M , it is again refracted from the Perpendicular PM by the Angle $RMG = \frac{1}{2}$ PMR . The Angle of Refraction RMG from Glass to Air, being $\frac{1}{2}$ the Incidence PMR (by *Exper. 5.*) But PMR is equal to LHM . Wherefore the first Angle of Incidence LHF by this Double Refraction, has lost first $\frac{1}{2}$ of it self LHM , and again $\frac{1}{2}$ of this Third RMG : But $\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{1}{2}$ is equal to $\frac{1}{4}$, wherefore the Angle of Incidence is diminished by its Half.

Tab. 6. Fig. 3. Let us therefore consider the Glass AB , *Tab. 6. Fig. 3.* without respect to its Thickness, or to the Refraction the Ray suffers at its egress on the lower side of the Glass into Air. But let us imagin, that by the first Refraction, the Ray were brought nigher the Perpendicular HF by half the Angle of Incidence LHF , and we shall find, that then the Ray shall cross the Axis at G ; so that GE shall be the Diameter of the Convexity. For LHG or GHF is equal to $\frac{1}{2}$ LHF or HFE the Incidence; but HFE is equal to $GHF + HGF$ (*32. 1. Eucl.*) therefore GHF and HGF are equal, each of them being $\frac{1}{2}$ HFE , and consequently their opposite Sides HF and GF are equal. Wherefore GE is the Diameter of the Convexity AEB . Which was to be Demonstrated.

Corollary.

In Plano-Convex Glasses, As $300 - 193 = 107$: To 193 :: So Radius of the Convexity : To the refracted Ray taken to its Concourse with the Axis; which in Glasses of large Spheres and small Segments is almost equal to the Distance of the Focus taken in the Axis. *Tab. 6. Fig. 1.* De-





Demonstration.

The Inclination = $ABC = OBE = BCD = 193$.

Refracted Angle = $OBD = 300$

Angle of Refraction = $EBD = BDP = OBD - OBE$
 $= 300 - 193 = 107$.

In the $\triangle BDC :: \angle BDC :: \angle BCD :: \text{Rad.} = BC : BD$.

That is,

$107 : 193 :: BC : BD = PD$ in Glasses of large Spheres.

Which was to be Demonstrated.

PORP. III. PROBL.

In a Double Convex-Glass ab , Tab. 7. Fig. 1. of equal or Tab. 7. Fig. 1.
 unequal Convexities; Let the Centres of the Spherical Sur-
 faces be k and q ; cd a Ray falling parallel to the Axis kqz ,
 and being Refracted at its Ingress at d , and at its Egress at i ;
 'Tis required to determin the Point of its Concourse with the
 Axis at z , from these Data, the Radii ky , qw , of the
 Convexities, and gd the Distance of the Point of Incidence
 from the Axis.

This is so easily done by Scale and Compass from what
 foregoes, that I shall not insist on a farther Explication there-
 of, but shall shew the more certain Method of tracing the
 Progress of this Ray by Calculation.

Produce cd directly to e , and di to m . We have $ky +$
 $qw - wy = kq$.

First therefore in the Right-angled Triangle qdg , gd and
 qd being given, we may find the $\angle gqd = qde$ and the side
 qg . Then $kq - qg = kg$.

Secondly, In the Right-angled Triangle kgd , there being
 given kg and gd , we may have the $\angle gkd = kdc$, and the
 side kd .

D 2

Thirdly,

Thirdly, As $300 : \text{to } 193 :: \text{so } S. \angle qde : \text{to } S. \angle qdi$.

Fourthly, $qde - qdi = edi$. And $180 - kdc - edi = kdi$.

Fifthly, In the Triangle kdi , the Angle kdi and the Sides ki, kd , being known, we have the Angle $kid = mil$. And the Angle ikd . Then $gkd - ikd = qki$.

Sixthly, As $193 : \text{to } 300 :: \text{so } S. \angle kid : \text{to } S. \angle zil$, then $zil - qki = kzi$.

Lastly, In the Triangle zki , ki and all the Angles being known, we find kz , from which subtracting ky , there remains zy , which was required.

If by this Method we calculate the Progress of a Ray through a Double Convex-Glass of equal Convexities; and the Thickness of the Glass be little or nothing in comparison of the Radius of the Convexity; and the Distance of the Point of Incidence from the Axis be but small, we shall find the Point of Concurrence to be distant from the Glass about the Radius of the Convexity nearly.

This our Third Proposition in most Dioptrick Writers is usually thus expressed.

Parallel Rays falling on a Double Convex-Glass of equal Convexities on both sides, are united with the Axis, about the Distance of the Radius of the Convexity from the Pole of the Glass, if the Segment be but 30 Degrees.

But their Demonstrations of this Proposition are usually perplexed enough, by reason of the smallness and multiplicity of Angles, which are expressed in their Figures, as also by the Thickness of the Glass, which they are forced to represent, and yet is to be neglected in their Demonstrations. I shall give as short a Demonstration as I can, after their Method, as follows.

Tab. 7. Fig. 2. Let a be the Centre of the Convexity bkl , b the Centre of the Convexity bil , db a Ray of Light produced directly to f parallel to the Axis ab . abe is a Right Line, and hbc is a Right Line. The first Angle of Inclination

tion is $c b d = h b f = e b f$, and the perpendicular is $h b$. By the first Refraction the Ray is deflected from its straight course $b f$ and becomes $b g$, approaching the perpendicular $h b$ by the Angle $f b g$, which is $\frac{1}{2}$ of the Inclination $e b f$. Produce $g b$ directly to z . Now the Angle of Incidence on the lower Convexity $b k l$ from Glass to Air is $a b z = e b g$, and the Perpendicular is $a b$ or $b e$: Wherefore the Ray instead of going forwards in $b g$ is now, by its Emerfion into Air, refracted from the Perpendicular $b e$, by the Angle $g b h$, which is therefore to be equal to $\frac{1}{2}$ the second Angle of Incidence $e b g$. by Exper. 5. and that $g b h$ is equal to $\frac{1}{2} e b g$, I thus demonstrate;

By the Suppofals in the Prop. }
and Common Geometry. }

By the Fig. — — — — —	1 $e b f = f b h$
From 1 and 2 — — — — —	2 $g b h = f b h - f b g$
Doct. Refract. Exper. 4. — —	3 $g b h = e b f - f b g$
From 3 and 4 — — — — —	4 $f b g = e b f - g b h$
Moreover by the Scheme —	5 $g b h = e b f - \frac{1}{2} e b f = \frac{1}{2} e b f$
4 \div 2 — — — — —	6 $\frac{1}{2} e b g = \frac{1}{2} e b f + \frac{1}{2} f b g$
From 6 and 7 — — — — —	7 $\frac{1}{2} f b g = \frac{1}{2} e b f$
From 5 and 8 — — — — —	8 $\frac{1}{2} e b g = \frac{1}{2} e b f + \frac{1}{2} e b f = e b f$
	9 $g b h = \frac{1}{2} e b g$

Which was to be Demonst.

Therefore it follows that the Ray by this second Refraction proceeds in $h b$, crossing the Axis in h the Centre of the Convexity. *Which was to be demonstrated.*

See the Demonstrations of this Proposition *Kepleri Dioptri.* Prop. 39. *Herigon.* Prop. 15. *Dechales* Pr. 21. lib. 13. *Hon. Faber* Prop. 43. Sec. 8. *Cherubin de Orleans la Dioptr. Oculaire*, Prop. 2. *Zahn Telefcop.* Fund 2. Syntag. 1. Cap. 4. Prop. 8.

Of Glaffes of unequal Convexities.

As for double Convex Glaffes of unequal Spheres on each Side, the foregoing Method of Calculation determines the Point
of

of Concourse exactly in them also: But the shorter Rule laid down by most Optick Writers, is this.

As the Sum of the Radii of both Convexities: to the Radius of either Convexity :: So the double Radius of the other Convexity: to the Distance of the Focus. Vid. Corol. Pr. 16.

Demonstration.

T. 7. F. 3.

T. 7. F. 3. Let the Axis of the Glass ib be gk , dla Ray of Light Parallel thereto, g the Centre of the Convexity ifb , b the Centre of the Convexity icb , gi and bi are right Lines. The first Angle of Inclination is $din = ibc$, and by the first Refraction the Ray is deflected into ik , crossing the Axis in k , so that kc is equal to $3cb$. The second Angle of Incidence from Glass to Air on the Surface ifb is the Angle lik : I say kim being made $\frac{1}{2}$ of lik , the Ray crosses the Axis in m , and it shall then be (according to the Rule) $gb:2fg::cb:mc$. Which I thus Demonstrate.

In the Δkim

$$1 \quad \angle kim : \angle k :: km : mi = mf \text{ in large Spheres.}$$

Double the Antecedents

$$2 \quad \angle kim = lik : \angle k :: 2km : mf :: km : \frac{1}{2}mf = \frac{1}{2}mc$$

Divide the second Step.

$$3 \quad \angle lik - \angle k = \angle g : \angle k :: km - \frac{1}{2}mc : \frac{1}{2}mc$$

Then in Δgik it shall be

$$4 \quad \angle g : \angle k :: ik = kc : ig = fg$$

From 3 and 4

$$5 \quad kc : fg :: km - \frac{1}{2}mc : \frac{1}{2}mc$$

Compound the 5th.

$$6 \quad kc + fg : fg :: km : \frac{1}{2}mc$$

Double the Consequents

$$7 \quad kc + fg : 2fg :: km : mc$$

Compound the 7th.

$$8 \quad kc + 3fg : 2fg :: km + mc = kc = 3cb : mc$$

That is otherwise

$$9 \quad 3cb + 3fg : 2fg :: 3cb : mc$$

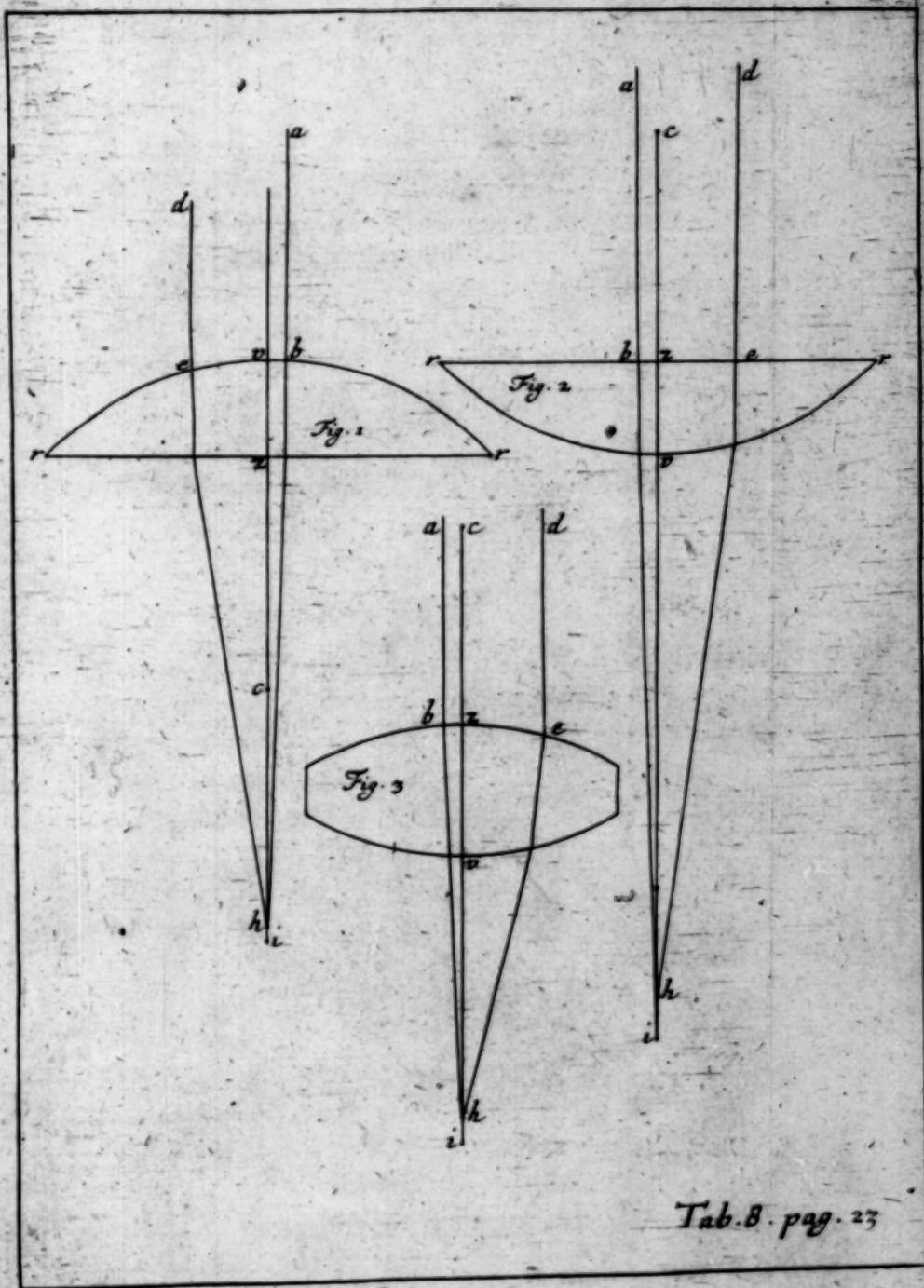
Divide the Ant. by 3.

$$10 \quad cb + fg : 2fg :: cb : mc$$

Which was to be Demonstrated.

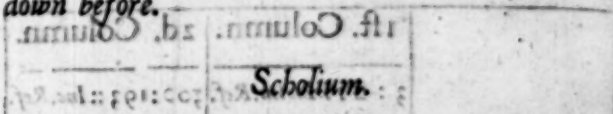
Vid. *Cherubin Dioptr. Ocul.* pag. 61, 62. *Hon. Faber Synop. Opt. Prop.* 43. Sec. 8, 9. *Dechales Prop.* 23, 24. *Zahn Telescop. Fund.* 2. Cap. 9. Prop. 38.

But



But this Rule will need but little farther Demonstration, if we consider, that it holds even in double Convexes of equal Convexities; for in them, as the Sum of the Radii of both Convexities: to the Radius of the Convexity :: so the double Radius of the Convexity: to the Distance of the Focus. And 'tis the same in Glasses of unequal Convexities, for these refract in the same Proportion to their Curvities as the other.

Wherefore from these three foregoing Propositions we lay these down as General Rules; That the Point of Convergency for Parallel Rays falling on a Plano-Convex Glass, is distant about the Diameter of the Convexity. On a double Convex of equal Convexities, 'tis about the semi-Diameter of the Convexity. And on a Double Convex of unequal Convexities, the Rule for finding it is what we have laid down before.



Before I proceed, it will be requisite here to note, That the Rays which fall nigher the Axis are not united thereto so near the Pole of the Glass, as those that fall farther from it; and the difference of the Points of Concourse is not so great, when the Convex side of a Plano-Convex Glass is turned to the Object, as when the Plain side is towards the Object.

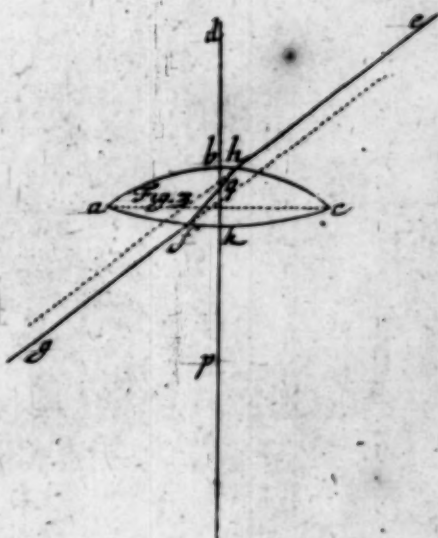
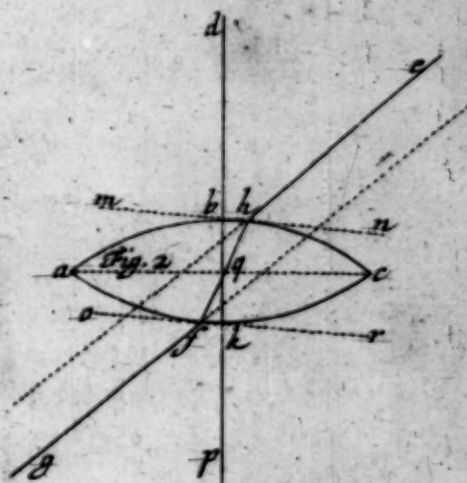
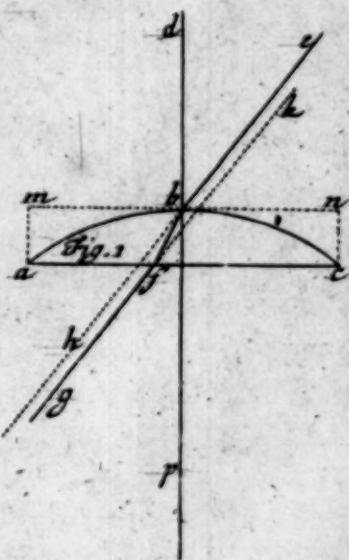
For this observe Tab. 8. F. 1, 2, 3. in each of which, the Radius of the Convexity cv is 2 Inches, or 20000 Parts (an Inch being 10000 Parts) the Arch rvr in the first and second Figures is a Quadrant, and consequently the Thickness of the Glass vz in each is 5858, being the versed Sine of 45° to the Radius 20000. But in the double Convex Fig. 3. the thickness of the Glass vz is $\frac{1}{2}$ of an Inch or 5000 such Parts. In each is a Ray falling on the Glass Parallel to the Axis cd , at the distance of $\frac{1}{2}$ of an Inch or 1000 such Parts as cd is 20000. A like Ray falling on C of an Inch, or 20000 such Parts

Tab. 8. F. 1,
2, 3.

Parts distant from the Axis, i is the Point wherein the Ray ab meets the Axis, h the point in which de crosses the Axis. Here I call hi the Depth of the Focus, which we see is greater in the second Figure, wherein the plain side is towards the Object, than in the first Fig. wherein the Convex Surface is towards the Object. For Proof of this, let the Angles of Incidence be to the refracted Angle (according to Kepler) as 3 to 2, Expressed in the first Column of the following Table; or let the Sines of the Incidence be to the Sines of the refracted Angles, as 300 to 193 (according to Mr. Newton) as is expressed in the second Column of the Table. Then by the foregoing Method of Calculation, delivered in the second and third Propositions, we shall find as in the following Table.

	1st. Column.	2d. Column.
	3 : 2 :: <i>Inclm. Ref.</i>	300 : 193 :: <i>Inc. Ref.</i>
For the Plano-Convex with the Convex Side to the Object.	$vi = 4.1930$ $vh = 4.1230$ Tab. 8. Fig. 1. $hi = 700$	$vi = 3.4762$ $vh = 3.4112$ $hi = 650$
For the Plano-Convex with the plain side to the Object.	$vi = 3.9910$ $vh = 3.7140$ Tab. 8. Fig. 2. $hi = 2770$	$vi = 3.5957$ $vh = 3.3259$ $hi = 2698$
For the Double Convex of equal Convexities.	$vi = 1.841$ $vh = 1.685$ Tab. 8. Fig. 3. $hi = 156$	$vi = 1.762$ $vh = 1.653$ $hi = 109$

From this Table 'tis manifest, first that the Ray ab , which falls nigher the Axis vt , is not united thereto so near the Pole of the Glass v , as the Ray de , which falls farther from the Axis; for ab is united to the Axis at the Distance vi , and de is united to the Axis at the Distance vh , but vi is greater than vh . Secondly,



Secondly, 'Tis evident that the *Focal Depth* bi is not so great in a Plano-Convex Glass, when the Convex Side is towards the Object, as in *Fig. 1. Tab. 8.* as when the plain Side is towards the Object, as in *Fig. 2. Tab. 8.* for by the Table bi is in the former but 650, but in the latter posture 'tis 2691, that is more than four times greater.

Whence I deduce this, that in viewing an Object by a Plano-Convex Glass, 'tis best turning the Spherical Side to the Object; and so likewise for burning by such a Glass, 'tis best turning the Convex Side towards the Sun.

OF OBLIQUE RAYS.

Premises to the Fourth Proposition.

Hitherto I have consider'd only the *Direct Rays*. I come now to the Consideration of *Oblique Rays*: And for this we must have Recourse to our foregoing 13th. and 17th. Definitions; as also to the first and second Experiments: By the 13th Definition, we know what *Oblique Rays* are: And by the 17th Definition, we learn that in every Parcel of *Oblique Rays*, flowing from the same Point of an Object upon a Glass; there is one certain Ray, which may be called the *Axis*, and that this *Axis* is as it were not at all refracted. But for a farther Declaration hereof, *Tab. 9. Fig. 1.* Let abc be a Plano-Convex Glass, with the Convex Side towards the Object, db the Axis of the *Direct Cone* of Rays, eb a Ray of the *Oblique Cone*, falling on the Vertex of the Glass at b , I say, this is the Axis of an *Oblique Cone*, and after its Double Refraction; first at its Ingrefs at b , and then at its Egrefs at f , it becomes fg Parallel to eb . This is Evident, if we imagine the Ray eb to fall on the Plane Glass $amnc$, for 'tis refracted the same way in one Case as in another. Wherefore the Ray $ebfg$ is as it were straight, as ebh or kfg ; so

Tab. 9. Fig. 1.

likewise we may imagine the Plane Side of this same Glass ac turn'd towards the Object, then pbd is the Axis of the *Direct* Cone, and gf the Axis of an *Oblique* Cone. We see therefore that a Plano-Convex Glass, being exposed with its *Convex Side* to the Object, the Rays that fall from the several Points of the Object, whose *Ingress* is at the Glasses vertex b , are the *Axes* of the respective Cones. But the *Plane Side* being towards the Object, the Rays from the several Points, which have their *Egress* at the Glasses vertex b , are the *Axes* of the respective Cones. And that amongst the Infinite Rays that proceed from a *Collateral Point* of an Object, and form an *Oblique* Cone, falling on the Plano-Convex Glass; there is some one certain Ray, that falls on the Vertex, and has its *Ingress* at b ; Or (if the Plane Side be towards the Object) has its *Emersion* at b , is manifest from this Reason only, that the Rays compounding such a Cone, are spread over the *Whole* surface of the Glass, and are as it were *Infinite* or *Indefinite*.

Tab. 5. Fig. 1.

In like manner, Tab. 9. Fig. 2. $abck$ is a Double Convex of Equal Convexities, $dbkp$ the Axis of the *Direct* Cone of Rays, eb a Ray of an *Oblique* Cone, which at its *Ingress* at b is refracted into bf , in such manner that it makes the Arches bb , kf , equal; then at its *egress* it becomes fg , Parallel again to eb , as if it had not been at all refracted, but had passed directly forward, or through a Plane Glass. This also will be manifest, if at the Point of Incidence b , and at the Point of Emersion f , we draw the Tangents mn , or ; for these Tangents are Parallel when bb and fk are equal Arches; and the Ray eb is no otherwise refracted by the Convex Glass, than if it had fallen on the Plane Glass mno .

And that amongst the Infinite Rays that proceed from a *Collateral Point* of an Object, and form an *Oblique* Cone, falling on the Double Convex $abck$; there is some one certain Ray, that has its *Incidence* so, that after its *Refraction* at

its

its Ingress, it may proceed so in the Glass, as to cut off equal Arches $b h$, $k f$, may be easily conceived; because the whole surface of the Glass is occupy'd by each Cone from each Point; so that the Point b (for instance) cannot miss of having amongst the rest its Ray, which must necessarily be so refracted.

What has been said concerning a Double Convex of Equal Convexities *Tab. 9. Fig. 2.* may be accommodated to a Double Convex of unequal Convexities *Tab. 9. Fig. 3.* For as in *Tab. 9. Fig. 2.* the Arches $b h$, $k f$, are to be equal (the Convexities being Equal) so in *Fig. 3.* the Arches $b h$, $k f$, are to be Similar; that is to say, $b h$ is to be as many degrees of its Circumference, as $k f$ is of its Circumference. Thus suppose in the same *Fig. 3.* The Arch $a b c$ to be struck with a Radius of two Inches, and the Arch $a k c$ to be struck with the Radius 4 Inches; these Radii being to each other as 2 to 1, $k f$ must be to $b h$ as 2 to 1, for then they shall be of equal degrees, each in its own respective Circumference, for here likewise in this third *Fig.* the Tangents at b and f , run Parallel to each other.

The Axes of all the Radius Cones, falling on a Plano-Convex with its Convex Side to the Object, Cross each other in the Vertex of the Convex surface, at their Ingress into the Glass, as in *Tab. 9. Fig. 1.* at b ; but if the Plane Side be turned towards the Object, they Cross in the Vertex of the Convex surface at their Egress from the Glass, as in the same *Fig. 1.* at b . *Tab. 9. Fig. 1.*

The Axes of all the Radius Cones, falling on a Double Convex of Equal Convexities, Cross each other in the Middle or Centre of the Glasses Axis. Thus in *Tab. 9. Fig. 2.* *Tab. 9. Fig. 2.* $d b p$ is the Line, which produced, would pass through the Centres of the two Convex surfaces; this Line I call the Axis of the Glass, let $b q$ be equal to $k q$, then q is the Point

wherein the Axes of all the Radius Cones Cross each other.

The Axes of all the Radius Cones, falling on a Double Convex of unequal Convexities, Cross each other in a Point within the Glass in its Axis, which divides the said Axis in the Proportion, that the Radius of one Convexity has to the Radius of t'other. *Tab. 9. Fig. 3.* Let the Radius of the Convexity $a k c$ be to the Radius of the Convexity $a b c$, as 2 to 1. Let $b k$ the Axis of the Glass be divided in q , so that $k q$ may be to $b q$ as 2 to 1, then q is the Point wherein the Axes of all the Radius Cones Cross each other.

The same may be understood of Concave Glasses.

And this is sufficient for explaining what I mean by the *Axis of an Oblique Cone of Rays.*

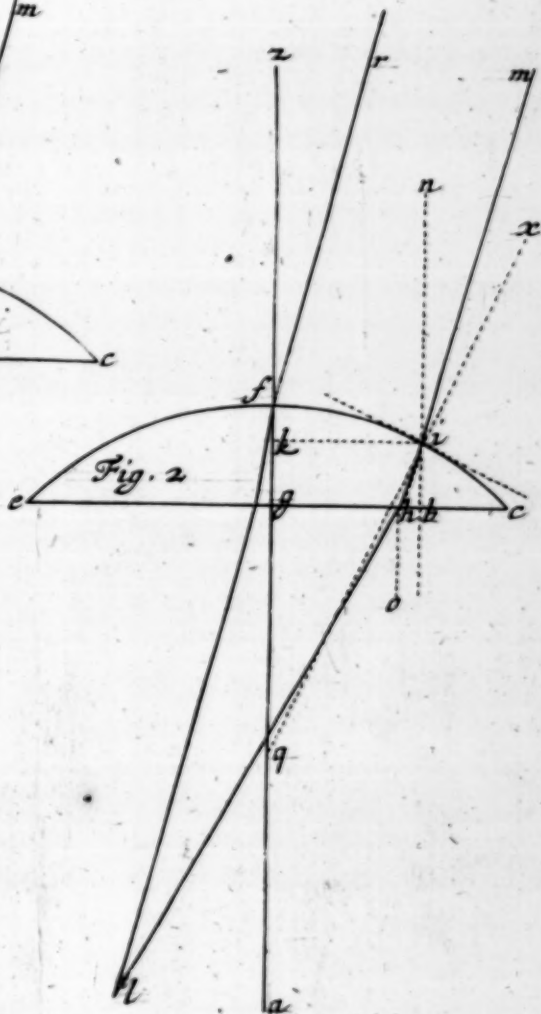
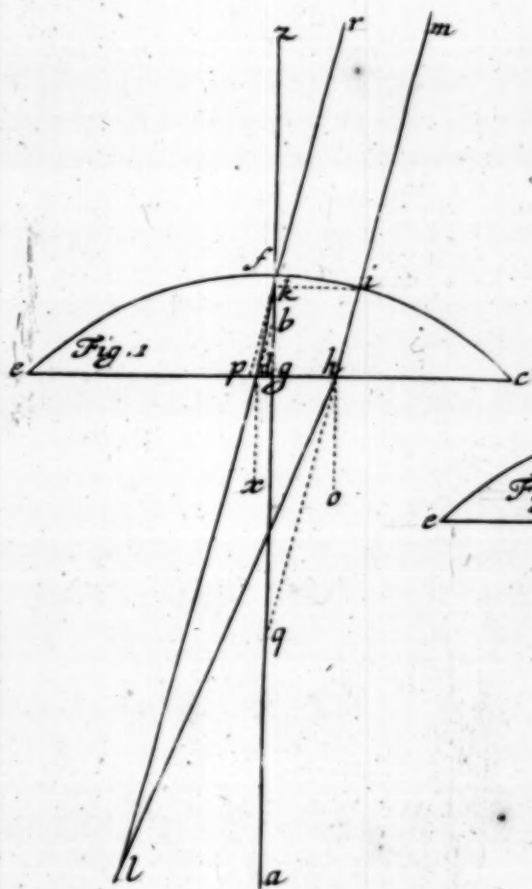
I come now to the Fourth Proposition.

P R O P. IV.

The Parallel Rays that proceed from each Collateral Point of an Object, and fall Obliquely on a Glass, are united with their Axis at the same Distance, as the Direct Parallel Rays are united with their Axis, viz. By a Plano Convex about the Distance of the Diameter; by a Double Equal Convex about the Distance of the Radius; and by a Double Unequal Convex, as is before determin'd.

I have hitherto proposed the Properties of Glasses Problematically, to be Demonstrated or rather traced out by a Geometrical Calculation of the Rays Progress. This Method, as 'tis the most Artificial, so 'tis the most Legitimate and Exact, and wholly New and Distinct from that of other Optick Writers. I shall first observe the same Method in this Proposition, proposing it Problematically thus, *Tab. 10. Fig. 1.* $e f c$ is a Plano Convex Glass, with the Convex Side towards the Object.

Let



Let there be given qf the Radius of the Convexity (as in the Table after the foregoing *Scholium*) 20000, and fg the Glasses thickness 5858, $zfq a$ is the Axis of the Glas it self, or of the Direct Cone of Rays, the Point of whose Concourse is in a , so that fa (as we have found in the foregoing Propositions) is 34112. Let rf be the Axis of an Oblique Cone of Rays, falling on the Vertex f ; and making the Angle of Incidence rfz ($= xdl$) of any given Quantity (as in this Example suppose $14^{\circ} 28' 40''$.) Let us suppose an other Ray mi Parallel to rf , and its Point of Incidence i at such a Distance from the Axis zfa , that being produced directly forwards, it may pass through the Centre q of the Convexity; that is to say, let us suppose kqi to be equal to zfr . Thus I propose it for ease of Illustration, for the Calculation may as truly be performed by supposing the Ray mi at any other Distance, as I shall declare presently.

From these things given, 'tis required to find the Point l in the Axis, where the Ray mi crosses it. For the Discovery whereof we proceed thus.

Draw ik Perpendicular to zq , then in the Right-angled Triangle kqi , we have qi , and the Angle kqi to find kq and ki ; then $qf - fg = qg$. And $kq:ki::qg:gb$.

Produce rf to p , then in the Right-angled Triangle fgp , we have fg , and the Angle $gfp = rfz$ to find the Side pg ; or thus, the Triangles gfp , gqb are Similar, then $gq:gb::gf:gp$. Say then, as 300 : to 193 :: so S. $\angle gfp$: to S. $\angle gfd$.

In the Right-angled Triangle fgd , we have fg , and the $\angle gfd$, to find the Side dg , then $dg + gb = db$.

Draw bo and dx Perpendiculars to ec , say then, as 193 : to 300 :: so S. $\angle obq = gqb$: to S. $\angle obl$. But $90^{\circ} - obl = dbl$. And $90^{\circ} + xdl$ or $zfr = bdl$.

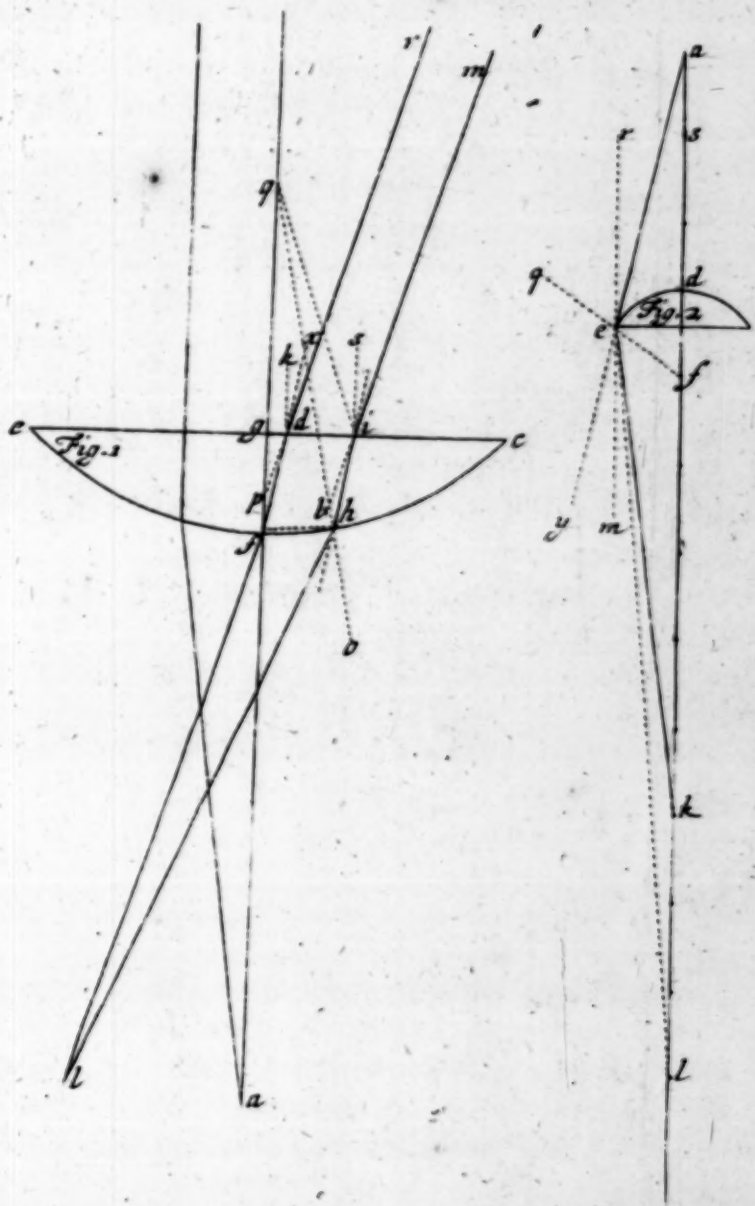
Lastly,

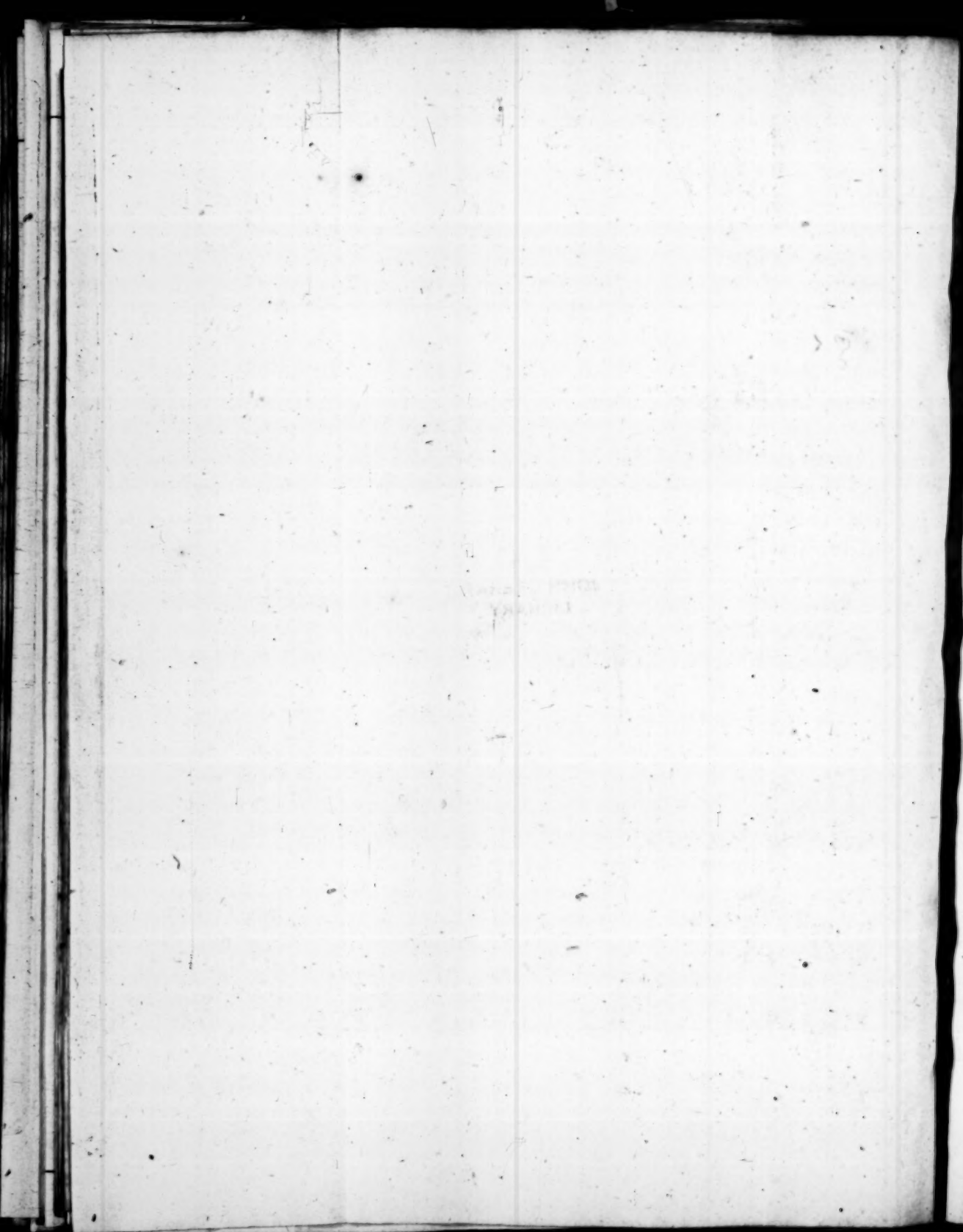
Lastly, In the Triangle bdl , we have the two Angles dhl and hdl , and the Side bd to find the Side dl ; which was required.

I have said before, that the same Calculation may be performed for any other Ray Parallel to rf , falling at any distance ki from the Direct Axis $zfqa$; supposing with the former Data (instead of the Angle $fq i$) (*Tab. 10. Fig. 1.*) the Distance ki (*Tab. 10. Fig. 2.*) be given. For then in *Tab. 10. Fig. 2.* draw nib Parallel to zq , and produce qi to x , then $nim = rfz$. In the Right-angled Triangle qki , having ki and qi , we get the Angle $kqi = nix$, and the Side kq . And $nix - nim = mix =$ to the Angle of Incidence. And $qf - qk = fk$, $fg - fk = kg = ib$, by which in the several small Triangles within the Glass, we may find the Angle of the Rays Incidence from Glass to Air, as in the immediate foregoing Example, and so onwards in like manner.

*Tab. 11.
Fig. 1.*

I shall propose one Case more in this Doctrine of *Oblique Rays*, and that is in *Tab. 11. Fig. 1.* where the Plane Side of the Glass is turn'd towards the Object. And that we may trace a Ray in this Case by Calculation, and find out a proper Ray for our purpose, we are to work a little backwards, thus, $qf = qb$ the Radius of the Convexity is given 20000, fg the thickness of the Glass is given 3858; And $qf - gf = qg = 14142$. Let df be a Ray within the Glass, whose Ingress is at d , so that dg is given 1000; and its Egress is at the Glasses Vertex f : In the Right-angled Triangle dgf , we have two Sides given to find the Angle dfg . Let kd be Perpendicular to ec , and produce fd directly to x : Then say, as 193 : to 300 :: so Sine of the Angle $kdx = dfg$: to Sine $\angle kdr$. And $kdr - kdx = xdr$. Wherefore rd is a Ray of Light falling on the Glass at d obliquely by the Angle kdr , which after its Immersion into the Glass proceeds onwards, and Emerges at the Vertex of the Glass f . This Ray at its Emer-





Emerſion at f ſhall again run Parallel to rd , by what foregoing; wherefore the Angle lfa is equal to the Angle kdr .

And thus much for the backward Operation, which we have Inſtituted to find a proper Ray to work with, that is, to find the Axis of an Oblique Cone, and its Angle of Inclination.

Let mi be another Ray Parallel to rd , and falling on the Glaſs at i , ſo that gi may be given 5000. 'Tis required to determine the Point l , where the Ray mi , after a Double Refraction at its Ingreſs at i , and at its Egreſs at h , meets the Axis $rdfl$.

Produce rd directly to p , and mi to b , and make si Perpendicular to ec ; Seeing therefore that mi is Parallel to rd , the refracted part ih , ſhall be Parallel to the refracted part df , and the Angle xdr ($=pdf$) ſhall be equal to bih , and the Angle kdr is equal to sim . Joyn fb .

Then in the Right-angled Triangle qgi , we have qg and gi to find the Angle $gqi = qis$, and the Side qi : But $qis + sim = miq$, and $180^\circ - miq = qib$. And $qib + bih$ is equal to qih .

Therefore in the Triangle qih we have the Sides qi and qh , and the Angle qih , to find the Angles iqh and qbi . But $gqi - iqb$ is equal to gqb or fqh .

Then in the Iſoſceles Triangle fqb , we have fq and qb equal to each other, and the Angle fqb to find the Side fb , and the Angle $fbq = qfb$.

Produce qb directly to o , then as 193 : To 300 :: So Sine Angle qbi : To Sine of the Angle obl . But $180^\circ - obl - fbq = fbl$. Alſo $180^\circ - qfb = bfa$. And $bfa + lfa = bfl$.

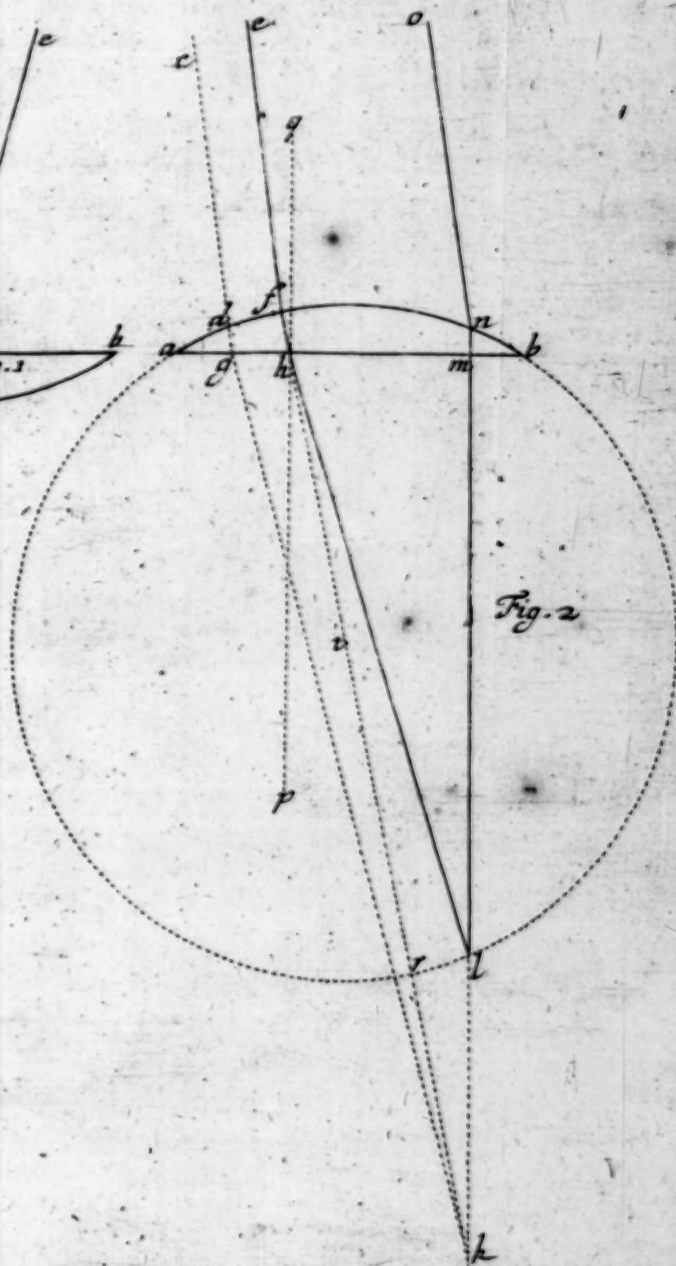
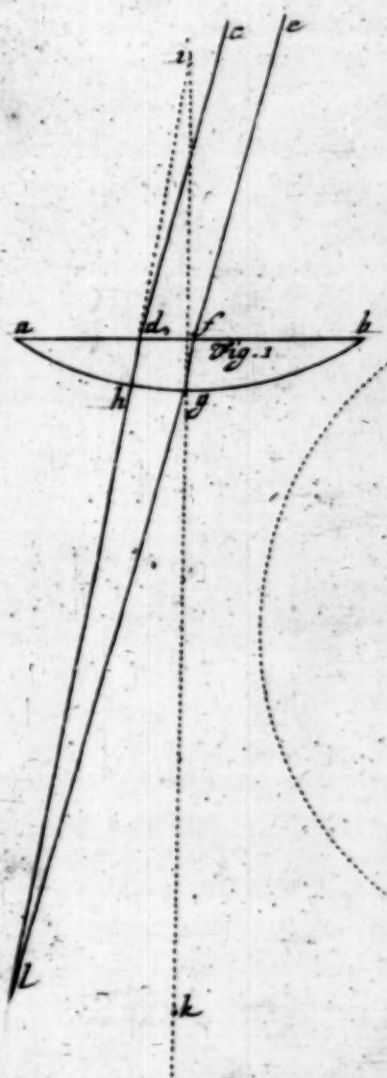
Laſtly, In the Triangle bfl , we have fb and all the Angles to find the Side fl , which was required.

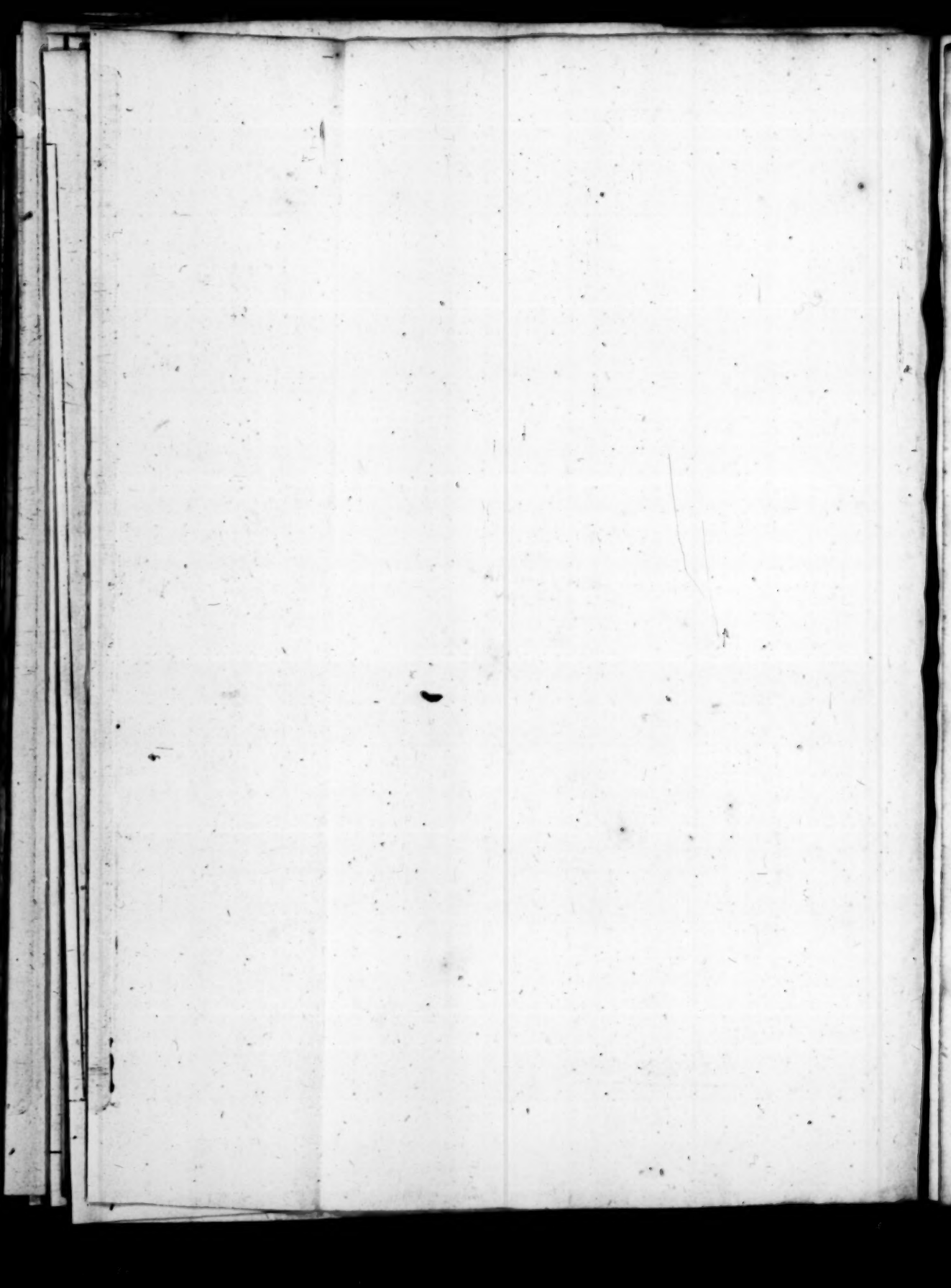
Common Demonstration.

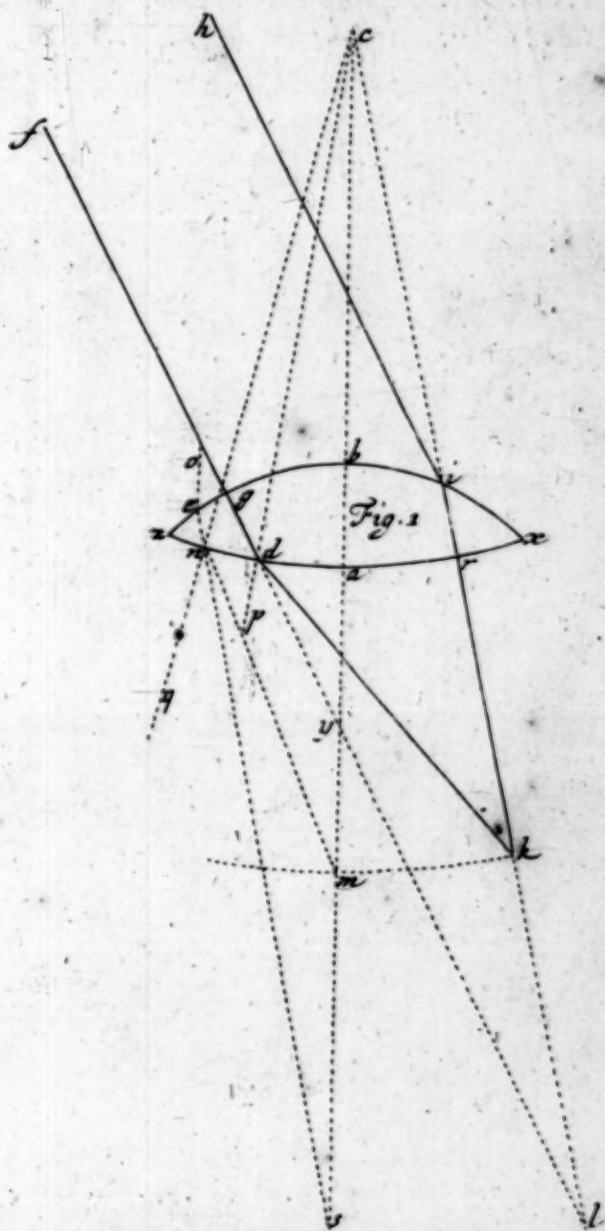
Tab. 12.
Fig. 1.

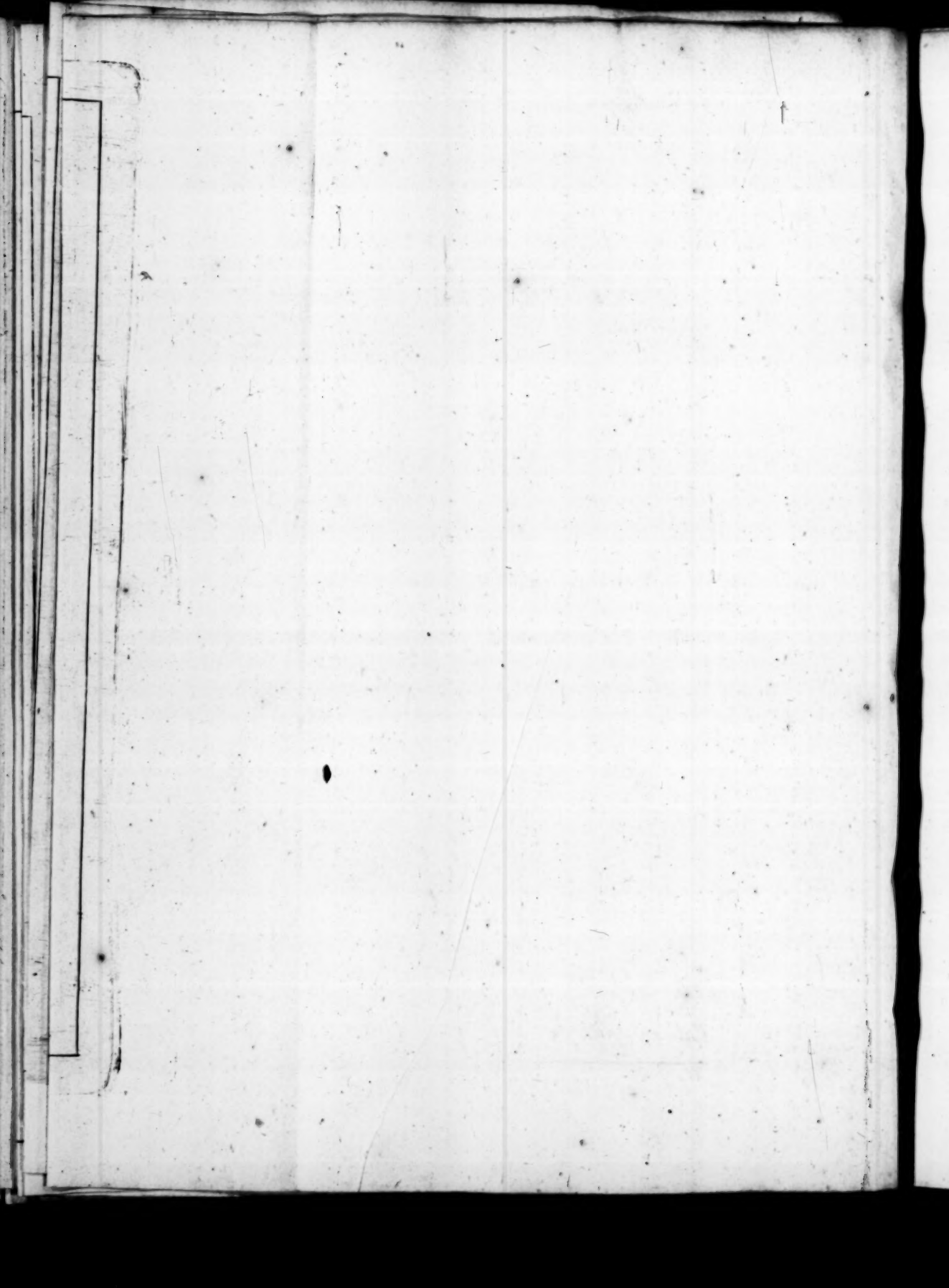
I shall now Demonstrate this Proposition the usual Way.
Tab. 12. Fig. 1. agb is a Plano-Convex, let the Plain surface ab be first turn'd to the Object: And let the two Parallel Rays cd, ef , fall Obliquely on the Plane surface ab , I say, that these Rays shall be united in l , so that hl shall be near equal to the Diameter of the Convexity, which we'll suppose gk .

Seeing the Angles efb, cdb , are equal, these two Rays have equal Angles of Inclination, and consequently shall have equal Correspondent Angles of Refraction within the Glass, and therefore the refracted Rays dh, fg , shall also be Parallel within the Glass: And because from the same Point in the Object, that transmits these two Rays cd, ef , there proceed Infinite Rays, which falling on the Glass, and being refracted therein, do occupy the whole Glass; some one of these Rays after its first Refraction, being produced backwards, shall pass through the Centre of the Convexity i . Let hd be this Ray produced directly both ways to i and l : Seeing therefore hd is Perpendicular to the surface agb , at h it suffers no Refraction in its Egress from the Glass. Wherefore let us consider the other refracted Ray fg , and what Refraction it suffers at the Point g on its Egress from the Glass: Here the Angle of Inclination is fgi equal to the Altern gih ; Let us suppose the refracted Angle kgl , which (by Experiment V) ought to be $\frac{1}{2}$ more than the Inclination $fgi = gih$. Wherefore in the Triangle $igl, li:lg::S. \angle lgi = S. \angle l g k: S. \angle lig$. But in these small Angles, as the Sines, so the Angles, and therefore if the Angle lgk be $\frac{1}{2}$ more than the Angle lig , the Side li is $\frac{1}{2}$ more than the Side lg or lh (for lg and lh are insensibly equal in Glasses of small Segments of large Spheres)









Spheres) wherefore hi being the Semidiameter, hl shall be the Diameter. *Which was to be Demonstrated.*

Secondly in *Tab. 12. Fig. 2.* Let the same Glass ab be turned with its Convex-Side to the Object, and thereon let the Parallel Rays cd, ef , fall obliquely to the Axis. These Rays shall be united together behind the Glass almost at the distance of the Diameter of the Convexity. T. 12. Fig. 2.

For amongst the parallel Rays that fall on the Glass, let us conceive one ef , that after its first Refraction at f , passing in the Glass in the Line fb , and being produced directly, would pass through the Centre of the Convexity i . If we conceive the Medium of Glass continued, we know that these two Rays ef, cd , would be united in k at the distance of a Diameter and half from the Glass (by *Prop. I.*). From the point k draw kn perpendicular to the plane surface of the Glass ab , and let the Ray no be drawn parallel to ef, cd ; this Ray no shall also, by its first Refraction, be directed towards the Point k ; and because within the Glass the Ray nm is perpendicular to the plain Surface ab , it shall proceed unrefracted through m, k . I shall now shew that fb by its second Refraction at its egress at b is refracted to l . Draw bl , and through b draw qbp perpendicular to ab . The Angle of Inclination within the Glass is $fbq = pbk = bkl$. And now I shall prove that the Angle kbl is $\frac{1}{2}$ the Angle of Inclination pbk or bkl . In the Triangle bkl , $s. \angle bkl : s. \angle kbl :: bl : kl$ (or kr , for they are nearly equal), but bl is almost double kl or kr , therefore the Angle bkl is almost double kbl . *Which was to be Demonstrated.* And therefore the parallel Oblique Rays have the Point of Concourse at the distance of a Diameter almost from a Plano-Convex Glass. *Which was to be Demonstrated.*

Vid. *L'Optique de Pierre Ango Livre 3. Sec. 69. &c.*

Thirdly, In *Tab. 13. Fig. 1.* zx is a double Convex, c is the Centre of the Convexity zax , y the Centre of the Convexity T. 13. Fig. 1.

F

zbx ,

zbx , cys the Axis of the Glas. 'Tis evident by what foregoes, that whatever Rays fall on this Glas parallel to the Glas's Axis are united thereto, at the distance of the Focus, (*viz.* about the Semidiameter in equal Convexities) as suppose at m . Wherefore let us consider two other Rays, parallel between themselves, but oblique to the Glas's Axis cs , such as fg , hi , I say these likewise, after a Double Refraction do concur about the distance of the *Principal Focus* (for so I call the Focus of the Rays parallel to the Glas's Axis.)

Seeing there are infinite parallel Rays, that fall thus obliquely on the Surface zbx , most certainly there must be one amongst the rest, which so falls thereon, that being produced directly, shall pass through the Centre y of the Convexity zbx : Let us conceive fg to be this Ray, so that fgy is a Right Line; let it be produced to l , so that yl may be equal to double gy .

From the Point l to c the Centre of the Convexity zax draw the Right Line lc , and where this Line cuts the Surface zbx , as in i , let the Ray hi fall parallel to fg . 'Tis evident, that by the first Refraction the Ray hi suffers on the Convexity zbx , it will concur with the Ray fg in l at the distance of a Diameter and half, for now $fgdyl$ may be considered as the principal Axis.

Let us suppose the principal Focus of this Glas to be at m . From the Centre c strike the Arch mk cutting cl in the Point k . The Ray hi shall be refracted at i , but the refracted Ray ir (seeing it is perpendicular to the Surface zax by Construction) shall proceed unrefracted directly to k ; but the Ray fgd being refracted in the Point d shall proceed refracted, and shall be united with the Ray ik in k . Which I thus prove,

From c draw the Line cdp , the Angle pdy is equal to the Angle of Inclination gdc in the Glas; and therefore if dk be

be truly the refracted Ray, the Angle $pd\gamma$ ought to be double the Angle γdk , and that it is so I thus prove.

Let us suppose the Ray oe Parallel to the principal Axis cs , this Ray by the first Refraction it suffers at e is directed to the Point s , so that bs is a Diameter and half, or triple by , and therefore equal to gl . The same Ray oe being refracted at e to n , and at n suffering a second Refraction is brought to concur with the Axis cs in m , the principal Focus by Supposition. In which Case, the Angle of Inclination in the Glass shall necessarily be enc , which is equal to qns , and the Angle of Refraction shall be snm ; and therefore the former qns shall be double the latter snm . But I say, as qns is double of snm : so pdl is double of ldk . Which I thus prove,

In the Triangle ncm | 1 $s. \angle cnm = s. \angle qnm : s. \angle nmc :: cm = ck : nc = cd$

Also in the Triang. dck | 2 $s. \angle cdk = s. \angle pdk : s. \angle dkc :: kc : cd$

From 1 and 2

| 3 $s. \angle qnm : s. \angle nmc :: s. \angle pdk : s. \angle dkc$

Then Sines and Angles being proportional

| 4 $qnm : nmc :: pdk : dkc$

By the Scheme

| 5 $qnm = qns + snm$. Also $pdk = pdl + ldk$.

From 4 and 5.

| 6 $qns + snm : nmc :: pdl + ldk : dkc$

Alternando 6.

| 7 $qns + snm : pdl + ldk :: nmc : dkc$

Moreover in $\triangle nms$.

| 8 $s. \angle smn = s. \angle nmc : s. \angle snm :: ns = dl : ms = kl$

Also in Triang. dkl

| 9 $s. \angle dkl = s. \angle dkc : s. \angle ldk :: dl : kl$

From 8 and 9

| 10 $s. \angle nmc : s. \angle snm :: s. \angle dkc : s. \angle ldk$

Sines and Angles prop.

| 11 $nmc : snm :: dkc : ldk$

Alternando 11

| 12 $snm : ldk : nmc : dkc$

From 7 and 12

| 13 $qns + snm : pdl + ldk :: snm : ldk$

Alternando 13

| 14 $qns + snm : snm :: pdl + ldk : ldk$

Dividendo 14

| 15 $qns : snm :: pdl : ldk$

From which 15th Step it follows, that if qns be double of snm :: so pdl is double of ldk . Which was to be Demonstrst.

Note, In this Demonstration we assume ns and dl to be equal, and ms and kl to be likewise equal, whereas they are not so exactly, for the Thickness and Breadth of the Glals causes some little difference, but in Glassses of small Segments and large Spheres, and where the Rays do not fall very obliquely, this Difference is so very inconsiderable, that in a Demonstration relating to a Physico-Mathematical Matter, they may well be considered as equal.

Scholium.

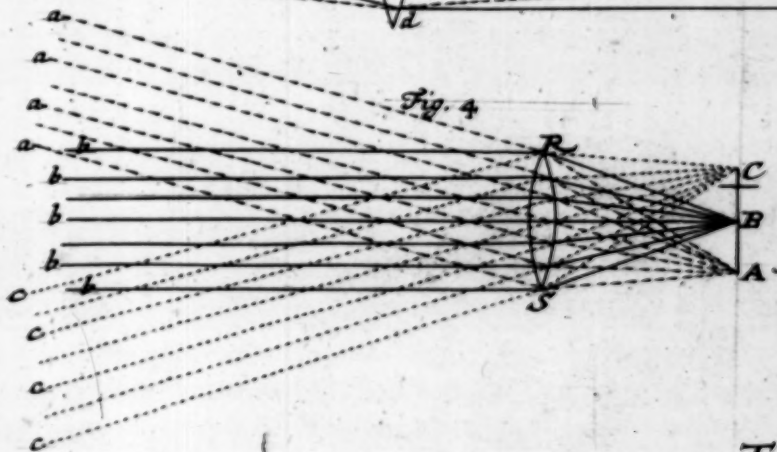
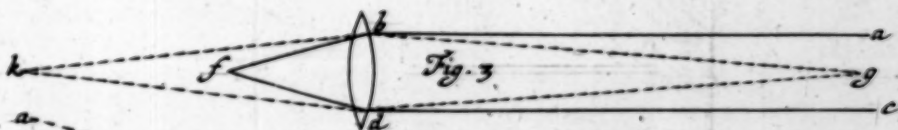
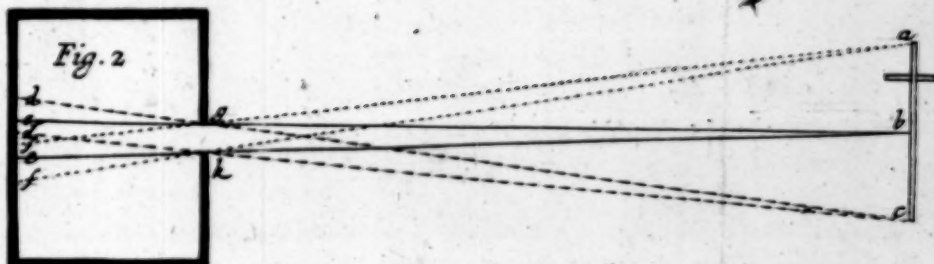
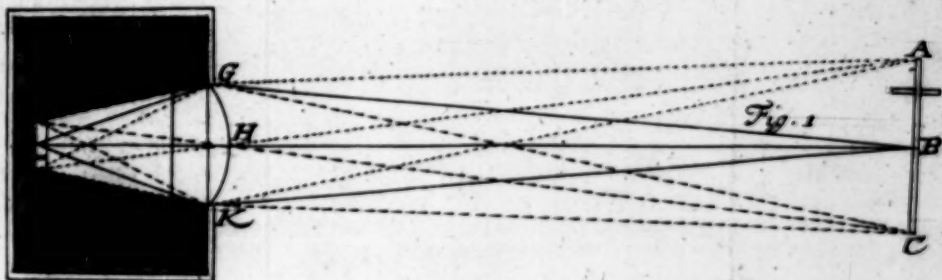
But tho the Proposition expresses, that the Parallel Rays which proceed from each collateral Point of an Object, and fall obliquely on a Glals are united with their Axis at the same distance, as the direct Parallel Rays; yet this is to be understood with some Limitation: For if the Rays fall very obliquely, their Union with the Axis is not so exact, but they are scattered and the Focal Depth is very great. As will appear by Calculation.

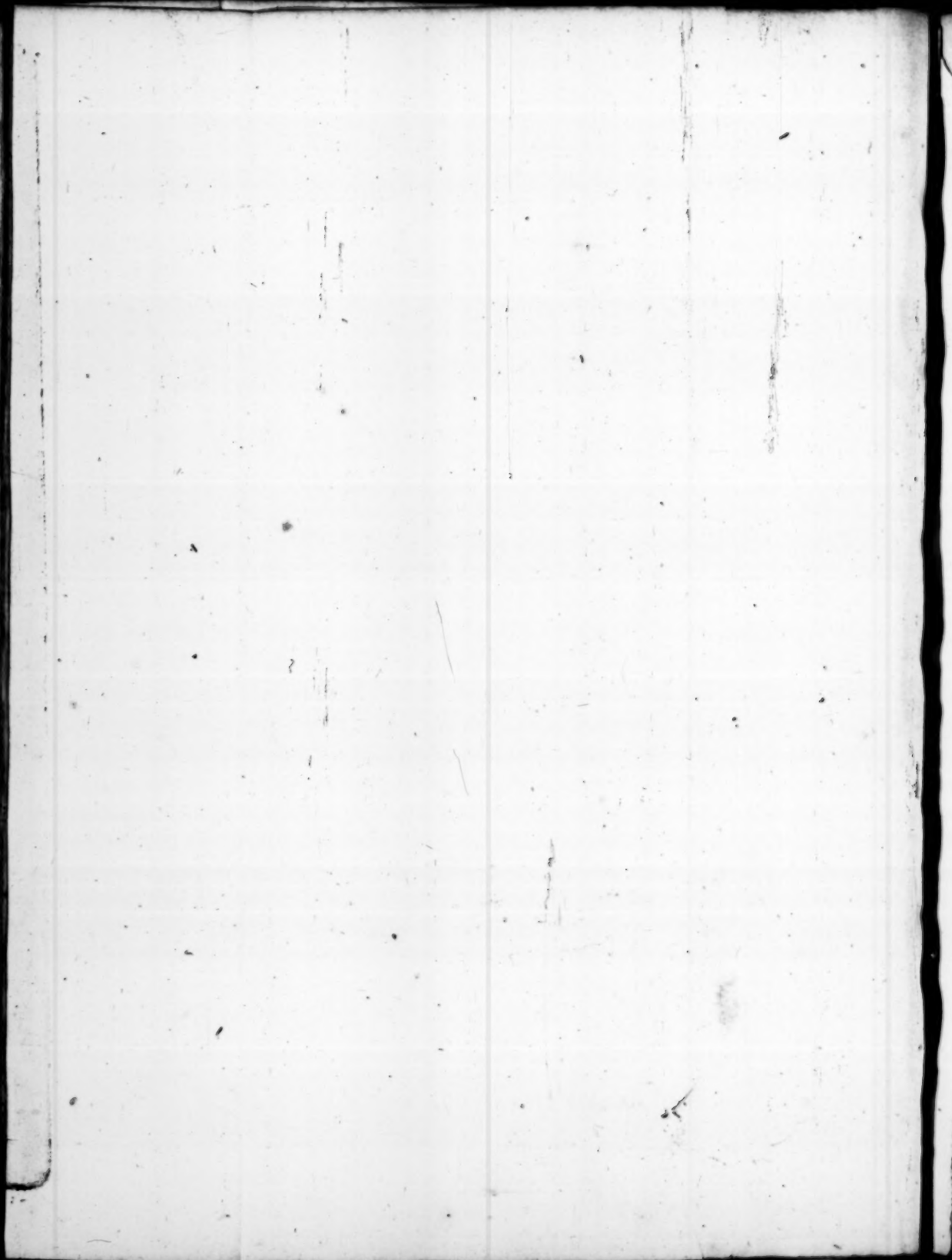
Of the Representation of outward Objects in a Dark Chamber; by a Convex-Glafs.

Tab. 14.
Fig. 1.

From what has been hitherto laid down results that admirable Appearance, and Effect of a Convex-Glafs duly placed in a small Hole in a Dark Chamber; Which therefore we shall here consider, *Tab. 14. F. 1.* ABC is a distant Object, which we are now to consider as very remote, so that tho the Figure, for want of Room in the Paper, expresses the Rays flowing from every single Point, as very much diverging; yet we are to imagine, that the Rays (for Instance) from the Point A run as it were Parallel to each other; the distance of the Object

BH





B H being supposed vastly great, in Comparison to the Glasses Breadth G K (But after the 5th. Proposition following, this Advertisment of the great distance of the Object will be needless, as therein shall appear) G H K is a Plano-Convex-Glass, receiving on its Surface a Cone of Rays from each single Point of the Object, but here for avoiding Confusion in the Scheme, we have only expressed the Cones flowing from the uppermost, middle, and lower Point of the Object, that is, from A, C, and B; and in each of these Cones neither have we expressed any more Rays than the Axis of the Cone (as for instance) A H, and the two extreme Rays of the same Cone A G and A K.

First therefore for the Rays that flow from the Point A. By what forgoes we know that the Axis of this Cone A H, after it has passed the Glass proceeds directly towards F, as if it were not at all refracted; and that the Rays A G, A K, are united with this Axis in a Point (suppose at F) distant from the Glass about the Diameter of its Convexity (the Glass being a Plano-Convex) and what is said of the Rays A G, A K, being united with their Axis, must be conceived of all the Rays that make up the whole Cone flowing from this single Point A: and they are all united likewise with the Axis in the Point F, forming another correspondent Cone G F K, having its Base on the Glass, and its Vertex in the Point F, making the Pencil of Rays F G A K F.

And so likewise, what is declared of the Point A, and the Rays that flow from it, and of their being united together in F, may be easily conceived of the Rays flowing from the middle Point B, and their being united in E; and may be also understood of the Rays flowing from C, and their union in D; and so of the Rays proceeding from any other Point in the Object A B C, and their being united in a correspondent Point in F E D. And this F E D we call the *Distinct Base, Focus, or Burning Point*. 'Tis called the *Dis-*

tinct.

stinct Base, because therein is a *Distinct Representation* of the Object. And supposing the Glass GK in the Hole of a dark Room, so that the Representation FED may be disturbed by no other luminous Rays from outward Objects, and a Paper were placed at DEF, there to receive the Image, we should thereon see the Object most livelily painted in its exact Shape and Colours. 'Tis called the *Focus*, or *Burning Point*, because the Glass being exposed to the Sun, it there collects the Rays from each Point of the Sun's Body, and paints its Image there most vividly, exciting a violent Heat, even to the inflaming of combustible Bodies.

But whereas in a *Scholium* after the Third Proposition, it is observed, that the Rays falling nigh the Axis are not united thereto so near the Glass, as the Rays that fall farther from the Axis. And consequently that all the Rays from one Point in the Object, do not unite in a correspondent Point in the *Distinct Base*, but that (as we have noted before) the Focus has some Depth, and consists not in an Indivisible Point. Yet this hinders not the Representation in the *Distinct Base* to be very lively and exact. For tho all the Rays from each Point are not united in an answerable Point in the Image, yet there are a sufficient quantity of them to render the Representation very perfect. And tho some Rays from other adjacent Points may a little intermix with those from its neighbouring Parts, yet these are so few, that they make no great disturbance.

N. B. Moreover in a *Scholium* following the Fourth Proposition, 'tis observed, that when the Incidence of the oblique Rays is very oblique, that then their Concourse with their Axis is not so very regular. From hence it proceeds, that the Representation in the *Distinct Base* is not so clear and exact towards the Extremes, as towards the Middle.

As for the reason of this surprizing Appearance in a dark Chamber, wherein there is a small Hole armed with a Con-

vex-Glass; it will be manifest, if we consider, that from every Point in the inverted Representation on the Paper, the Rays proceed to the Eye that looks at it, exactly in the same manner, as from the Object it self. For let us imagin the Point A to be *Blue*, B to be *Yellow*, and C *Red*. Because all the Rays flowing from A are separated by themselves, and fall in the Image on the Point F, where no other Rays intermix with them to disturb them, they must needs there represent *Blue*. For passing through the Glass, it being a Diaphanous Colourless Medium, does not at all alter them (notwithstanding the Appearance of the *Prisme*,) and the Point F in the Paper being enlightened only with Rays of *Blue* Light, and of it self being *White*, indifferent to all Colours, or rather indeed of no Colour, must needs represent *Blue*; and so E *Yellow*, and D *Red*. As to the *Shape* of the Object, it must needs be expressed exact likewise; for every Physical Point in the Object sending out a Cone of Rays falling on the Glass, and these being formed into another Correspondent Cone on the other side the Glass, determining their Vertices in Correspondent Points of the Image; the Eye must necessarily perceive on the Paper the lively Image of the Object, being it is affected in the same way by *This*, as by the Object it self. For from the Object it self the Eye receives only a Cone of Rays from each of its Points, and so it receives a Cone of Rays from each Point in the Image on the Paper; each Point in the Image on the Paper being enlightened by the Rays it receives through the Glass; and consequently the Paper, being an Impolite Surface, reflects these Rays into the whole circumjacent Medium. *Kid. Schol. Prop. 52. Gregorii Optic. Promat.*

And now that I am on the Consideration of this Appearance, it may not be improper to consider the Effect of a Plain small Hole without a Glass in a dark Room. *Tab. 14. Fig. 2.* ^{*Tab. 14. Fig. 2.*} represents a dark Chamber, in whose side there is a small Hole

gk.

gk , $\frac{1}{2}$ an Inch wide; abc is a distant Object. Now because there is no Glass in the Hole gk , the Rays from each Point of the Object pass through the Hole directly strait, and represent on the Wall within, at $dedef$, a faint and confused Image of the Object: Because here by the narrowness of the Hole, the Rays from some Points are hindered from intermixing with the Rays from some other Points; as here the Rays from the Point a do not intermix on the Wall with the Rays from the Point c ; for we see dd do not intermix with ff . But then the Representation is confused; for tho some Rays are hereby hindered from intermixing with some other Rays, yet other Rays from nigh adjoining Points do intermix with their neighbouring Rays, as here the Rays from a , and those from b are blended together in fe , and so those from c and b in de . And if the Hole be made so narrow as to hinder this communication of Rays in a great measure (for 'tis absolutely impossible to hinder it altogether) then the opening admits so few Rays from each Point, that the Image is obscure and imperceivable.

Tab. 14.
Fig. 2.

There is another Particular relating to this Appearance in a dark Chamber, which seems not so clearly treated of in those Optick Writers which I have consulted. Particularly *Zahn* in his *Ocul. Artif. Fund. 1. Synt. 3. Cap. 2. Sec. 8.* gives no satisfactory Account thereof. The Matter is thus; in *Tab. 14. F. 1.* if the Image DEF in the dark Chamber be received on a *Speculum* or Looking-Glass instead of a White Paper, the Eye perceives a most dilute and faint Image on the *Speculum*: But if the *Speculum* be placed any where between GK and DEF , and a Paper placed as far distant from the *Speculum* before it, as the distinct Base DEF is behind the *Speculum*; then the *Speculum* shall reflect on the Paper the distinct Image of the Object. The dilute and very faint Image, which the Eye perceives on the *Speculum* in the first Case, proceeds from the Imperfect Politure of the *Speculum*; For if it were a most exquisitely smooch

smooth Surface, even that faint Image it self would not appear. But as tis impossible by the Industry of Man to procure such, and all Miroirs partake in some measure of a little Roughness, therefore it is that we have that dilute Image. And the reason we have no such vivid Representation from a Speculum (considered as a perfectly polish'd Surface) as from a Paper, as also the reason of the appearance in the second fore-mentioned posture of the Speculum, does so depend on the Doctrine of Reflection or Catoptricks, that I shall pass it over in this place, it being manifest to any one meanly versed in that Doctrine. *Vid. Schol. Prop. 92. Gregorii Optic. Promot.*

Of Nigh Objects, or Diverging Rays.

We have hitherto considered the Radiating Points of *Distant* Objects both *Direct* and *Oblique*, the Rays from each single Point whereof do proceed as it were Parallel. I come now to the Consideration of *Nigh* Objects or *Diverging* Rays, and for these let us look back on the 5th, 6th, 7th, 8th, and 9th Definitions.

When the Rays from each Point of an Object run as it were Parallel, we may imagin that they may be sooner or easier brought together, than when they *Diverge* considerably, by coming from a *nigh* Object. And that therefore a Convex Glais unites the Parallel Rays *nigher* to it self, than *Diverging* Rays.

Tab. 14. Fig. 3. *ab, cd* are Parallel Rays flowing from one Point of a distant Object, these are united by the Convex-Glasis *bd* in its Focus at *f*. Let *bg, dg* be Diverging Rays flowing from the Point *g*, these are united in the Point *k*, farther from the Glais than the Point *f*. T. 14. Fi. 3.

For we may conceive (to omit the farther Proof of this by Geometrical Calculation), that the Parallel Rays have not so great a Reluctancy to be brought together, as the Diverging

ing Rays; and therefore the Refractive Power of the Glass has not so much to do to bring *those* together, as to bring *these*. And consequently it performs the *First* sooner than the *Latter*; or in a *shorter* space, after the Rays Emerge from behind the Glass. And this the *more*, according as the Object $\&$ approaches *nigher* and *nigher* to the Glass, till at last it be so nigh, that the Rays, after they have passed the Glass, become *Parallel* or *Diverging*, as I shall shew in the following Propositions.

Definitions.

The Focus wherein the Parallel Rays of a *Distant* Object are united, I call the *Focus*, simply without any Addition, sometimes the *Absolute Focus*, or *Solar Focus*.

The Focus wherein the Rays from a *Nigh* Object, or the Diverging Rays are united, I call the *Respective Focus*.

The foregoing Precepts, which I have given for calculating the Progress of a Ray through a Spherical Glass, are abundantly sufficient to find the Point of Union of Rays from Nigh Objects, or of Diverging Rays; The Distance of the Object or point of Divergency from the Glass (together with the foregoing Data) being given. I shall not therefore enlarge thereon, but Instead thereof I shall give these following short Rules in this Matter.

PROP. V.

In *Convex-Glasses*, When a *Nigh Object* is Placed more *Distant* than the *Focus*; The Rule for Determining the *Distinct Base*, or *Respective Focus* is This,

As the Difference, between the Distance of the Object, and Focus:
Is To the Focus, or Focal Length ::

So the Distance of the Object from the Glass:

To the Distance of the *Respective Focus* or *Distinct Base* from the Glass.

Demon-

Demonstration.

In *Tab. 11. Fig. 2.* Let ed be a Plano-Convex Glass, whose absolute Focus we know is about a Diameter of the Convexity, Let sd be this Diameter. a is a Radiating Point, f the Centre of the Convexity, fe the Radius of the Convexity produced directly to g . ae a Ray falling on the Glass, produced directly to y . Here we see the Angle of Inclination, or Incidence of the Ray ae is gea .

Let it then be made — — —
I say k is the respective Focus of the Ray ae .

Let kl be made $= \frac{1}{2}kd$.
I shall first Demonstrate, that by virtue of the first Refraction which the Ray suffers at its entrance on the Convex-Side of the Glass at e , 'tis directed as if it proceeded strait towards l .

For to the Consequents of the Analogy in the first Step add their Halfs, and it shall be — — —

And compounding the ad —

Here we see sf is equal to three Semidiameters of the Convexity, that is, to thrice fe .

The Angle of Refraction is yel if therefore we prove that yel is $\frac{1}{2}$ of gea the Angle of Incidence, it will be manifest, that by the first Refraction the Ray is directed towards l .

$$1 \quad as : sd :: ad : dk$$

$$2 \quad as : sf :: ad : dl$$

$$3 \quad af : sf :: al : dl$$

G a In

In order to the Proof hereof,
we lay down these Suppositions.

1. In the Triangle $a e l$ we suppose $l e$ and $l d$ equal, because we suppose the Glass of the least Thickness imaginable, and the Segment of a large Sphere.

2. We suppose likewise, that the Angles of Incidence are all very small, that so Sines and Angles may be proportional. Tho we could not express this truly in the Figure.

Wherefore, for the Demonstration of the forgoing Position, in the Triangle $a e l$ it is — — —

But the Angle $a e l$ is the Complement of the Angle $y e l$ to 180° . Therefore the Sine of the Angle $a e l$ is equal to the Sine of the Angle $y e l$.

Wherefore the 4th Step runs thus

But in these small Angles, as the Sines are, so are the Angles. Therefore the 5th stands thus —

Then from 3 and 6 it follows —

Then in the 7 Triple the Antecedent on this side, and subtriple the Consequent on the other side, and it will be — — —

Moreover, in the $\triangle a e f$ it is —

But the Sine of the Angle $a e f$ is equal to the Sine of the Angle $q e a$, being Complements to 180° .

$4 a l : l e :: l d :: s \angle a e l : s \angle a$

$5 a l : l d :: s \angle y e l : s \angle a$

$6 a l : l d :: s \angle y e l : \angle a$
 $7 \angle y e l : \angle a :: a f : s f = 3 e f$

$8 3 \angle y e l : \angle a :: a f : e f$
 $9 a f : e f :: s \angle a e f : s \angle a$

Where-

Wherefore the 9 may stand thus $10 a f : e f :: s \angle q e a : s \angle a$

But Sines and Angles in these small Angles being proportional, it follows from the 8th and 10th Steps, That $11 s \angle y e l : \angle a :: \angle q e a : \angle a$.

Wherefore from the Analogy in the 11th, it is evident, that the Angle of Inclination or Incidence $q e a$ is thrice the Angle of Refraction $y e l$; seeing three times the Angle $y e l$, and the Angle $q e a$, bear the same Proportion to the same Angle a . And this was the first thing to be proved; and consequently the Ray $a e$ by its first Refraction at its Point of Incidence is directed towards the Point l .

It remains to be Demonstrated secondly, that the Ray, at its Eruption on the plain side of the Glas into Air, is refracted into ek .

For the Proof of this draw $r e m$ Parallel to the Axis. Now the Angle of Incidence from Glas to Air shall be $m e l$, the Angle of Refraction $l e k$, which we shall prove to be half the Angle of Inclination $m e l$; or we shall prove that the Angle $m e k$ is thrice the Angle of Refraction $l e k$. Which being Demonstrated, 'tis certain the Ray is refracted into ek .

For the Demonstration hereof we retain the Series of our former Steps, and in the Triangle $l e k$ we have it

$$12 l e = l d : l k :: s \angle e k l : s \angle l e k$$

But $s \angle e k l$ is equal to $s \angle e k f$

being Complements to 180° .

Therefore the 12th. Analogy

may stand thus

$$13 l d : l k :: s \angle e k f : s \angle l e k$$

And the Angle $e k f$ is $= \angle m e k$

Wherefore

$$14 l d : l k :: s \angle m e k : s \angle l e k$$

Now the Angles being as the Sines, and $l k$ being by Construction the third Part of $l d$, it follows from the 14th Step, that

that the Angle lek is the third Part of the Angle mek . Wherefore the Angle lek is the Angle of Refraction agreeable to the Angle of Inclination mel . Which was to be Demonstrated.

Example of this Rule.

Of this Rule I shall give this short Example. Let the Focus of a Glas be 48 Inches; the Distance of the Object 156, their Difference is 108. Then $108 : 48 :: 156 : 69 =$ to the respective Focus.

By this Rule we may view very nigh Objects with long Telescopes, converting them as it were into Microscopes, only by lengthning them. But of this more hereafter.

Also from hence I shall deduce a Method of measuring distances at one Station: but hereof also more hereafter.

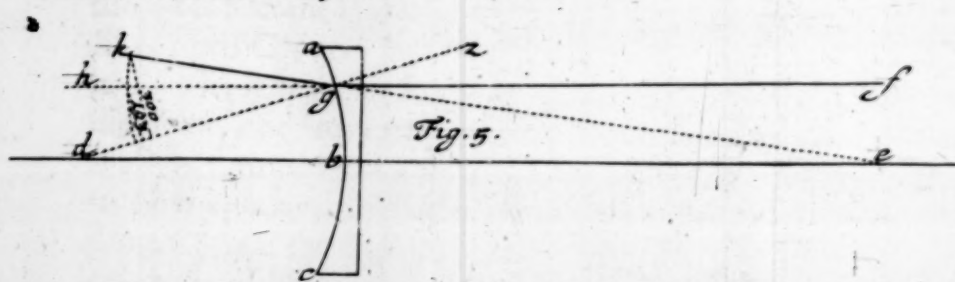
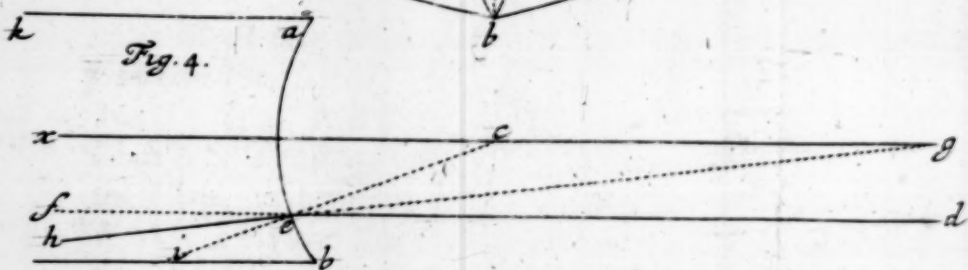
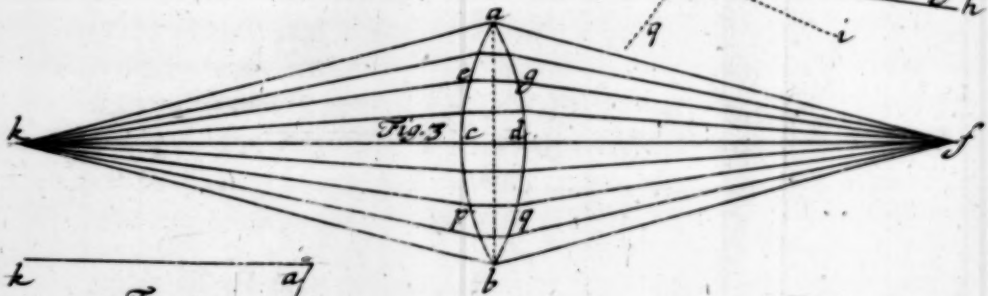
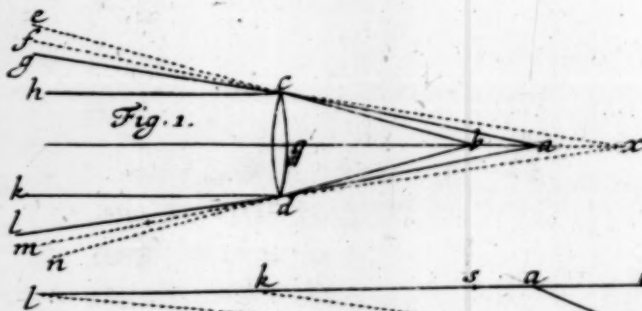
PROP. VI.

An Object being placed in the Focus of a Convex Glas, the Rays from each Point thereof, after passing the Glas become Parallel;

Tab. 14.
Fig. 4.

Tab. 14. F. 4. A B C is an Object placed in the Focus of the Glas R S. The Rays from each Point of this Object flow upon the Glas, as here we have expressed those from the Points A B C; I say the Rays from each of these Points after passing the Glas become Parallel, those from A being $a a a$, from B $b b b$, from C $c c c$. For let us imagine a distant Object to send its Parallel Ray, $a a a$ from its upper Point, $b b b$ from its middle Point, $c c c$ from its lower Point: These (by what foregoes) shall be formed by the Glas into the distinct Base A B C. Let us now conceive this distinct Base, A B C, to be made the Object; and the Case is most plain, that its Rays must needs be remitted back again in the same manner as they were received

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ceived from the distant Object on the side abc , that is, Parallel. For the Progress of a Ray is reciprocal. *per Exp. 8. Vid. Prop. XXVII.*

PROP. VII.

If an Object be placed nigher a Convex-Glass than its Focus, the Rays from each single Point thereof after they have passed the Glass, do proceed onwards diverging; But do not diverge so much as before they entred the Glass.

For if it be as much as the Glass can do, by its refractive Power to reduce to a Parallelism the diverging Rays from an Object placed in its Focus; it must needs be more than it can do, to reduce to a Parallelism the Divergency of the Rays, from an Object placed nigher to it than its Focus; The Divergence in this latter Posture being much greater than in the former, and consequently not so easily reduced by the refractive Power of the Glass. In *Tab. 15. Fig. 1.* cd is a Tab. 15.
Fig. 1. Convex-Glass in whose Focus at a there is a radiating Point, from which proceed the Rays acf , adm , which meeting with the Glass, instead of going onwards to f and m , are reduced thereby to cb , dk , Parallel to each other. b is another radiating Point nigher the Glass than its Focus a , we may plainly perceive the Rays bc , bd , diverge more than the Rays ac , ad . Wherefore bc , bd , instead of proceeding directly onwards to e and n , are reduced to cg , dl ; But cg and dl do yet diverge, but not so much as bc , bd .

Definition.

Imaginary Focus.

In the same Figure, draw gc , ld , directly to Cross in x , I call x the *Imaginary Focus*; which is determined by the following Proposition.

PROP.

PROP. VIII.

In Convex-Glasses when an Object is placed nigher the Glass than the Focus, the Rule for finding the Imaginary Focus is this,

As the Difference between the Distance of the Object from Glass, and the Glasses Focus :

Is to the Glasses Focus ::

So the distance of the Object from the Glass :

To the Distance of the Imaginary Focus from the Glass.

Demonstration.

In Tab. 15. F. 2. Let ed be a Plano-Convex-Glass, whose absolute Focus is a Diameter of the Convexity, let sd be this Diameter; a is a radiating Point, f the Center of the Convexity $fe (=fd)$ the Radius of the Convexity produced to q , ae a Ray falling on the Glass produced directly to i .

We here suppose the Thickness of the Glass to be inconsiderable; and also, that the Breadth of the Glass is so little, that the Angles of Incidence shall be so small, that Sines and Angles may be proportional.

Let it then be made according to the Rule, $as:sd::ad:dk$. I say kd is the Distance of the Imaginary Focus of this Ray ae , after it has passed the Glass: That is, it shall be refracted, and proceed onwards in eb , as if it came directly from the Point k , keb being a strait Line.

Let kl be made $=\frac{1}{2}kd$. I shall first Demonstrate, that by virtue of the first Refraction the Ray suffers at its Entrance on the Convex-Side of the Glass at e , 'tis refracted so as if it proceeded directly from l , that is to say, 'tis refracted into eg , leg being a Right Line.

'Tis

'Tis first manifest, that the Angle of Incidence is qea ; if therefore we prove, that the Angle of Refraction $ieg = lea$ is $\frac{1}{2}$ of qea , we shew hereby, that by the first Refraction, the Ray is broken as if it proceeded directly from l .

That we may prove this, add to the Consequents of the foresaid Analogy their Halfs, — — — — —

And it shall be — — — — —

Inverting, and Dividing this Second — — — — —

That is by the Scheme — — — — —

Moreover in the Triangle $ae f$ it is

But $s\angle fea = s\angle qea$ being Complements to 180° — — — — —

And in these small Angles, Sines

and Angles being proportional,

the 6th shall be — — — — —

Subtriple the Antecedent on this

side, and triple the Consequent

on t'other — — — — —

Then from the 4th and 8th Steps it

follows — — — — —

Moreover in the Triang. ela it is

But $s\angle lae = s\angle eaf$ being Complements to 180° — — — — —

And Signs and Angles being pro-

portional — — — — —

From the 12th and 9th Steps 'tis

manifest that — — — — —

$$1 \quad as : sd :: ad : dk$$

$$2 \quad as : sf :: ad : dl$$

$$3 \quad sf - as : sf :: dl - ad : dl$$

$$4 \quad af : sf :: al : dl$$

$$5 \quad af : ef :: s\angle fea : s\angle eaf$$

$$6 \quad af : ef :: s\angle qea : s\angle eaf$$

$$7 \quad \angle qea : \angle eaf :: af : ef$$

$$8 \quad qea : eaf :: af : 3ef = sf$$

$$9 \quad qea : eaf :: al : dl$$

$$10 \quad la : le = ld :: s\angle lea : s\angle lae$$

$$11 \quad la : ld :: s\angle lea : s\angle eaf$$

$$12 \quad al : dl :: \angle lea : \angle eaf$$

$$13 \quad qea : eaf :: lea : eaf$$

Wherefore from the 13th Step 'tis evident that $\frac{1}{2} qea$ is equal to lea , they having the same Proportion to the same Angle eaf .

H

Which

Which was the first thing to be proved: and consequently 'tis manifest that the Ray ae , by its first Refraction at its Incidence on e , is refracted into eg , as if it came directly from l , leg being a Right Line.

It remains to be Demonstrated secondly, that the Ray upon its Eruption on the plain side of the Glass into Air is refracted into eh , as if it proceeded directly from k , keh being a Right Line.

Draw mer Parallel to the Axis lf . Now the Angle of Incidence from Glass to Air is $ger = mel = ela$. — The Angle of Refraction is $geh = lek$, which I shall prove to be half the Angle of Inclination mel ; Or I shall prove that the refracted Angle $mek = reb$ is thrice the Angle of Refraction lek .

Which being Demonstrated, 'tis certain the Ray is refracted into eh , as if it came directly from k , and consequently k is the Imaginary Focus of the Ray ae . But this we shall thus Demonstrate.

In the Triangle lek —————	14 $le = ld : lk :: s\angle lke : s\angle lek$
But $s\angle lke = s\angle ekf$ being Com-	15 $ld : lk :: s\angle ekf : s\angle lek$
plements to 180° . —————	
But $\angle ekf = \angle mek$ therefore, the	
15th is —————	16 $ld : lk :: \angle mek : lek$

And by Construction ld was put equal to $3lk$, therefore from this 6th step 'tis manifest, that the Angle mek is equal to Thrice the Angle lek . Which was to be Demonstrated.

Corollary.

Concerning Convex-Glasses exposed to Converging Rays.

Hitherto we have spoken of Convex-Glasses exposed either to Parallel or to Diverging Rays. Let us now consider their

T. 15. F. 2. Property exposed to Converging Rays. Tab. 15. Fig. 2. Let he be a Ray of Light falling on the Convex-Glass ed , and converging

verging directly towards the point k ; dk the distance of this Point of Convergence from the Glass, and sd the Focal length of the Glass being given, 'tis required to find da , the distance at which this Ray converges after passing the Glass.

We know that the Progress of a Ray is reciprocal (by Exp. 8.) and therefore, that if the Ray ae be refracted into eb , the Ray be shall be refracted into ea .

Wherefore it being by Prop. VIII. — — — $ds - da = as : sd :: ad : dk$
 Alternating we have it — — — — — $sd : dk :: ds - da : ad$
 And Compounding — — — — — $sd + dk : dk :: ds : ad$

From which last Analogy this Rule arises, for solving the Problem in the Corollary.

As the Sum of the Focal Length of the Glass and the distance at which the strait Ray converges directly — — — $ds + dk$:

To the said Distance at which the strait Ray converges dk ::

So the Focal Length of the Glass — — — ds :

To the Distance at which the refracted Ray converges — — — ad :

Observation.

Dechales in his first Book of Dioptricks, Prop. 48. gives a Rule for solving this 8th Proposition: But therein he is the most egregiously mistaken as ever Man was, that pretended to Demonstration, and commits the most notorious Error that can be imagined. And yet herein he wants not his Followers, for Zahn in his *Telescopium Fundam.* 2. Syntag. 1. Cap. 4. Prop. 13 transcribes Dechales, and copies out even his *Errata Typographica*, besides the chief and great Error of the whole Solution. Which shews that Zahn was either very careless, or else that he understood nothing of the Matter, as indeed he seems to do very little, for he is a mere blind Transcriber from others.

Tab. 15.
Fig. 2.

Dechales Rule in his fore-cited Proposition is this. *As the Sum of the Distance of the Object from the Glass, and half the Glass's, Focus: Is to the Glass's Focus :: so the Distance of the Object from the Glass: to the Distance of the Imaginary Focus beyond the Distance of the Object:* That is, in Tab. 15. F.2. $af:sd::ad:ak$. But how false this is, shall appear not only from his mistaking one Angle for another in his Demonstration, but from the following Examples, which I have taken the pains to calculate for a more full Confutation of Dechales, and Illustration of my own Rule.

1. Example.

Inch.

Given in T. 15. F.2. $\left\{ \begin{array}{l} \text{Plan.-Conv. Gl. Foc. } 144,00 = sd = 2ef \\ \text{Rad. of the Convexit. } 72,00 = ef = fd \\ \text{Distance of the Object } 48,00 = ad = az \\ \text{Breadth of the Glass } 1,00 = ez \end{array} \right.$

By my Rule $\left\{ \begin{array}{l} dk = 7200 \\ kl = 3600 \\ dl = 10800 = dk + kl \\ al = 6000 = dl - da \end{array} \right.$

By Dechal. Prop. 48. $\left\{ \begin{array}{l} dk = 10560 \\ kl = 5280 \\ dl = 15840 = dk + kl \\ al = 11040 = dl - da \end{array} \right.$

Tab. 15.
Fig. 1.

Let us now try by Calculation, which of these is the Truth. Tab. 15. Fig. 2.

In the Right-angled Triangle efz , we have the Sides ef and ez , to find the Angle $efz = 0^\circ 47' 40''$ and its Complement to 90° $fez = 89^\circ 12' 20''$.

Then in the Right-angled Triangle $ea z$, we have az , and ez to find the Angles and Side ae .

Thus

Thus $\angle eaz = 1^\circ 11' 40''$
 $\angle aez = 88^\circ 48' 20''$
 $ae = 4801$

Then $aez + fez = aef = 178^\circ 0' 40''$

And Compl. aef to $180^\circ = qea = \text{Incl.} = 1^\circ 59' 20''$

Then as $3 : 1 :: \angle qea : \angle ieg = lea = 0^\circ 39' 50''$ Ang. of Ref.

Moreover, $eal = 180^\circ - eaz = 178^\circ 48' 20''$

And $ela = 180^\circ - eal - lea = mel = ger = 0^\circ 31' 50''$

Then in the Triangle eal , we have two Angles, lea and ela , and the Side ae , to find the Side al . Thus,

As $s \angle ela = 0^\circ 31' 50''$ Log. C. Ar. 2.0333979726

To $ae = 4801$ — — — — — 3.6813317060

So $s \angle lea = 0^\circ 39' 50''$ — — — — — 8.0639630497

To $al = 6007$ — — — — — 3.7786927283

which is sufficiently agreeable to my $al = 6000$, but differs vastly from Dechales $al = 11040$.

I now proceed to find the Side dk , which by my Rule is 7200 , but by Dechales is 10560 .

$\angle qez = qea + aez = 1^\circ 59' 20'' + 88^\circ 48' 20'' = 90^\circ 47' 40''$

$\angle qem = qez - 90^\circ = 0^\circ 47' 40''$

$\angle mel = ale = ger = 0^\circ 31' 50''$

This is the Angle of Inclination or Incidence from Glass to Air, on the plane side of the Glass.

Then, As $2 : 3 :: \angle ger = 0^\circ 31' 50'' : \angle her = mek = ekd = 0^\circ 47' 40''$.

And $kez = 90^\circ - ekd = 89^\circ 12' 20''$.

Then in the Triangle kez , all the Angles, and the Side ez are given, to find kd or kz (we supposing the Thickness of the Glass inconsiderable). Thus,

Rad. — — — — —

To $ez = 100$ — — — — — Log. 2.0000000000

So Tang. $\angle kez = 89^\circ 12' 20''$ 11.8580312668

To $kd = 7208$ 3.8580312668

Which

Which $k d$ answers sufficiently exact to what my Rule gives it 7200, the Eight Parts coming in for the Thickness of the Glass, and the Angles not being calculated more accurately than to 10" Seconds *Numero rotundo*. But this is much different from what *Dechales* Rule gives it 10560.

Example.

Given in *Tab. 15. F. 2.* { Glass's Focus = 14400 = $s d = 2 e f$
 { Rad. of the Conv. = 7200 = $e f = f d$
 { Dist. of the Object = 5763 = $a d = a z$
 { Bread. of the Glass = 100 = $e z$

By my Rule { $d k = 9608$
 { $k l = 4808$
 { $d l = 14412 = d k + k l$
 { $a l = 8549 = d l - a d$

By *Dechales*, 48 Prop. { $d k = 12165$
 { $k l = 6082$
 { $d l = 18247 = d k + k l$
 { $a l = 12484 = d l - a d$

The Thickness of the Glass $z d$ is but $\frac{1}{7200}$ and therefore inconsiderable, it being the Versed Sine of the Angle $e f z$. We suppose therefore $k d$ and $k z$ equal.

Then by Calculation, *ex. Datis*, we find as follows,

1	$e f z =$	0° 47' 40"
2	$f e z =$	89 12 20
3	$e a z =$	0 59 40
4	$a e z =$	89 00 20
5	$a e =$	5764
6	$a e f =$	178 12 40

$$7 | q e a = 1^{\circ} 47' 20'' = \text{Inclination}$$

$$8 | l e a = 0^{\circ} 35' 40'' = \text{Refraction}$$

$$9 | e a l = 179^{\circ} 0' 20''$$

$$10 | e l a = 0^{\circ} 24' 0''$$

$$11 | a l = 8566$$

Which $al = 8566$ is but 17 more than my Rule gives it, which (considering the Inaccuracy of the Calculation of the Angles, being only *Numero rotundo* to about 10" Seconds) proves my Rule sufficiently true, and shews *Dechales* as false, which makes $al = 12484$.

Moreover, By my Rule dk is $= 9608$, and by *Dechales* 12165. The Angles requisite to find dk we have thus,

Before found—	$1 q e z = 90^{\circ} 47' 40''$
	$2 q e m = 0^{\circ} 47' 40''$
	$3 a l e = 0^{\circ} 24' 0'' = g e r = m e l = \text{Inclina}$
	$4 m e k = 0^{\circ} 36' 0'' = h e r = e k d.$
	$5 k e z = 89^{\circ} 24' 0''$
	$6 k d = k z = 9550$

Which $kd = 9550$ is but 58 less than my Rule gives it; and this small Discrepancy may easily arise from the Error of 10" Seconds in the Calculation of the Angles. But *Dechales* gives it 12165.

I have chosen a Plano-Convex Glas to demonstrate this Rule, which holds as true in double Convexes, but would in them be more difficult and intricate to demonstrate.

PROP. IX.

In double Convex-Glasses of equal or unequal Convexities, the Rays proceeding from the Distance of a Diameter of one Convexity, are united at the Distance of a Diameter of the other Convexity.

T15. F3.

In Tab. 15. Fig. 3. Let us conceive the Glass ab divided by the Pointed Right Line ab into two Plano-Convex Glasses acb and adb . Let the Point k be distant from the Glass acb the Diameter of the Convexity acb . By what foregoes, Prop. II. The Rays $ka, ke, kc, kp, &c.$ shall be sent parallel through the Glass acb , so that they shall become eg, cd, pq . Then falling Parallel on the Plano-Convex Glass adb , they are united thereby in f , at the distance of its Diameter: By the foregoing Doctrine. Prop. II.

Of Concave-Glasses.

Hitherto we have treated of Spherical Convex Glasses only. We now proceed to the Consideration of Concave-Glasses.

Definition.

Virtual Focus, or Point of Divergence.

T15. F5.

In Tab. 15. Fig. 5. abc is a Plano Concave Glass, whose Axis is de , fg is a Ray falling thereon parallel to the Axis de , d is the Centre of the Arch abc . This Ray after it has passed the Glass at its Emerision at g , instead of proceeding directly to h , is refracted from the Perpendicular dg , and becomes gk . Draw gk directly to cross the Axis in e . I call the Point e the *Virtual Focus*, or *Point of Divergence*.

Concerning

Concerning Concaves, there is little to be said after our foregoing Method for Calculating the Progress of a Ray falling on a Spherical Convex; for this may easily be accommodated to Concaves.

I shall therefore slightly pass these over, and shall only lay down their Properties in brief, with their usual Demonstrations, referring to Calculation for a more accurate Scrutiny.

P R O P. X.

A Ray falling parallel to the Axis from Air, on the Concave Surface of a Glass Medium, has its Virtual Focus by its first Refraction at the Distance of a Diameter and half of the Concavity.

Tab. 15. Fig. 4. *abkb* is a Glass Medium terminated by the Concave Surface *ab*, *de* a Ray of Light falling parallel to the Axis *gcx*. I say the Ray *de*, by its Refraction at the Point of Incidence *e*, is so refracted into *eb*, as if it proceeded directly from *g* the distance of a Diameter and half of the Convexity.

Let *c* be the Centre of the Concave-Arch *ae b*, draw *ce* directly to *i*, produce *de* directly to *f*, and *be* directly to *g*. The Angle $ced = fei$ is the Angle of Inclination, the Perpendicular is *ei*. By what foregoes, the Ray passing from Air a Rare Medium into Glass a Dense Medium, is refracted towards the Perpendicular by the Angle feb , which must be $\frac{1}{2}$ of the Inclination fei . Therefore $feb = deg = egc$ is half $bei = gec$. Wherefore in the Triangle *ceg*, the Angle *ceg* being double the Angle *egc*, the Side *cg* that subtends *ceg*, shall be double the side *ce* that subtends *egc*. For we suppose these Angles to be so small, that Sines and Angles, or Sides and Angles are proportional. Which was to be Demonstrated.

PROP. XI.

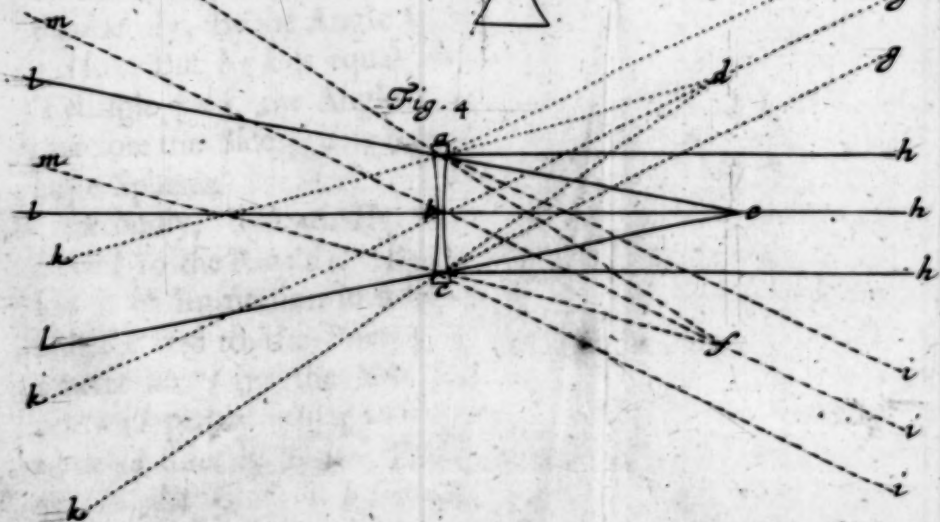
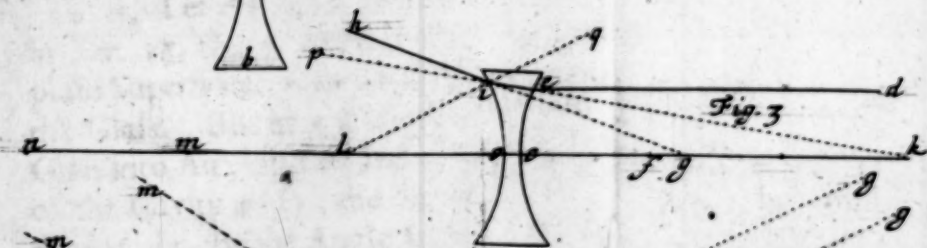
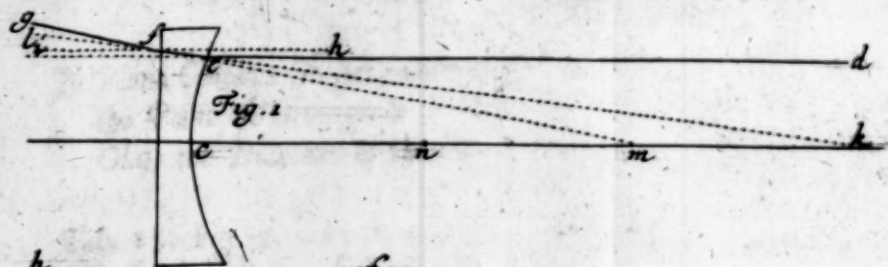
In Plano-Concave Glasses, when the Rays fall parallel to the Axis, the Point of Divergence or Virtual Focus is distant from the Glass the Diameter of the Concavity.

T.15.F.5. *Tab. 15. Fig. 5.* abc is a Plano-Convex Glass, whose Axis is ed , fg is a Ray of Light falling on this Glass parallel to its Axis. This Ray after it has passed the Glass at g , is so refracted into gk , that gk being produced directly backwards, it shall intersect the Axis in e , so that eb shall be the Diameter of the Cavity abc .

First, Let the plain Side be turned towards the Object, as in *Tab. 15. Fig. 5.* Because the Ray falls perpendicular on the plain Superficies, it is not at all refracted at its Immersion into the Glass. But at g it emerges from the Concave Side of the Glass into Air; and its Inclination is gdb (d being the Centre of the Cavity abc) and 'tis now refracted from the Perpendicular dg , by the Angle $h g k$, which is half the Inclination gdb . But $h g k$ is equal to $f g e = ged$. Wherefore in the Triangle gde , the Angle ged is half the Angle gde , and therefore the Side gd is half the Side $ge = eb$ in Glasses of large Spheres.

T.16.F.1. Secondly, *Tab. 16. Fig. 1.* Let the Concave Side ec be turned to the Ray de . By the first Refraction the Ray suffers at its Immersion into Glass at e , 'tis refracted from e to f , and directed to the Point k , so that ck is thrice the Semidiameter nc (by the Xth hereof). Through the Point f draw ib perpendicular to the plain Side of the Glass, and continue ef directly to l . The Angle of Inclination of the Ray within the Glass is $bfe = ifl = dek = ekc$. Now the Ray emerging from Glass to Air, the second refracted Ray ought

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ought to recede from the Perpendicular if , so that the Angle gfi must be thrice the Angle gfl . Produce gf directly to m . Then gfi shall be equal to $fm c$. Wherefore in the Triangle kfm ; As the Sine of the Angle $k m f$, or the Sine of the Angle $f m c$: To the Sine of the Angle $f k m$:: So $f k$: To $f m$, and (neglecting the Thickness of the Glass) So is $k c$: To $m c$. And Angles and Sines being proportional, $k c$ shall be to $m c$ as 3 to 2; but $k c$ is Thrice the Semidiameter, therefore $m c$ is the Diameter. Which was to be Demonstrated.

Corollary.

In Plano-Concave Glasses, as $300 - 193 = 107 : 193 ::$ So the Radius of the Concavity: To the Distance of the Virtual Focus.

Demonstration.

Tab. 15. F. 5. The Inclination is $h g d = g d b = f g z = 193$. Tab. 15. F. 5. Refracted Angle is $z g e = k g d = 300$.
 Ang. of Refr. $k g h = k g d - h g d = 300 - 193 = 107 = f g e = g e d$.
 In the Triang. $g d e$. As $s \angle g e d$: To $s \angle g d e$:: So Rad. $= g d$: To $g e$.
 That is ———— $107 : 193 :: g d : g e = e b$ in Glasses of large Spheres and small Segments.

PROP. XII.

In Double Concaves of equal Cavities, Parallel Rays have their Virtual Focus, or Point of Divergence at the Distance of the Radius of the Concavity.

Tab. 16. F. 2. ab is a Concave of equal Cavities, that is, Tab. 16. Fig. 2. $m c$ the Radius of the Cavity $e c$, is equal to $g n$ the Radius of the Cavity $n i$. Let the Ray $d e$ fall thereon parallel to the Axis $k c$. This Ray, after it has suffered a double

double Refraction, shall proceed in ib , as if it came directly from m , the Centre of the Cavity.

Let ck be made three times cm , draw $keil$. By the first Refraction the Ray is propagated into ei , as if it proceeded from k (by Prop. X.). From the Centre g draw the Perpendicular gif . The Angle of Inclination of the Ray within the Glafs is $eif = lig$. The Ray must therefore at its Egress at i be refracted, so that the Angle of Refraction $hil = kim$ must be half the Inclination lig . Wherefore in the Triangle kig , As the Sine of the Angle kig , or of the Complement to 180° viz. lig : To the Sine of the Angle at g :: So $kg = 4$: To ik or $nk = 3$. And as the Sines, so are the Angles, seeing they are supposed very acute: Therefore, As 4 : To 3 :: So $\angle lig$: To the $\angle g$: Add $hil = 2$ to $lig = 4$, we have $big = 6$. Wherefore $\angle big$: Is to the Angle g :: As 6 : To 3 :: or 2 : To 1 . But the Sine of the Angle big is equal to the Sine of the Angle gim ; and consequently in the Triangle gim , As Sine of the Angle gim : To the Sine of the Angle g :: So gm : To im . Therefore gm : Is to im :: As 2 : To 1 . So that neglecting the Thickness of the Glafs, mi may be taken for the Radius of the Cavity, and gm is double thereto. Wherefore tis Demonstrated, that m is the Virtual Focus. Which was to be Demonstrated.

P R O P. XIII.

In double Concaves of equal or unequal Concavities, the Virtual Focus or Point of Divergence of the Parallel Rays is determined by this Rule :

As the Sum of the Radii of both Concavities :

Is to the Radius of either Concavity ::

So the Double Radius of t^h other Concavity :

To the Distance of the Virtual Focus.

Tab.

Tab. 16. Fig. 3. fc is the Radius of the Concavity ec , make kc thrice fc , lo is the Radius of the Cavity oi ; let lm be put equal to lo , and mn be put $=lm$, that is, on shall then be equal to thrice ol .

I say the Ray de , after a double Refraction, shall proceed in ib , as if it came directly from g , and that it shall be, As fl the Aggregate of the Semidiameters (neglecting the Thickness of the Gläs): To fc the Radius of one Concavity :: So om the Diameter of t'other Concavity: To gc the Distance of the Point of Divergence.

By the first Refraction the Ray proceeds in ei , as if it came directly from k , $keip$ being a Right Line (Prop. X).

Then the Inclination within the Gläs is $qie=pil$. And the second Angle of Refraction is $pih=gik$, which is to be half the Angle of Inclination pil .

Wherefore Sines and Angles being proportional, and the Thickness of the Gläs neglected, we thus proceed in the Demonstration.

In the Triang. gik we have it — — — — 1 $gik=hip:gki::kg:gi=gc$

Double the Antecedent on this side, and halve the Consequent on t'other — 2 $gik=pil:ikl::kg:\frac{1}{2}gc$

Dividing the Second — 3 $pil-ikl(=ilk \text{ 32. 1 Eucl.}):ikl::gk-\frac{1}{2}gc:\frac{1}{2}gc$

Then in the Trian. kli — 4 $ilk:ikl::ki=kc:il=lo$

From the Third and Fourth it follows — — — 5 $kc:ol::kg-\frac{1}{2}gc:\frac{1}{2}gc$

Compounding the Fifth — 6 $kc+ol=kl:ol::kg:\frac{1}{2}gc$

Double the Consequents — 7 $kl:2ol=om=ln::kg:gc$

Compound. the Seventh — 8 $kl+ln=kn:om::kg+gc=kc:gc$

Alternate the Eighth — 9 $kn:kc::om:gc$

But kn is equal to $3fl$

that

that is, thrice the Aggregate of the Semidiameters fc and lo . Also kc is equal to thrice the Semidiameter fc : and as the Triples are, so are their Thirds. Wherefore from the 9th. Step it follows —

$$10fl : fc :: om : gc.$$

Which was to be Demonstr.

Example.

Let fc be = 8 Feet or 96 Inches

lo = 10 Feet, or 120 Inches.

Then fl shall be = to 216 Inches.

Say then $fl = 216 : fc = 96 :: 240 = 2lo : 107 = gc$.

Of thus, $fl = 216 : lo = 120 :: 192 = 2fc : 107 = gc$.

Corollary.

It follows from the four last Propositions, that supposing a Ray falling on a Concave Glass, and tending towards the virtual Focus, after it has passed the Glass, it shall proceed Parallel to the Axis. Because the Progress of Light through Glasses is reciprocal. *Vid. Prop. VI.*

Scholium.

It is before Demonstrated in *Prop. IV.* that the Focus or Point of Convergence of a Convex-Glass, for the Parallel Rays that fall *obliquely* thereon is at the same Distance, as the Focus of the *direct* Parallel Rays. The same may be Demonstrated concerning the Virtual Focus or Point of Divergence in Concave Glasses; but 'tis sufficient only to illustrate this Matter by *Tab. 16. Fig. 4.* Let us imagine the uppermost Point of a distant Object

Tab. 16.

Fig. 4.

Object to send the Parallel Rays ga, gb, gc , on the Concave-Glass abc , and its middle Point to send the Rays ba, bb, bc ; and the lowermost Point to send the Rays ia, ib, ic . Here are therefore three Radious Cones, having their Vertices in their respective Points of the Object, and their Bases on the Surface of the Concave-Glass: the Axes of these Cones, are $gdbk, hebl, ifbm$: These, by what foregoes preliminary to the Prop. IV. may be conceived to pass the Glass as it were unrefracted. But the other Rays of each Cone are refracted; that is, ga is so refracted at a into ak , as if it proceeded directly from d the Virtual Focus or Point of Divergence for this oblique Cone. And so gc is refracted at c into ck , as if it proceeded directly from d . And in like manner ba, bb, bc , are refracted into al, bl, cl , as if they came from e . And the Rays ia, ib, ic , are propagated in am, bm, cm , as if they proceeded directly from f . So that def is the Virtual Focus of the Glass exposed to the Object, that sends these direct and oblique Cones of Rays.

Converging Rays.

The Properties of Concaves exposed to *Converging* Rays are shewn in these two next following Propositions.

P R O P. XIV.

In Concave Glasses, if the Point to which the incident Ray converges, be distant from the Glass farther than the Virtual Focus of Parallel Rays, the Rule for finding the Virtual Focus of this Ray, is this:

As the difference between the distance of this Point from the Glass, and the distance of the Virtual Focus from the Glass:

Is to the distance of the Virtual Focus ::

So the distance of this Point of Convergence from the Glass:

To the distance of the Virtual Focus of this Converging Ray.

Tab.

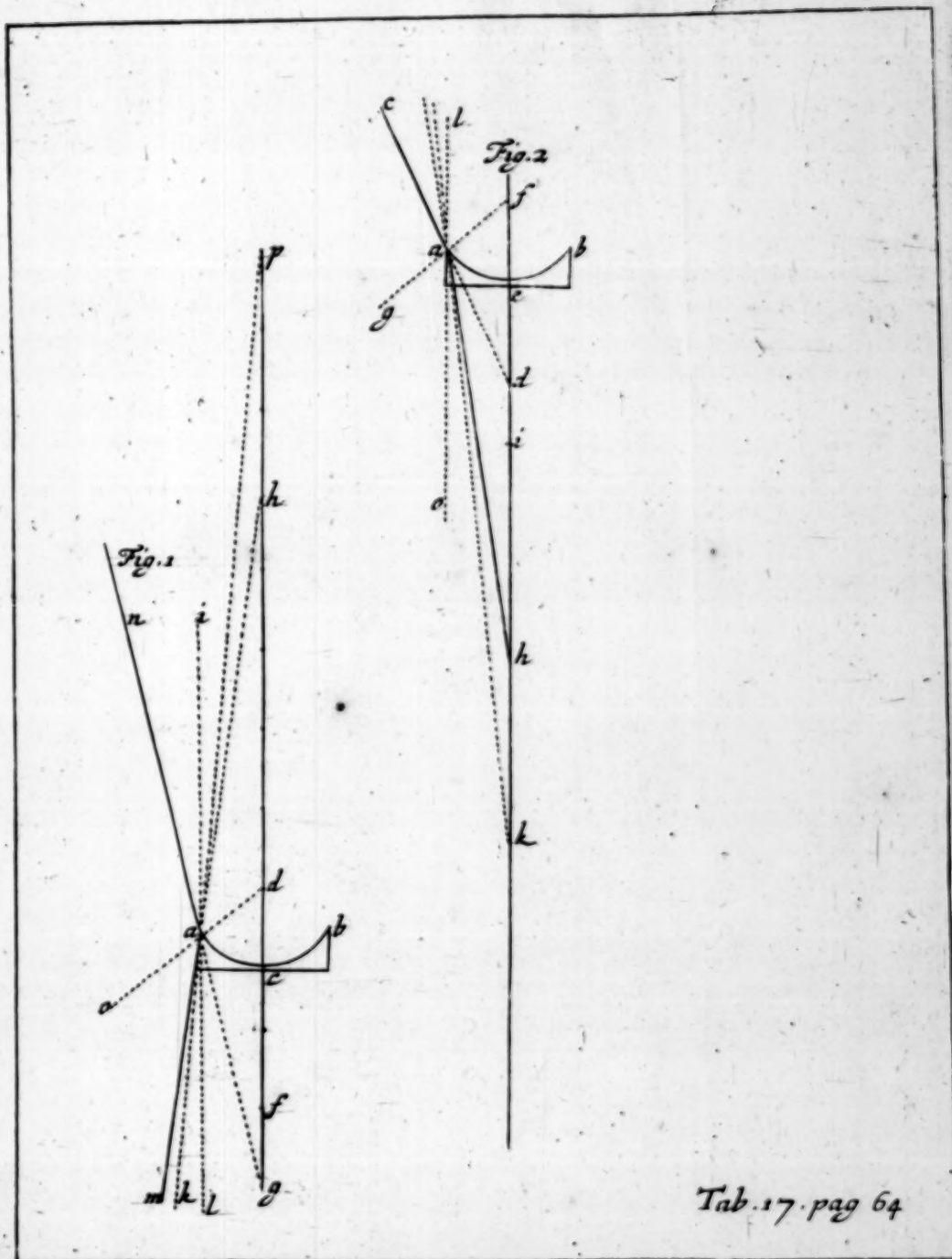
T. 17. F. 1. *Tab. 17. Fig. 1.* ab is a Plano Concave Glass, d the Centre of the Concavity, dao a right Line, cf the length of the Virtual Focus, which (by what foregoes, *Prop. XI.*) is equal to $2ad$ or $2dc$; so that fd is equal to $3dc$ or $3ad$. Let na be a Ray of Light converging directly towards the Point g . If it be made according to the Rule in the Proposition, $gf:fc::gc:cb$. Then b shall be the Virtual Focus of the Ray na , that is the Ray na after the double Refraction it receives by the Glass shall proceed onwards in am , as if it came directly from b , mab being a Right Line.

The Proof hereof is after the same manner with the *Prop. V.* beforegoing; and we suppose (as in that) the Thickness of the Glass to be nothing in Comparison to the Focal Length; and likewise that the Angles of Incidence are so small, that they are as the Sines; also that the Breadth of the Glass is so little, that it makes the Angles agc , adc , abc , apc , &c. very small and Proportionable to their Sines.

Demonstration.

Let ph be made half bc , and consequently the third Part of pc . I say by virtue of the first Refraction which the Ray suffers at its entrance into Glass at a , 'tis refracted into ak , as if it proceeded directly from p , pak being a Right Line.

To prove this we must observe that the Angle of Inclination or Incidence is $nad = oag$. If therefore it be proved, that the Angle of Refraction $gak = nap$ is $\frac{1}{3}$ of the Angle nad , our first Position will be manifest, *viz.* That the Ray by the first Refraction is bent into ak .



Tab. 17. pag 64

First therefore it is by Position 1 $gf:fc::cg:ch$
 Add to the Consequents their

Halves — — — — — 2 $gf:fd::cg:cp=pa$

Compounding the second — 3 $gf+fd=gd:fd::cg+cp$
 $=gp:pa$

Also in the Triangle gad — 4 $gd:da=dc::s\angle dag=$
 $s\angle nad:s\angle g$

And Sines and Angles being
 proportional, it follows from
 the 4th. — — — — — 5 $\dagger nad:\angle g::gd:3ad=fd$

It then follows from the 3th.
 and 3d. — — — — — 6 $\dagger nad:\angle g::gp:pa$

Moreover in the Triangle gap 7 $gp:pa::s\angle gap=s\angle nap:$
 $s\angle g$

And Sines and Angles being
 proportional — — — — — 8 $gp:pa::\angle nap:\angle g.$

From the 6th and 8th it follows 9 $\dagger nad:\angle g::\angle nap:\angle g.$

From which 9th Step 'tis manifest that the first Angle of
 Refraction, nap , is equal to \dagger of the first Angle of Incidence
 nad . Which was first required to be Demonstrated.

We are secondly to Demonstrate, that by virtue of the se-
 cond Refraction the Ray receives on the plane side of the Glas
 at its Egress into Air, 'tis refracted into ma , as if it came direct-
 ly from b . Draw ial Parallel to the Axis bg , and Perpen-
 dicular to the plane side of the Glas; The Angle of Incidence
 is $lak=iap$; the Angle of Refraction is $mak=hap$; the re-
 fracted Angle $mal=hai$.

We are therefore to prove, that the refracted Angle mal is
 equal to triple the Angle of Refraction mak .

In the Triangle pab we have this Analogy $pa=pc:pb::$
 $s\angle ahp=s\angle abc=s\angle hai=s\angle mal:s\angle hap=s\angle mak$.
 But pc is thrice pb , therefore (Sines and Angles being proporti-

onal) the Angle $m a l$ is thrice the Angle $m a k$. Which was to be Demonstrated.

P R O P. XV.

Tab. 17.
Fig. 2.

In Concave Glasses $a b$ (T. 17. F. 2.) if the Point d to which the Incident Ray $c a$ converges, be nigher to the Glas than the Virtual Focus of Parallel Rays i ; the Rule to find where it crosses the Axis at h , is this,

As the Excess of the Virtual Focus more than this Point of Convergency $i d$:

To the Virtual Focus $i e$:

So the distance of this Point of Convergency from the Glas $d e$:

To the distance of the Point where this Ray crosses the Axis $h e$.

In the forementioned Figure $a b$ is a Plano-Concave Glas, f the Center of the Cavity $i e$ the Length of the Virtual Focus of Parallel Rays, which we know is a Diameter of the Cavity, that is, $= 2 f e$. On this Glas there falls the Ray $c a$ converging directly to the Point d . If it be made $i d : i e :: d e : h e$, I say h is the Point in the Axis, where the Ray $c a$, after its double Refraction, crosses it.

Demonstration.

For the Proof thereof we suppose all that we laid down as supposed in the last Proposition. Draw $f a$ directly to g , and make $h k$ equal to $\frac{1}{2} b e$. I shall first shew, that by virtue of the first Refraction, which the Ray suffers at a , 'tis refracted directly towards k . Here the first Angle of Incidence is $c a f$, $= g a d$. If therefore we prove that $k a d$ is equal to $\frac{1}{2} c a f$, our first Position shall be manifest, that the Ray is refracted first towards k .

For

For by the Rule it is — — —	1 $id: ie:: ed: eb$
Add to the Consequents their	
Halves — — — — —	2 $id: if:: ed: ek$
Inverting and dividing the	3 $if-id=df: if:: ek-ed$
Second	$= dk: ek$
Moreover in the Triang. dfa	4 $df: af:: s\angle fad=s\angle caf:$
	$s\angle adf$
From the 4th Step it follows	5 $\angle caf: \angle adf:: df: af$
	$= if$
By the 3d and 4th it is — —	6 $\angle caf: adf:: dk: ek$
Also in the Triangle akd —	7 $kd: ka=ke:: s\angle kad:s\angle kda$
	$= s\angle adf$
Sines and Angles being prop.	8 $kd: ke:: \angle kad: \angle adf.$
From the 6th and 8th it follows	9 $\angle caf: adf:: kad: adf$

Wherefore from this 9th Step 'tis manifest that kad is equal to $\angle caf$, which was first to be Demonstrated.

I shall next demonstrate, that by virtue of the second Refraction the Ray proceeds in ab .

Draw oa Parallel to the Axis fk , and therefore (by Supposition 2d) perpendicular to the plane side of the Glass. Here the Angle of Incidence from Glass to Air shall be oak . I am therefore to prove that the Angle of Refraction kab is equal to $\angle oab$ the refracted Angle. Thus,

In the Triangle kab it is $ka=ke:kb:: s\angle abk=s\angle abe, = s\angle oab: s\angle kab$

And Sines and Angl. being prop. $ke: kb:: oab: kab$

But kb is $\frac{1}{2}$ of ke and therefore the Angle kab , is $\frac{1}{2}$ of the Angle oab . Which was to be Demonstrated.

DIVERGING RAYS.

Corollary.

Concerning Concave-Glasses exposed to Diverging Rays.

The Virtual Focus of a Concave Glass exposed to Parallel Rays is found by Prop. X, XI, XII, XIII. And the Properties of a Concave exposed to Converging Rays are shewn in Prop. XIV. and XV.

Let us now consider a Ray Diverging from the Axis, as *T. 17. F. 2.* suppose a right Object (*T. 17. F. 2.*) in *b*, whose Ray *ba* diverges from the Axis *be*, and so falls on the Concave Glass *ab*. Let us conceive this right Object *b* either farther from, or nigher to the Glass, than the Glasses Virtual Focus. This Ray after passing the Glass diverges more, than before it entred it. And the Rule for determining the Point *d*, from which this Ray after passing the Glass does as it were directly Diverge, is this,

As the Sum of the distance of the Object from the Glass and the Glass's Virtual Focus:

To the Distance of the Object from the Glass::

So the Glasses Virtual Focus:

To the distance of this Point of Divergency from the Glass:

Demonstration.

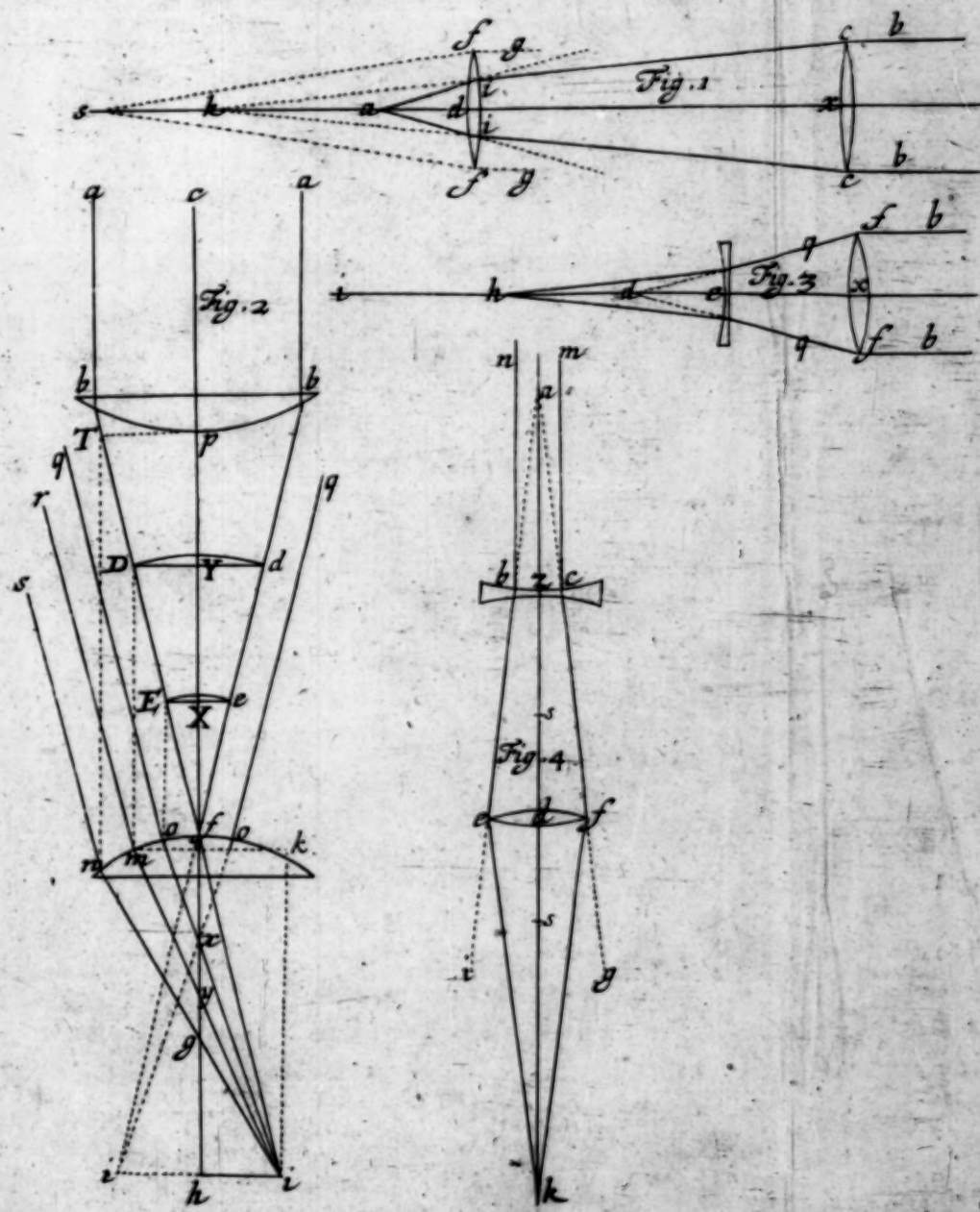
By the foregoing XVth Prop. — $id: ie:: de: eb$

And Alternating — — — — $ie: eb:: id (= ie - de): de$

And Compounding — — — — $ie + eb: eb:: ie: de$

Which last Analogy gives the forementioned Rule in Words as is manifest by comparing it with the Scheme.

PROP.



PROP. XVI. PROBL.

The Focal Lengths of two Convex-Glasses being given, the longer being placed before the shorter at any given Distance less than the longer Focal Length; 'Tis required to determine the Distance of the Distinct Base from the shorter Glass.

This is solved from the VIII. Proposition hereof.

Tab. 18. Fig. 1. cxc is a Convex-Glass uniting the Parallel Rays bc, bc , in its Focus at k ; fdf is a shorter Convex uniting the Parallel Rays gf, gf , in its Focus at s . In the Problem, kx the longer Focal Length, and sd the shorter Focal Length, and dx the Distance of the Glasses, are given; To find da , the Point wherein the Rays ci, ci , are united with the Axis by the Glass ff . T. 18, F. 1

By the 8th. Proposition preceding, the Focal Length ds of the Convex-Glass ff being given, and the Distance of an Object da being given less than the Focal Length ds , to find the Distance dk of the Imaginary Focus; the Rule is, As the Difference between the Distance of the Object da , and the Focal Length ds , that is, $ds - da = as$: To the Focal Length ds :: So the Distance of the Object from the Glass ad : To the Distance of the Imaginary Focus from the Glass dk .

Now whereas in the 8th. Proposition da is given, and dk sought; in this Problem da is sought, and dk is given; for dk is equal to the Difference between the Focal Length of the longer Glass kx , and the Glass's Distance xd .

Wherefore by the foresaid 8th. Proposition, and its Corollary, it being $ds - da = as$: ds :: ad : dk . It shall be also alternando ds : dk :: $ds - da$: ad . And Compounding $ds + dk$: dk :: ds : ad . But in this Problem the Three first Terms of this last Analogy are given, and the last ad required.

Where-

Wherefore by this Analogy, the last Term $a d$ is found. And according to this Analogy we frame this Rule for solving this Problem.

As the Focal Length of the shorter Glass + the Difference between the Focal Length of the longer, and the Glasses Distance :

To the Difference between the Focal Length of the longer and the Glasses Distance ::

So the Focal Length of the shorter Glass :

To the Distance of the Distinct Base from the Glass.

Example.

Let the longer Focal Length be $x k = 20$, the shorter Focal Length $d s = 8$. The Distance of the Glasses $d x = 15$.

Then $x k - d x (= d k) + d s : d k :: d s : a d$

In Numb. $20 - 15 (= 5) + 8 = 13 : 5 :: 8 : 3,07692$.

Schol. 1. The same Solution holds, if the shorter Glass be placed before the longer, and the longer Glass at any Distance from the shorter less than the Focal Length of the shorter.

Let the shorter Focal Length be $= 8$, the longer $= 20$, and the Glasses Distance $= 6$; then $8 - 6 = 2$; and the Proportion is $8 - 6 + 20 = 22 : 2 :: 20 : 1,8182 =$ to the Distance of the Compound Distinct Base (for so I'll call it) of these two Glasses from the last and longer Glass.

Corollary.

From hence is manifest the Rule which is before laid down after our 3d Proposition for Determining the Focus of a double Convex-Glass. The Rule is this, *As the Sum of the Radii of both Convexities : To the Radius of either Convexity :: So the double Radius of t'other Convexity : To the Distance of the Focus.*

For

For Example, Let the Radius of one Convexity be 20, and of t'other 8. Then $28 : 8 :: 40 : 11,4286$. Or, $28 : 20 :: 16 : 11,4286 = \text{Focus}$. Let us then conceive this double Convex-Glass to be divided into two Plano-Convex Glasses, formed on the same Spheres as the two Sides of the double Convex, viz. One on a Sphere whose Radius is 20; and t'other on a Sphere whose Radius is 8; and these two Glasses placed touching each other on their plain Sides, the larger Focal Length would be 40, and the shorter Focal Length would be 16, and the Distance of the Glasses would be *Nothing*. Then by the Rule of the XVI. *Prop.* $16 + 40 = 56 : 40 :: 16 : 11,4286 = \text{To the Compound Focus}$.

Scholium 2.

By this Proposition, and by the Corollaries of *Prop. XXVI.* hereafter, the Eight first Propositions of *Dechales Second Book of Dioptricks* are manifestly solved.

Another Solution of the same Problem.

My esteemed Friend, the Learned and Ingenious Mr. JOHN FLAMSTEED, *Reg. Astron.* has very neatly solved this Problem, and communicated the Solution thereof to me, as follows.

Tab. 18. Fig. 2. Let bpb represent a Convex-Glass, whose Focus is at f ; let ab be two Rays falling upon it parallel to the Axis cf , which shall therefore be collected in f ; let nfk be another Convex-Glass of any different Sphere, whose Focus is h , its Distinct Base ih . 'Tis required to find, at what distance from the Point f , the Distinct Base shall be formed, the Glass nfk being placed any where nearer the Glass bb , as suppose in X , Y , or p .

Through

Through the Points X, Y, p , draw the Lines $E X e, D Y d, T p$, at Right Angles to the Axis. And from the Points E, D, T , wherein they cut the refracted Ray $b f$, draw the Lines $E o, D m, T n$, parallel to the Axis. And from the Points o, m, n , where they fall on the Glas $n f k$, draw the Lines $o q, m r, n s$, parallel to the refracted Ray $b f$; these represent this Ray $b f$ falling upon the Glas $n f k$ placed at X, Y, p .

Produce $b f$ through the Glas to the Distinct Base, which it cuts in i , we may now consider $q o, r m, s n$, as three parallel Rays proceeding from the same Point of an Object infinitely distant, which by the known Properties of Glasses, shall be collected into the same Point i of the Distinct Base with the Ray $b f$. Draw therefore the Lines $o i, m i, n i$, intersecting the Axis in the Points x, y, g . Now, because the Focus is that Point, where the Ray falling upon the outward Glas is by the second Glas collected or made to converge with the Axis, the Points x, y, g , shall be the Foci sought; that is, $f x$ shall be the Compound Focal Length for the Glas $n f k$ placed in X , $f y$ when 'tis placed in Y ; and $f g$ when 'tis placed in p , or when both the Glasses touch.

Now that we may find these Focal Lengths $f x, f y, f g$, draw $m k$ parallel to the Distinct Base $i b i$, and $i k$ parallel to the Axis $f b$. We suppose the Thickness of the Glasses to be as little as possible, and therefore the Lines representing the Rays, passing through them, are *strait*, and not at all bent.

Wherefore the Triangles $D Y f, b i f$ are similar. Also the Triangles $m k i, m f y$ are similar.

It shall therefore be — — —	1	$f Y : f b :: Y D (= m f) : b i$
Converting and Compound.	2	$f Y + f b : f Y :: m f + b i : m f$
But by the Scheme — — —	3	$m f + b i = m k$
Therefore the 2d is thus —	4	$f Y + f b : f Y :: m k : m f$
And by the sim. $\Delta m k i, m f y$ —	5	$m k : m f :: k i : f y$
From the 4th and 5th it foll.	6	$f Y + f b : f Y :: k i : f y$

Now

Now by Position $bf = pf$ is the longer Focal Length, and $fb = ki$ is the shorter Focal Length, and Yp the Distance of the Glasses; and therefore $fY = pf - pY$ is the longer Focus, — the Distance of the Glasses; and fy is the Distance of the Distinct Base from the inward Glafs. From all which, the foregoing 6th Analogy, being resolved into Words, gives his Rule for solving the Problem,

As the longer Focus — the Glasses Distance + the shorter Focus :

To the longer Focus — the Glasses Distance ::

So the shorter Focus :

To the Distance of the Distinct Base from the inward Glafs.

Or thus,

As the Aggregate of the shorter Focus and the Difference between the longer and the Glasses Distance :

To the Difference between the longer Focus and Glasses Distance ::

So the shorter Focal Length :

To the Distance of the Distinct Base from the inward Glafs.

Which is the very same Canon with that I have given, deduced from the 8th Proposition hereof.

This Problem is of considerable Use in Dioptricks, being the Foundation of an excellent sort of Telescope much used in *England* for the Night. But it seems no Optick Writer has solved it. *Honoratus Faber* has clearly omitted it. And *Dechales*, though he offers at our foresaid VIII. Proposition, on which our Solution of this Problem is founded; yet missing so enormously as he has done in that, he could not pretend to the Solution of this. And had he rightly solved our VIII. Proposition, I question whether he would have pushed it on so far as to solve thereby this Problem; for in the two next succeeding Problems, he has rightly the Propositions on which their Solutions depend, but leaves the Problems themselves untouched.

PROP. XVII. PROBL.

A Convex-Glass being given, with a Concave of a larger Sphere, the Concave being placed behind the Convex, at any Distance less than the Focal Length of the Convex; 'Tis required to find the Place of the Compound Focus, or Distinct Base of these two Glasses.

T. 18. F. 3.

This Problem is solved by the XV. Proposition hereof. *Tab. 18. F. 3.* The Convex-Glass x is placed so before the Concave-Glass e , that this Convex would unite the Parallel Rays bf , bf , in its Focus at d , and the Rays qd , qd , would cross the Axis in d , unless the Concave e were interposed. Which Concave is supposed to have its Virtual Focus at i more distant than d the Distance of the Focus of the Convex x .

Wherefore here is a Ray q incident on the Concave e , and converges towards a Point d , nigher the Glass e than the Virtual Focus of parallel Rays i ; and 'tis required to determine the Point h , where it crosses the Axis. The Rule by the XV. Proposition is this,

As the Excess of the Focus of the Concave more than this Point of Convergency id :

To the Focus of the Concave ie ::

So the Distance of this Point of Convergency from the Glass de :

To the Distance of the Point where this Ray crosses the Axis he :

Now in our present Problem, the three first Terms of this Analogy are given, and the last required. For the Problem gives dx the Focal Distance of the Convex, and ie the Focus of the Concave, and ex the Distance of the Glasses. Hence we have $de (= dx - ex)$ the Distance from the Concave of the

the Point d , to which the Ray converges. And we have id ($=ie-de$) the Exceſs of the Virtual Focus of the Concave beyond the Point of Convergency d .

Wherefore by the foreſaid Analogy it being, As $id:ie::de:be$. The Rule for Solving this Problem is this,

From the Focal Length of the Convex ſubtract the Glaſſes diſtance. Mark the difference, then ſay,

As the Focal Length of the Concave — this difference :

To the Focal Length of the Concave ::

So this difference :

To the diſtance of the Diſtinct Baſe from the Concave.

1. Example.

Focus Convex $=dx=20$

Focus Concave $=ie=60$

Glaſſes Diſtance $=ex=10$

Hence — — — $=de=10$ Diff.

And — — — $=id=50$

Then $id=50:ie=60::de=$

$10:be=12$.

2. Example.

Focus Conv. $=12$

Focus Conc. $=15$

Glaſſes Diſt. $=2$

Hence $de=10$ Difference

And $id=5$

Then $5:15::10:$

$30=be$.

PROP. XVIII. PROBL.

The Focal Lengths of a Convex-Glaſs and a Concave-Glaſs being given, and the Concave placed towards the Object, diſtant from the Convex, ſo that the Sum of the Concaves Focal Length and Glaſſes diſtance may be greater than the Focal Length of the Convex, and this diſtance of the Glaſſes being given alſo; 'Tis required to determin the diſtance of the Diſtinct Baſe from the Convex.

This is performed by the V. Proposition hereof. *Tab. 18.*
T. 18. F. 4. *Fig. 4.* Let bzc be a Concave, on which there fall the parallel Rays nb, mc , which are so refracted by this Glass, that after passing it, they proceed on in be, cf , as if they came directly from the Point a the Virtual Focus of the Concave, (by Prop. X. XI. XII. hereof) but in e and f meeting with the Convex-Glass, whose Focus is on each side it at s, s , the Rays diverge not to i and g , but are refracted, so that they cross the Axis in k .

We may therefore conceive these Rays, not as passing through the Concave bc , but as proceeding directly from the nigh Point a ; and then to determine the Point k , where they cross the Axis, the Rule is in the Prop. V. thus,

As sa *the difference between the Focus* sd *of the Convex-Glass* ef , *and the distance* da *of the Object from the Glasses :*

To the Focus sd *of the Convex-Glasses ::*

So the distance da *of the Object from the Glasses :*

To the distance of the respective Focus dk .

Now in this Problem, the three first Terms of this Analogy are given, and the fourth dk is required. For the Sum of the Concaves Focal Length and the distance of the Glasses, that is, $za + dz$ is equal to da ; and da — the Convex's Focal Length that is $da - sd$ is equal to sa .

Wherefore we solve the Problem by this Canon.

From the Sum of the Focus of the Concave and distance of the Glasses subtract the Focus of the Convex, that is, $za + dz - sd = sa$. Keep this difference, then say,

As this difference sa :

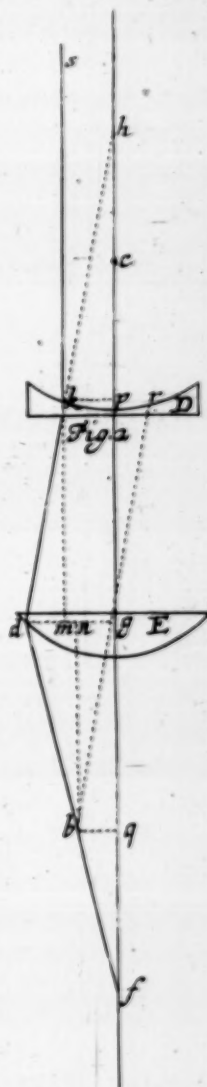
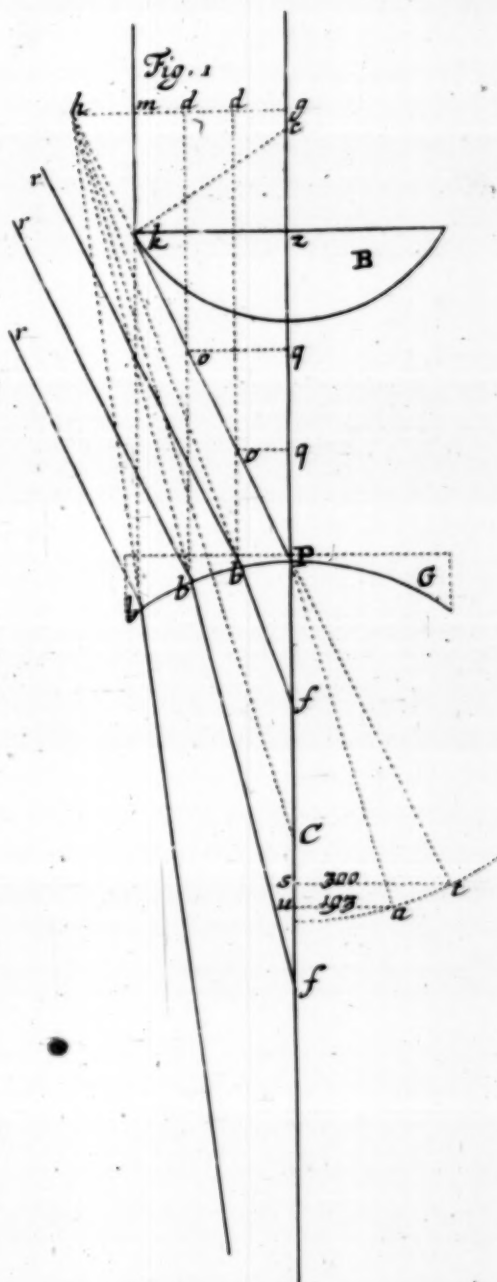
To the Focus of the Convex sd ::

So the Sum of the Concave's Focus and Glasses distance $za + dz = ad$:

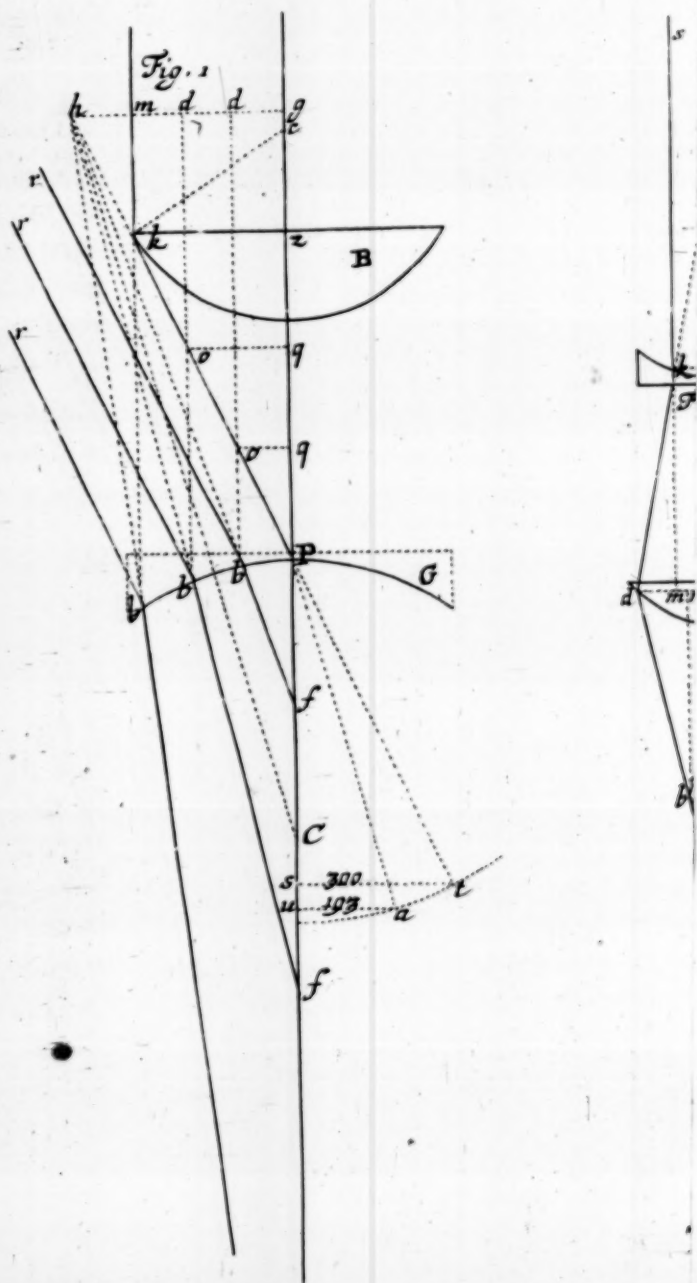
To the distance of the Focus from the Convex dk .

Which is required by the Problem, and is the very Analogy of our Prop. V.

An



Tab. 19. pag 77.



Another Solution of the Two last Problems in Prop. XVII. and XVIII.

These Two last Problems were likewise communicated to me, most ingeniously solved by my forementioned Honoured Friend Mr. J. FLAMSTEED. The Solutions I shall give in his own Words at large, as I have them in a Letter to me Dated from the *Greenwich-Observatory*, Jan. 17. 1683. as follows.

The First Problem is this, *A Convex Glass being given, with a Concave of a larger Sphere; the Concave being placed at any distance less than the Focal Length of the Convex from it, and remoter from the Object; To find the Place of the Focus or Distinct Base.*

The Second Problem is, *From the same Data; the Concave being placed nigher the Object; To find the Distinct Base.*

For the First Problem, *Tab. 19. Fig. 1.* B is a Convex-Glass, T. 19. F. 1. P its Focus, *mk* a Ray of Light falling upon it parallel to its Axis, which will therefore converge into its Focus at P. G is a Concave-Glass of a larger Sphere, placed upon the Focus of the Convex; CP the Radius of the Sphere whereon the Concave is formed. And we are always to conceive the Thickness of both these Glasses where the Rays of Light pierce them as small as possible.

Draw the Line *kP*, this shews the way of the Ray *mk* after its Emerfion from the Convex B. Produce it both ways at pleasure to *t* and *b*. Then conceive the Concave G placed any where betwixt P and B, as in *q, q*; From which Places draw the Lines *qo, qo*, at Right Angles to the Axis. And from the Points *o, o, k*, wherein they intersect the refracted Ray *kP*; draw the Lines *ob, ob, kb*, falling on the Concave G in *b, b, b*. From these Points again draw the Lines *br, br, br*, parallel to the refracted Ray *kP*; these shall represent

present the Ray. kP falling on the Concave G placed at q, q ; and also when it touches the Convex B . Let these then be consider'd as so many parallel Rays, proceeding from the same distant Point. Wherever they shall intersect the Axis after they have passed the Concave, there shall be the true Focus for both Glasses, which I shall call the *Compound Focus*. To find which we are to remember;

That, as the Parallel Rays of Light falling on a Convex-Glass, are all collected very nearly in the same Point of the distinct Base; so also the Rays of Light falling Parallel to each other on a Concave, after they emerge from it, diverge and separate, as if they had all proceeded from the same Point, which I therefore call the *Point of Divergency* (*This, in my foregoing Propositions, I term with most Optick Writers, the Virtual Focus*) Which Point in a Concave-Glass is just as far distant from it, as the Point of *Convergency*, or Focus of a Convex of the same Sphere or Spheres is distant from it. We know the Focal Point is some little nigher the Glass in the Parallel Rays that fall *widest* from the Axis, than in those that fall near it; so is the Point of *Divergency* for Concaves. But the difference is so small in Glasses of a large Sphere (such as in these Problems we suppose) that for all Rays falling on the Glass Parallel to each other, it may be taken as a *simple Point*. Such therefore we are to conceive it; and then to find this Point of Divergency, The Rule is (*Corol. Prop. XI.*) $300 - 193 = 107 : 193 ::$ So CP the Radius of the Plano-Concave-Glass: To Pb : Which in Glasses of a large Sphere, and where the Inclination of Pb is small, will be insensibly different from Pg ; hg being perpendicular to the Axis. Or we may determine this Point of Divergence Geometrically thus, On the Centre P with any Radius Pt , on the void side of the Figure, strike the Arch at ; then make it $300 : 193 :: ts : au$. Draw Pa , and from C the Centre of the Concave draw Ch Parallel to aP . The Point h , wherein it cuts the Line Pk produced

duced upwards, shall be the Point of Divergency sought.

Lay a Ruler over the Point b , and the Points b, b, b ; this, where it cuts the Axis $g p$ produced, as in f, f , shall give all the possible Compound Foci.

Then bg being perpendicular to the Axis, produce the Lines bo , till they cut it in d, d .

Then the Triangles bPg , oPq are similar: likewise the Triangles bfg , bfP are similar also.

It will then hold <i>ob similia</i>	1	$bP : oP :: bg : dg$
<i>Triangula bPg, oPq</i>		
Dividing the First — —	2	$bP - oP : oP :: bg - dg : dg$
That is, — — — —	3	$bo : oP :: bd : dg = oq$
Moreover by <i>Simil. Triang.</i>		
bfg, bfP — — — —	4	$bd : dg :: bb : bf$
From the 3d and 4th it follows — — — —	5	$bo : oP :: bb : bf$
And Compound the 5th — —	6	$bo + oP : oP :: bb + bf : bf$
That is, — — — —	7	$bP : oP :: bf : bf$
And Dividing the 7th — —	8	$bP - oP : oP :: bf - bf : bf$
That is, — — — —	9	$bP - oP : oP :: bb : bf$
But $bP = gP = bb$, and so likewise $oP = qP$, (the Inclination of kP , as is said before, being supposed very small; and the Glasses of small Segments of large Spheres.) Wherefore the 9th is thus — — — —	10	$gP - qP : qP :: gP : bf$

But by Position	{	Focus of the Concave	11	$= gP = bP$
		Focus of the Convex	12	$= zP$
		Distance of the Glasses	13	$= zq$
		(Fo. Convex — Gl. dist.		$= qP = zP - zq$

Wherefore

Wherefore the 10th. Analogy, by the 11th, 12th, and 13th Steps, resolved into Words gives this Rule,

As the Focus of the Concave — Focus of the Convex + distance of the Glasses:

To the Focus of the Convex — distance of the Glasses ::

So the Focus of the Concave:

To bf = to the distance of the Distinct Base from the Concave.

And this Canon of Mr. Flamsteed is the very same with ours in the preceding 17th Proposition deduced from the 15th hereof.

But if the Concave be placed next the Object: Which is his Problem II, and our XVIIIth Proposition, my forementioned Learned Friend solves it thus.

T.19. F.2. Tab.19. F.2. D is a Concave, E a Convex, bp the Focal Length of the Concave, $gp = gq$ the Focal Length of the Convex. Let sk be a Ray of Light falling into the Concave at k . From the Point of Divergency b , through k draw the Line kd ; this shall shew the refracted Path of the Ray sk after it has passed the Glass D. From q the Focus of the Convex E erect qb perpendicular to the Axis bf ; and through g draw the Line rgb Parallel to kd , intersecting the distinct Base of the Convex E in b , through which draw db , continuing it till it intersect the Axis in f : There shall be the compound Focus or distinct Base for these Glasses thus posited. Draw now the Line dg at Right Angles to the Axis; produce sk till it cut dg in m ; and draw bn Parallel to qg .

Then the Triangles hdg , hkp , bgq , are similar. Also the Triangles bdn , fbq , are similar. And $bp + pg = bg$.

It shall therefore be — — —	1	$bg : gq :: dg : ng = bq$
And converting the First —	2	$bg : bg - gq :: dg : dg - ng$
Inverting the second — —	3	$bg - gq : bg :: dg - ng : dg.$
That is, by the Scheme —	4	$bg - gq : bg :: dn : dg.$

Then

Then by the similar Triangles dnb, dgf — — —

From 4 and 5 it follows —

Then bg being = to $bp + pg$, the 6th is thus

$$5 \quad dn : dg :: nb = gq : gf$$

$$6 \quad bg - gq : bg :: gq : gf$$

$$7 \quad bp + pg - gq : bp + pg :: gq : gf.$$

$$\text{Focus of the Concave — } 8 = bp$$

$$\text{Focus of the Convex — } 9 = gq$$

$$\text{Distance of the Glasses — } 10 = pg$$

But by Position { Focus of Concave + Glas-

$$\text{ses dist. — — — — } 11 = bg = bp + pg$$

$$\text{Dist. of the distinct Base from Convex — — } 12 = gf$$

From all which the 7th Analogy may be thus expressed in words,

As the Focus of the Concave + the distance of the Glasses — the Focus of the Convex :

To the Focus of the Concave + the distance of the Glasses ::

So the Focus of the Convex :

To the Distance of the distinct Base from the Convex :

Which is the same Rule with that which I have before given in *Prop. XVIII*, deduced from the 5th hereof.

And these two last Propositions do naturally lead us to the Consideration of *Meniscus-Glasses*.

MENISCUS-GLASSES.

That is called a *Meniscus-Glass*, which is Convex on one side and Concave on t'other. 'Tis so named from its Resemblance to the New Moon or *Lunula*, in Greek *μηνίς*.

In *Meniscus-Glasses* either the Semidiameter of the Convexity and Concavity are equal or unequal: If they are equal the Ray that falls thereon Parallel to the Axis, after Refraction pro-

M

ceeds

Observ.

ceeds again Parallel: If they are unequal, then either the Semidiameter of the Convexity is less than the Semidiameter of the Cavity, and then the Convexity prevails, and the Glass has a *Real Focus*: Or else the Semidiameter of the Cavity is less than the Semidiameter of the Convexity, and then the Cavity prevails, and the Glass has only a *Virtual Focus*. But of these more fully in the following Propositions; This *Observation* being premised, That the Focus of a *Meniscus*, the Semidiameter of whose Convexity is less than the Semidiameter of the Concavity, and these two Semidiameters being given, is easily determined by *Prop. XVII* preceding. For in that Proposition, two Glasses are given, one a Convex, and t'other a Concave, and the Focal Length of the Convex is shorter than that of the Concave. And whereas in that *Prop.* there is given a distance between the Glasses, let us conceive this Distance to be *nothing*, that is, let us imagine these Glasses to *touch*; and their Results a *Meniscus* whose Focus is determined by the same Rule in that *Prop.* Which is this,

As the Focal Length of the Concave — the Focal Length of the Convex + the Glasses Distance:

To the Focal Length of the Concave::

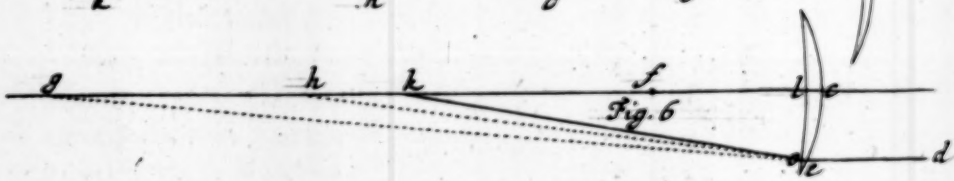
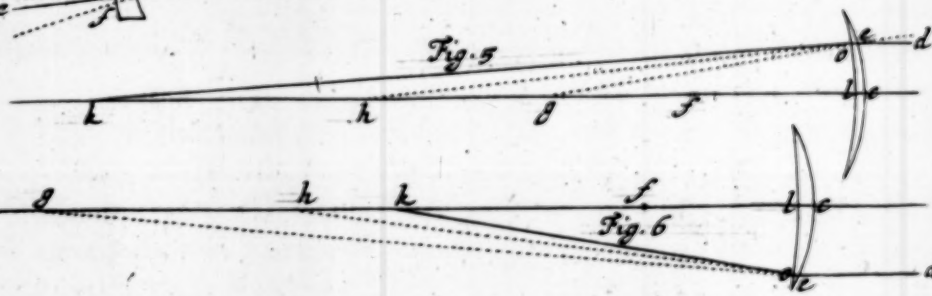
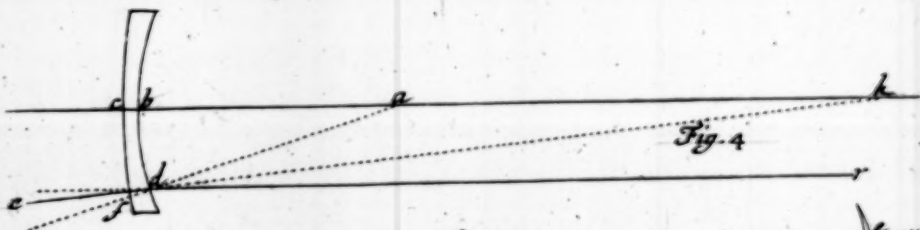
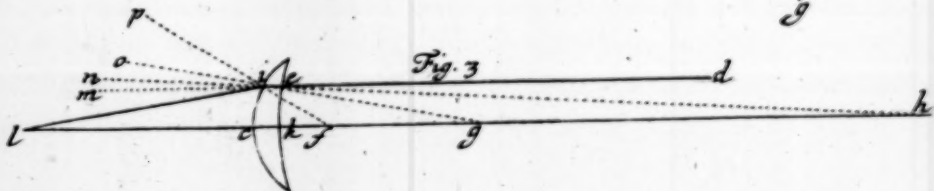
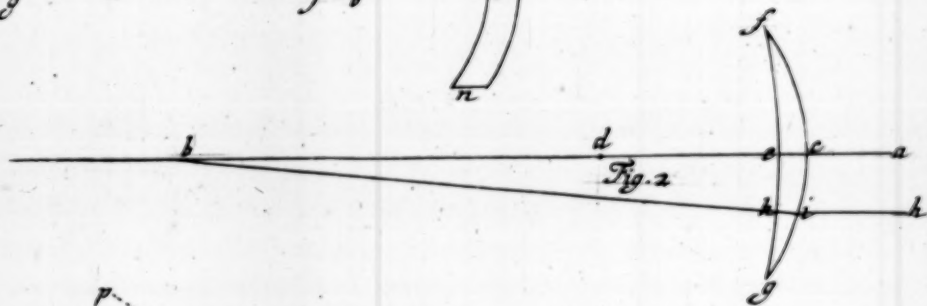
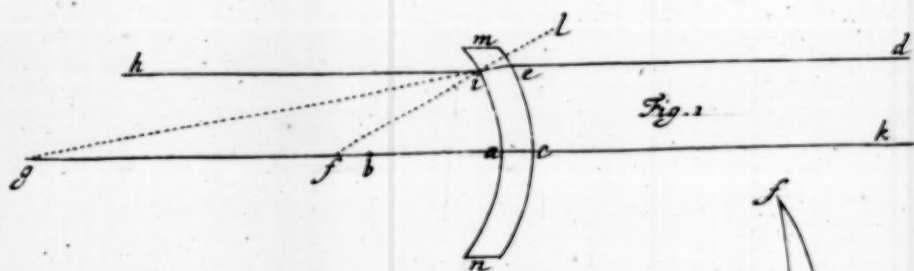
So the Focal Length of the Convex — the Glasses Distance:

To the Distance of the Distinct Base from the Concave.

But because we suppose, in the Case of a *Meniscus*, the two sides of the Glass to touch, that is, the distance of the two Glasses to be *nothing* (for every *Meniscus* may be conceived divided into two Glasses, a Plano-Convex, and a Plano-Concave) or the Thickness of the Glass to be inconsiderable in Comparison of the Focal Length. The forefaide Rule applied to a *Meniscus* shall be this,

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As the Focal Length of the Concave — the Focal Length of the Convex :

To the Focal Length of the Concave ::

So the Focal Length of the Convex :

To the Distance of the Focus of the Meniscus.

Now if we have given the Semidiameters of the Convexity and Concavity of a *Meniscus*, we have given the Focal Lengths of the Plano-Convex and Plano-Concave of which this *Meniscus* is compounded. For these Focal Lengths may be assumed the Doubles of the Semidiameters, wherefore Halfs being as their Wholes, the foresaid Rule for finding the Focus of a *Meniscus* may be thus exprest,

As the Difference of the Semidiameters of the Convexity and Concavity :

To the Semidiameter of the Concavity ::

So the Diameter of the Convexity :

To the Focal Length.

Which is the Common Rule assigned by Optick Writers. But of this more fully hereafter *Prop. XXIII.* And tho this Rule in General were sufficient for determining the Foci of *Meniscus-Glasses*, yet I shall enlarge more fully thereon in the following Propositions.

P R O P. XIX.

In a Meniscus, if both spherical Superficies have the same Diameter, the Ray that falls thereon Parallel to the Axis, after its second Refraction proceeds again Parallel.

Tab. 20. F. 1. mn is a Meniscus, bc the Semidiameter of the Convexity ec, fa the Semidiameter of the Concavity ia, de a Ray of Light Parallel to the Axis kg. I say this Ray after a double Refraction, one at e, and t'other at i, proceeds in ib Parallel to kg.

We here suppose the Thickness of the Glass inconsiderable in respect of the Semidiameter of the Sphere on which 'tis form'd; and therefore we wholly neglect it.

We suppose also that Sines and Angles and Sides in these small Angles are Proportional.

Let $gc = ga$ be made Triple $bc = fa$, the Ray de by its first Refraction at its Entrance into the Glass at e is refracted towards g , ieg being a Right Line; at i the Ray emerges from the Glass on the Concave Surface ia . Draw fil . Now the Angle of Inclination within the Glass is $lie = fig$, and if the Ray be refracted into ib , the Angle of Refraction is $hig = igf$, which ought therefore to be $\frac{1}{2}$ the Inclination fig , and, that it is so, I thus prove.

In the Triangle gif , As gf : To if :: So the Angle gif : To the Angle igf . But gf is double if , therefore $\angle gif$ is double $\angle igf$. Which was to be Demonstrated.

P R O P. XX.

In a Meniscus, if the Semidiameter of the Concavity be triple the Semidiameter of the Convexity, the Focal Length is equal to the Semidiameter of the Concavity.

This is most evident, if the Convex-side be turned to the Ray, as *Tab. 20. Fig. 2.* bi is a Ray parallel to the Axis ab , dc the Semidiameter of the Convexity $fcig$, $be = 3 dc$ the Semidiameter of the Concavity $fekg$. The Ray, by the first Refraction it suffers on the Convex Surface at i , is directed to concur with the Axis at a Diameter and an half of the Convexity, that is, in b . But b is the Centre of the Concavity; wherefore the Ray ik within the Glass falls perpendicularly on the Concave Surface $fk g$; and therefore is not at all refracted by its Emerision from the Glass, but proceeds onwards directly in ikb . Which was to be Demonstrated. But

But let the Concave-side be towards the Ray, as *Tab. 20.*

Fig. 3. fc is the Semidiameter of the Convexity ic , $gk = 3fc$ T. 20. F. 3. the Semidiameter of the Concavity ek , de a Ray parallel to the Axis, produced directly to m . Let lc be made equal to gk , I say the Focus shall be at l , lc being equal to the Semidiameter of the Concavity. Draw geo . The Angle of the Rays Inclination on the Concave Surface is $deg = oem$ (*Ang. ad Verticem*) and the Ray by the first Refraction it receives on the Concave Surface at e is so refracted as if it came from the Point b distant a Diameter and half of the Concavity then draw $bein$ directly. Here $men = deb = ebg$ is the Angle of Refraction, which is therefore $\frac{1}{2}$ of the Inclination $deg = egk = oem$. But we suppose, as the Angles so are the Sines, and so are the Sides. Wherefore in the Triangle egb , $s\angle egh = s\angle egk$: $s\angle ebk :: eb = kb : eg$, that is, $\angle egk : \angle ebk :: kb : eg$. But the Angle egk is equal to $3\angle ebk$, and therefore kb is equal to $3eg$, and consequently gb is double eg or gk . From f the Centre of the Convexity draw fip , this is perpendicular to the Convex Surface ic . Wherefore seeing gb is double gk , and gk is triple fc or fi , bf shall be octuple fi (by the Scheme). And the Second Angle of Inclination bif on the Convex Surface shall be octuple of the Angle at b , because their Correspondent subtending Sides are as 8 to 1. But bif is equal to pin (*Ang. ad Vert.*) And because at the Rays Egress from Glass to Air, the Angle of Refraction nil ought to be half the Inclination pin , so let it be made; and then seeing the Angle pin is octuple the Angle at b , its half nil shall be quadruple the Angle at b ; from the Angle nil subtract the Angle $nim = dib = \angle b$, there remains the Angle $mil = ilf = 3\angle b$; wherefore the Angle at l and $egk = 3ebk$ are equal. And then in the Triangle elg (neglecting the Thickness of the Glass) el and eg , or cl and kg the Semidiameter of the Concavity are equal. Which was to be Demonstrated.

Observation.

Observation.

This is the 28th. *Prop.* of *Dechales* First Book of *Dioptricks*, which he there huddles over in a most confused manner, omitting several Steps in the Demonstration. *Zahn* has the same in *Fund. 2. Syntag. 1. Cap. 7. Prop. 32.* and transcribes from *Dechales* not omitting his very Errors.

PROP. XXI.

In a Meniscus, the Semidiameter of whose Convexity is triple the Semidiameter of the Concavity, The Virtual Focus is distant the Semidiameter of the Convexity.

T. 20. F. 4. *Tab. 20. F. 4.* ab is the Semidiameter of the Concavity bd , $kc = 3ab$ is the Semidiameter of the Convexity cf , rd a Ray parallel to the Axis kc . This Ray by the first Refraction it receives on the Concave Surface at d , is refracted into de , as if it came directly from the Point k (by *Prop. X.*), wherefore de falls perpendicular on the Convex Surface (by *Def. 10.*) and consequently is not refracted at its egress from the Glass (by *Exp. 3.*) wherefore the Virtual Focus is at k . Which was to be Demonstrated.

PROP. XXII.

In whatever Meniscus wherein the Semidiameters of the Convexity and Concavity are unequal; The General Rule that assigns the Focus either Real or Virtual is this,

As the difference of the Semidiameters :

To either of the Semidiameters, whether of the Convexity or Concavity ::

So is the Diameter of the other Surface :

To the Focus Real or Virtual.

(Case

(Case 1.) I shall demonstrate this *Proposition* in Three several Cases, and the First shall be, Where the Semidiameter of the Concave is greater than the Semidiameter of the Convex, but less than triple the Semidiameter of the Convex.

(Case 2.) Wherein the Semidiameter of the Concave is greater than triple the Semidiameter of the Convex.

And in these Two Cases the Glas has a *Real Focus*.

(Case 3.) Wherein the Semidiameter of the Concave is less than the Semidiameter of the Convex, and then the Glas has only a *Virtual Focus*.

'Tis here supposed, that the Breadth of the Glas is so little, and the Angles so small, that Sines and Angles and Sides may be proportional. For an Angle, we often assume its Complement to 180°, as having the same Sine.

'Tis supposed likewise that the Thickness of the Glas is inconsiderable, and therefore 'tis wholly neglected.

First therefore for the First Case. *Tab. 20. Fig. 5.* Let *de* be T20. F. 5.
a Ray parallel to the Axis *ck*, *fc* is the Semidiameter of the Convexity *ec*; *gl* is the Semidiameter of the Concavity *ol*. Make *hc* triple *fc*. Then by the first Refraction the Ray suffers at *e*, 'tis directed towards *b* (by *Prop. I.*). Wherefore the Angle of Inclination upon its Emerfion from the Glas is *gob*. Let the Angle of Refraction *bok* be made half *gob*. Then *k* is the Focus, and it being so made, I say, the difference of the Semidiameters = *fg* : Is to *fc* the one Semidiameter :: As *2 gl* the other Diameter : To *ck* the Focal Length.

Demonstration.

In the Triangle <i>hog</i> — —	1 $\angle hog : \angle obg :: hg : go = gl$
And therefore — — —	2 $\angle hog = bok : obg :: hg : 2 gl$
Moreover in the Triang. <i>bok</i>	3 $bok : obg :: bk : ok = kl$
From 2 and 3 it follows —	4 $hg : 2 gl :: bk : kl$

By

By permuting the 4th. — —	5	$2gl : kl :: hg : bk$
Compounding the 5th. —	6	$2gl + kl : kl :: hg + bk = gk : bk$
But kl is equal to $kg +$	7	$2gl + kl = 2gl + kg + gl =$
$+ gl$, therefore ———		$3gl + gk$
Wherefore the 6th. runs thus	8	$3gl + gk : kl :: gk : bk$
By permuting the 8th. — —	9	$3gl + gk : gk :: kl : bk$
Dividing the 9th. ———	10	$3gl : gk :: kl - bk = hl : bk$
And permuting the 10th. —	11	$3gl : hl = 3fl :: gk : bk$
From 11 and 6 it follows —	12	$3gl : 3fl :: 2gl + kl : kl$
And consequently from 12	13	$gl : fl :: 2gl + kl : kl$
Dividing the 13th. ———	14	$gl - fl = fg : fl = fc :: 2gl :$
		$kl = kc$

For the Thickness of the Glass is neglected, and therefore we assume $kl = kc$.

But this 14th. Analogy $fg : fc :: 2gl : kc$. Is the Proposition which was to be Demonstrated.

Note, Zahn does again in this Proposition transcribe from Dechaless, and copies the very Mistakes of the Press. But in our foregoing Demonstration all is rectified.

Corollary.

Because ——— $fg : fc :: 2gl : kc$

It shall also be — $fg : gl :: 2fc : kc$

For if it be — $fg : fc :: 2gl : kc$

Then Permuting $fg : 2gl :: fc : kc$

Then halve the first Consequent and double the last Antecedent, and it shall be $fg : gl :: 2fc : kc$. Which was to be Demonstrated.

T20.F.6. In the 2. Case, Let fc , Tab. 20. Fig. 6. be the Semidiameter of the Convexity ec . Make cb equal to $3fc$. Let gl be the Semidiameter of the Concave lo , de a Ray parallel to

to the Axis, which by the first Refraction tends towards b (by *Prop. I.*) Now the Angle of Inclination on the Concave Surface is hog ; for go is the Perpendicular. Wherefore the Ray by the second Refraction from Glass to Air is bent from the Perpendicular by half the Angle of Inclination; Let kob be this Angle of Refraction $= \frac{1}{2} hog$. I say then, that As gf : To lf or fc (neglecting the Thickness of the Glass) :: So $2gl$: To ck or lk .

Demonstration.

In the Triangle gob — —	1 $\angle obg$ or ohl : $\angle goh$:: $og =$ gl : gb .
And therefore — — —	2 ohl : $\angle goh = hok$:: $2gl$: gb
Moreover in the Triang. hok	3 ohl : hok :: $ok = lk$: kb .
From 2 and 3 it follows —	4 $2gl$: gb :: lk : kb .
Permuting 4 — — —	5 $2gl$: lk :: gb : kb .
Compound the 4th. — —	6 $2gl + gb$: gb :: $lk + kb$: kb .
Permute the 6th. — — —	7 $2gl + gb$: $lk + kb$:: gb : kb .
From 5 and 7 it follows —	8 $2gl + gb$: $lk + kb = lb$:: $2gl$: lk .
Compound the 8th.	9 $2gl + (gb + lb =) gl = 3gl$: $lb = 3lf$:: $2gl + lk$: lk .
And from the 9th. it follows	10 gl : lf :: $2gl + lk$: lk .
And dividing the 10	11 $gl - lf = gf$: lf :: $2gl$: lk .

Which was to be Demonstrated.

Corollary.

Because — — — gf : lf :: $2gl$: lk .
It shall be also — — — gf : gl :: $2lf$: lk , by the
foregoing Corollary.

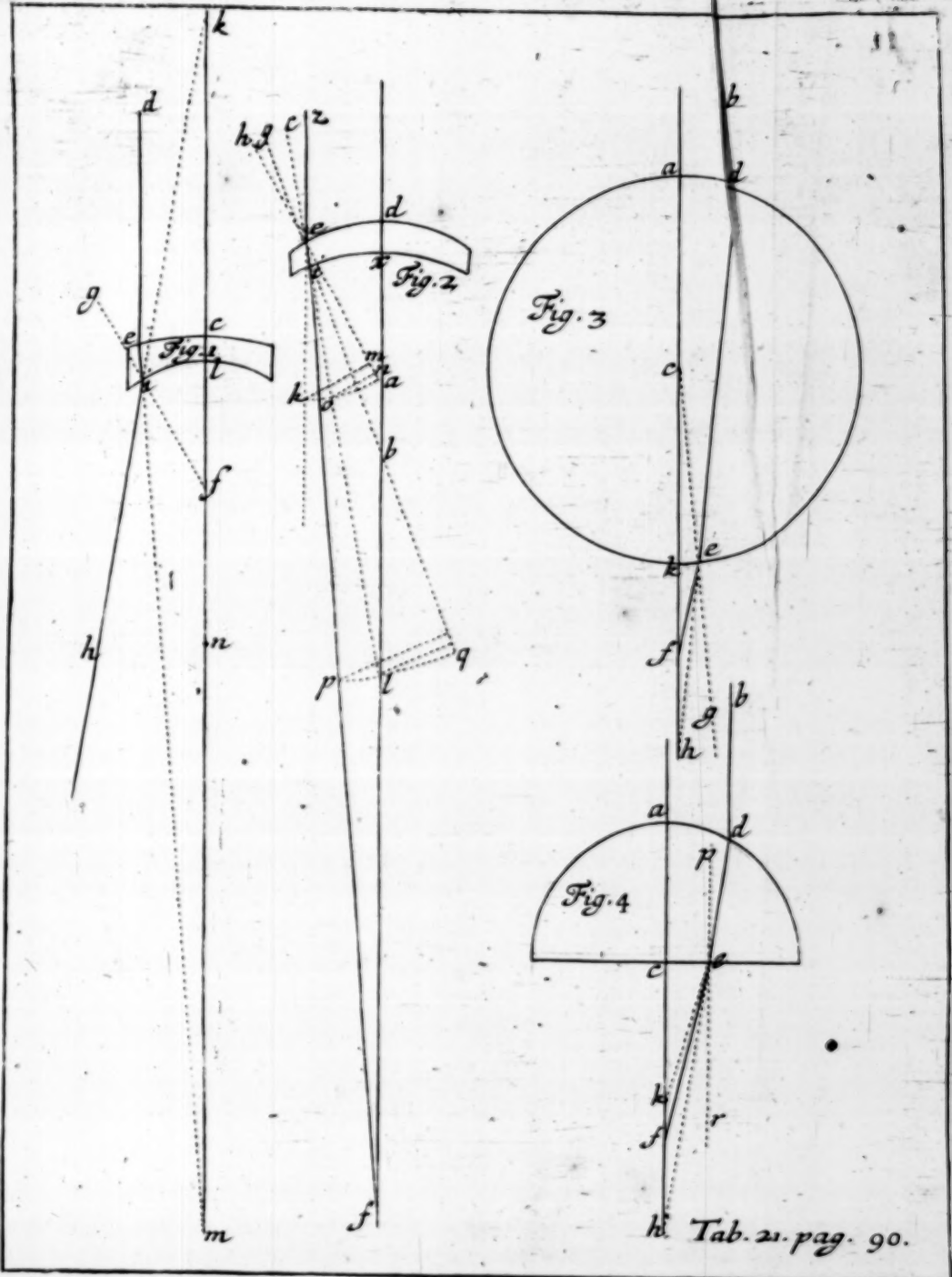
T. 21. F. 1. Lastly in the third Case, *Tab. 21. f. 1.* *fl* is the Semidiameter of the Concavity *il*, *nc* the Semidiameter of the Convexity *ec*. Let *cm* be made Triple *nc*. *de* is a Ray Parallel to the Axis *km*. this Ray by its First Refraction proceeds in *eim*; at *i* it emerges into Air on the Concave Surface. Draw the Perpendicular *fig*. The Angle of Inclination on this Concave Surface is $\angle ig = \angle fim$, *ad Vert.* Let the Angle of Refraction $\angle him$ be made equal to $\frac{1}{2} \angle fim$ as it ought to be. Then, I say, the Ray Proceeds after its Second Refraction in *ih*, as if it came directly from the Virtual Focus *k*, and that then it shall be, as $nf : nc :: 2 fl : kc$.

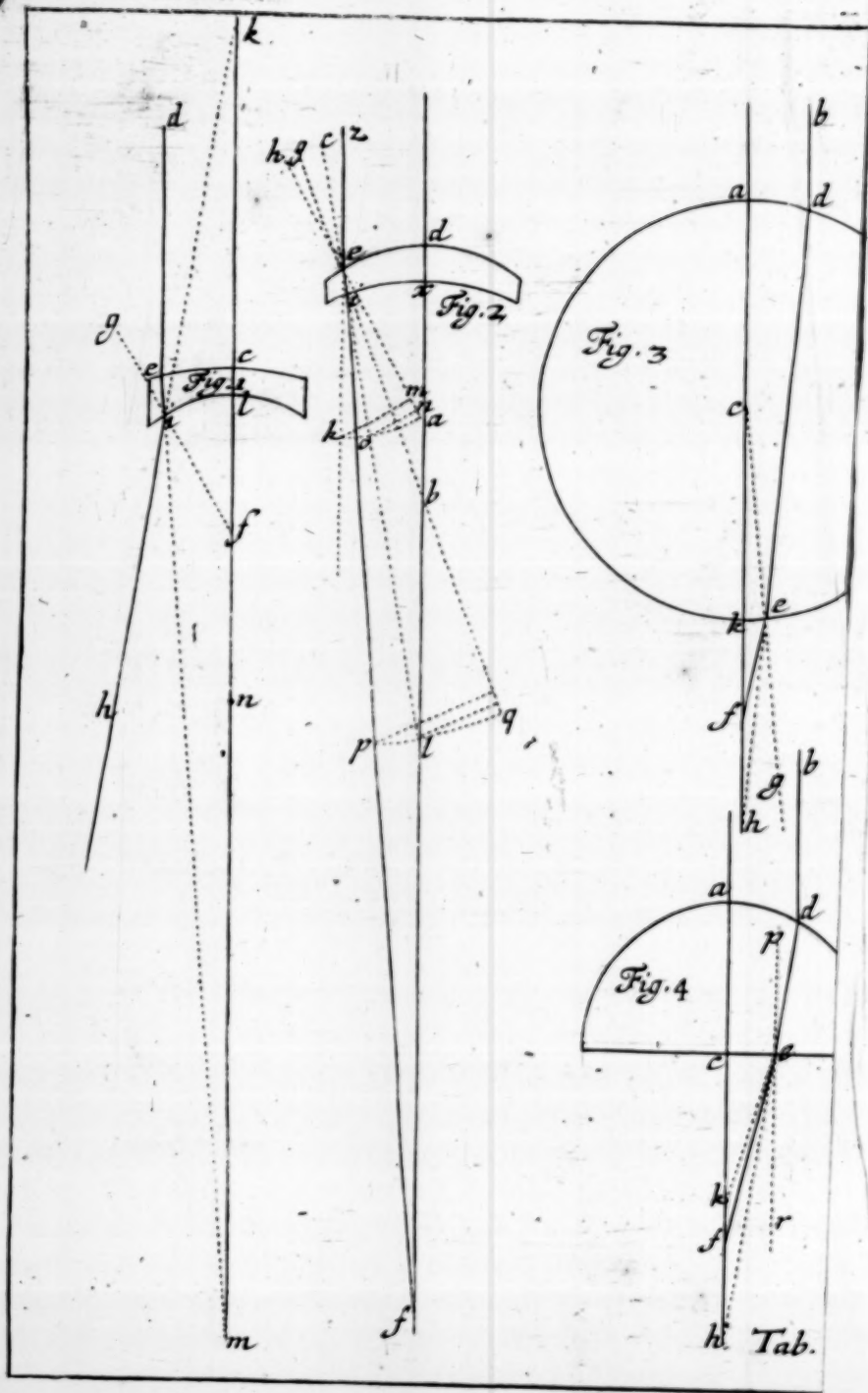
Demonstration.

In the Trian. <i>mif</i> — —	1	$\angle mif : \angle m :: fm : fi = fl$.
Therefore — — — —	2	$\frac{1}{2} mif = him : \angle m :: fm : 2 fl$.
Also in Trian. <i>mik</i> — —	3	$\angle mik$ or its Compl. $\angle him :$ $\angle m :: mk : ik = kl$.
From 2 and 3 — — —	4	$fm : 2 fl :: mk : kl = kc$ neg- lecting the Thickness.
Dividing 4 — — — —	5	$fm - 2 fl : 2 fl :: mk - kl =$ $ml = mc : kc$.
But by Position in the Scheme	6	$fm - 2 fl = ml - 3 fl$.
Wherefore the 5th Runs thus	7	$ml (= mc) - 3 fl : 2 fl :: ml$ $= mc : kc$.
And Permuting 7 — — —	8	$mc (= 3 nc) - 3 fl : mc ::$ $2 fl : kc$.
And therefore — — — —	9	$nc - fl = nf : \frac{1}{3} mc = nc ::$ $2 fl : kc$.

Which was to be Demonstrated.

Corol-





Corollary.

Seeing it is — — — $nf:nc::2fl:kc.$
 It shall be also — — — $nf:fl::2nc:kc.$

Vid. Preceding Corollaries.

PROP. XXIII. PROBL.

The Semidiameter of the Convexity being given, 'tis required to find the Semidiameter of the Concavity, that a Meniscus formed with this Convexity and Concavity may unite the Parallel Rays at a given distance. *Vid. Barrow Lect. Opt. 14. pag. 102. &c. Edit. Lond. 1672. 4to.*

Tab. 21 F. 2. $ae=ad$ is the given Semidiameter of the Convexity, df is the given Distance, at which the Parallel Rays are to be united, ze produced to k is a Ray Parallel to the Axis df . Draw aeb on the Point of Incidence e as a Centre at any Interval, strike the Arch $k oa$, and make the sine of the Angle of Incidence km , to the Sine of the first refracted Angle on , as 300 to 193, or as 14 to 9. Draw eol produced the other way to c . The Ray by the first Refraction is directed to l ; in the Line le take any Point at Pleasure i , so that ei may be the Thickness of the Glass which is to be formed; which Thickness is here represented considerable, for Illustration sake, but it being really of no Moment in Glasses of small Segments of large Spheres, 'tis neglected altogether in Demonstration. Draw fi which we suppose equal to fd . And make the Angle liq : To the Angle $lif::$ As 193: To 107. Draw qi which cuts the Axis in b . On b as a Centre strike the Arch of the Concavity ix . I say $bi=bx$ is the Semidiameter of the Concavity, which added on t'other side the Glass to the former given Convexity, shall unite the Parallel Rays at the given distance df . T21.F.2.

Demonstration.

By the first Refraction the Course of the Ray is through *eil*, but at *i* it meets the Concave Surface, and emerges from thence into Air. Now the second Angle of Inclination is *bil*, for *bi* is perpendicular to the Concave Surface. But the Angle *lif* is (by Construction) To the Angle *liq*:: As 107 : To 193. Wherefore *lif* is the second Angle of Refraction agreeable to the second Angle of Inclination *bil* (by Exp. 6.) Wherefore *ipf* is the refracted Ray. Which was to be Demonstrated.

Calculation of an Example.

Let the Semidiameter of the Convexity $ae=ad$ be given 10000. and the Focal Distance $df=fi$ 63000.

Let us suppose the first Angle of Inclination kea to be $5^{\circ}.0'.0''$.

Then as 300 : to 107 :: $s.\angle kea : s.\angle ila$, which is the first Angle of Refraction = $1^{\circ}.46'.50''$. Wherefore $ilf = 178^{\circ}.13'.10''$.

Also (by Prop. 1.) As 107 : 300 :: $ae=10000$: To $el=il=ld=28037$.

Then in the Triangle *ifl*, we have the three sides and the Angle *ilf*, to find the Angle, $ifl = 0^{\circ}.47'.30''$ and $fil = 0^{\circ}.59'.20''$.

Say then, As 107 : To 193 :: $s.\angle fil : s.\angle liq$ which we shall find to be $1^{\circ}.47'.0''$.

And $fil+liq = fib = 2^{\circ}.46'.20''$. Also $180^{\circ} - fib - ifl = ibf = 176^{\circ}.26'.10''$.

Lastly in the Triangle *fib*

As $s.\angle ibf =$	$176^{\circ}.26'.20''$	Log. comp. ar.	1.2064785633
To $s.\angle ifb =$	$0^{\circ}.47'30''$	— — — —	8.1404059077
So $if =$	63000	— — — —	4.7993405495
To $bi =$	14000	— — — —	4.1462250205

Where-

Wherefore I say that a *Meniscus*, whose Convexity hath the Semidiameter 10000, and the Concavity the Semidiameter 14000, has its Focal Length 63000.

Further Proof.

To Prove this by some of the foregoing Rules for finding the Focus of a *Meniscus*. Let us conceive such a *Meniscus* to consist of a Plano-Convex Glass, and a Plano-Concave Glass. And the Semidiameter of the Plano-Convex to be 10000, and the Semidiameter of the Plano-Concave 14000. Then to find the Focal Lengths of these Glasses separately, say,

By *Corol. Prop. II.* $107 : 193 :: 10000 : 18037 = \text{Fo. Conv.}$

And by *Corol. Pr. XI.* $107 : 193 :: 14000 : 25252 = \text{Fo. Conc.}$

Diff. 7215

Then by the Observation Preliminary to *Prop. XIX.* $7215 : 25252 :: 18037 : 63128 =$ to the Focal length of such a *Meniscus*. Which comes sufficiently nigh 63000, considering the foregoing Angles are Calculated only in Round Numbers to every 10" seconds.

What is here performed by supposing the First Angle of Inclination $ke a = 5^\circ. 0'. 0''$, may be likewise done supposing it of an other Inclination, and as the Angle is smaller it will be the more exact.

P R O P. XXIV.

An intire Glass-Sphere Unites the Parallel Rays at the Distance almost of half its Semidiameter behind it.

'Tis here supposed that the Incident Rays do not fall distant from the Axis more than 20 or 30 Degrees.

Tab.

T.21. F.3.

Tab. 21. f. 3. ak is a Glass-Sphere. On the Point d there falls the Ray bd Parallel to the Axis ak . I say this Ray after a Double Refraction concurs with the Axis in the Point f , so that fk is almost half the Semidiameter of the Sphere.

c is the Centre of the Sphere, make kb equal to the Semidiameter ck , then ab is a Diameter and Half. And the Ray bd by the First Refraction proceeds in de directly towards the Point b . But at e it emerges from Glass to Air. Draw the Perpendicular ceg , the Angle of this second Inclination is $ced = geb$. Now the Ray instead of proceeding directly to b is refracted from the Perpendicular eg by the Angle feb , which is therefore to be half ced . And that it is so, I thus prove. ced is equal to $ecb + ebc$ (32. 1. *Eucl.*) but ecb and ebc are to sense equal, their Subtenses eb and ec being very near equal: wherefore ebc is half ced . But ebc is equal to feb (their Subtenses fe and fb being as to sense equal; for fk by supposition is equal to fb , and fe is very near equal to fk , therefore fe is almost equal to fb) and consequently the Angle feb is half ced or geb . Which was to be Demonstrated.

PROP. XXV.

A Glass Hemisphere Unites the Parallel Rays at the Distance of a Diameter and one third of a Semidiameter from the Pole of the Glass.

T.21. F.4.

Tab. 21. f. 4. adc is a Glass Hemisphere, bd a Ray of Light Parallel to the Axis ab . This Ray after a Double Refraction Crosses the Axis in the Point f , so that fc is a Semidiameter and one third of a Semidiameter. Make ck and

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and kb each equal to ca . By the first Refraction at d , the Ray is directed towards b in deb . At e the Point where it emerges from the Glas, draw per Perpendicular to the Surface ce . Now the second Angle of Inclination is $dep = ber$ (15. 1.) $= kbe$ (29. 1.) $= keb$. (For ke being nighly equal to kc , 'tis also nighly equal to kb ; Then in the Equilateral Triangle keb , the Angle keb shall be nighly equal to the Angle kbe) And the Angle of Refraction is feb . And this we are to prove equal to $\frac{1}{2}$ the Inclination ber or ebf . Seeing kb is equal to ck , and kf is $\frac{1}{3}$ of kb by Construction, it follows that cf is double fb (for $ck + kf = cf = \frac{2}{3} + \frac{1}{3} = 1$, and $fb = \frac{1}{3}$, but $\frac{2}{3}$ is = to double $\frac{1}{3}$) But ef is nighly equal to cf ; therefore ef is nighly double fb . Then in the Triangle feb , as the Sides, so the Angles (by supposition) therefore the Angle ebf is nighly double the Angle feb . Which was to be Demonstrated.

Definition.

The Projection of an Object in the Distinct Base of a Convex Glas, I call the *Image*.

PROP. XXVI.

As the Distance of the Object from the Glas :
To the Distance of the Image from the Glas ::
So the Diameter of the Objects Magnitude :
To the Diameter of the Image.

Tab. 22. f. 1. abc is an Object, kdm a Convex Glas T22.F.1.
in the Hole of a Dark Chamber, efg the Image formed
by this Glas on a White Paper in the Distinct Base. I say,
As

As cd the Distance of the Object from the Glass : To ed the Distance of the Image from the Glass : : So ac the Objects Magnitude : To ge the Magnitude of the Image.

To shew this, we are to remember the Premises to the IV Proposition. For by them we shall find, that the Axes ad , and cd of the Luminous Cones kam , kcm , may be consider'd as passing the Glass unrefracted. Because that after they have passed the Glass and become dg , de , they proceed Parallel to their Course before they entred the Glass. So that in Glasses of large Spheres, and small Segments, the thickness of the Glass being inconsiderable in respect of the Focal Distance, we may neglect it. And then we have here two Similar Triangles adc , ged ; For $\angle adc$ is equal to $\angle edg$ (15. 1. *Euch.*) and the Side eg is Parallel to the Side ac (by supposition) and consequently the Angle cad is equal to the Angle egd (29. 1.) and acd is equal to the Angle ged . Wherefore it shall be $cd : ed :: ac : ge$. Which was to be Demonstrated.

Note. From hence appears the Mistake committed by Monsieur Comiers in *DeBlegny's Zodiacus Medico-Gallicus*. An. 3tio. pag 117. Who says, *Diameter Imaginis Solis in Distinctâ Basi obtinebit ad minimum in sua Diametro Funem seu Chordam Dimidii Gradus magni Circuli Sphæræ, cujus Vitrum est segmentum.* Whereas it should be, *Magni Circuli Sphæræ, cujus Radius est Distantia Focalis Vitri.*

What is here Demonstrated concerning the Real Image of a Convex Glass may be accommodated to the Virtual Image of a Concave.

Scholium.

But if we are yet more scrupulous, and will consider also the thickness of the Glass; Then the Point within the Glass

Glas from whence the Distance of the Distinct Base is to be counted, may be thus determin'd in a Plano-Convex Glas. *Tab. 22. f. 2.* *k d m* is a Plano-Convex Glas with T. 22. F. 2. its Convex Side towards the Object *a b c*; *a d*, *c d*, the Axes of Radius Cones falling Obliquely on the Glas in its Pole *d*; Which (if the Glas were not interposed) would proceed directly on to *p, p*, but by the Glas are now refracted into *d i*, *d i*; But at *i i* meeting with the Second surface of the Glas, instead of going strait onwards, they are again Refracted into *i e*, *i g*, Parallel to *a d*, *c d*. Let *e i*, *g i* be produced backwards till they Intersect the Axis *b f* in *x*, I say *x* is the Point in the Glasses Axis, from whence the Distance of the Distinct Base is to be reckon'd. And as the Thickness of the Glas *d z*: To the Distance of this Point *x* from the Inner surface *z x*: So is the Co-tangent of the Angle of Incidence from Glas to Air: To the Co-tangent of the Refracted Angle.

For to the Point of Incidence *i* draw *l i r* Perpendicular to *k m*. Here the Angle of Incidence from Glas to Air is *l i d = i d z* and the Complement of *i d z* is *d i z*, whose Tangent is *d z* (making *i z* Radius) and the Refracted Angle is *r i e = i x z* whole Complement is *x i z*, and the Tangent hereof is *x z*.

In Double Convexes the Case is something different, but 'tis needless to enlarge any farther thereon.

Wherefore we see, that if we use a Plano-Convex Glas with its Convex-side towards the Object, and if we allow for its Thickness, we are to compute the Distance of the Object from the Pole of the Glas *d*, and the Distance of the Distinct Base from the Point *x*. But supposing the Plane Side towards the Object *e f g*, then we are to reckon the Distance of the Object *e f g* from the Point *x*, and the Distance of the Distinct Base *a b c* from *d* the Pole of the Glas. *Vid. Prop. XXVII.* O *Corol-*

Corollary 1.

It follows from this *Prop. XXVI.* That the Diameter of the Sun subtending an Arch of 32 Minutes in a Great Circle of the Heavens, the Diameter of the Sun's Image, Represented in the Distinct Base of a Convex-glass, subtends an Arch of 32 Minutes in a Circle, whose Radius is the Distance of the Distinct Base from the Glass.

But *Tab. 23. Fig. 1.* *b* is a Convex-Glass, *ab* a Ray of Light proceeding from the Sun's Eastern Limb, *db* a Ray from its Centre, *cb* a Ray from its Western Limb. Let the single Glass *b* Represent the Image of the Sun in the Distinct Base *bqi*, and let us suppose the Sun's Diameter to be 32 Minutes. Draw *bb*, *bq*, *bi*. Let *exzf* be an other Plano-Convex-Glass, so placed behind the Glass *b* that both these Glasses together (according to *Prop. XVI.*) may Represent the Sun's Image in the Distinct Base *kpo*: 'Tis required to find the Breadth or Diameter *ko* of this Image, from these Data, *bbq* the Angle of the Sun's Semidiameter 16 Minutes, *bq* the Focal Length of the Glass *b*, *bf* the Distance of the Glasses, *pf* the Distance of the Distinct Base from the Glass *f*, *eg* the Radius of the Convexity *exz*, or instead thereof, the Focal Length of the Plano-Convex Glass *exzf*, for having one we may easily obtain the other.

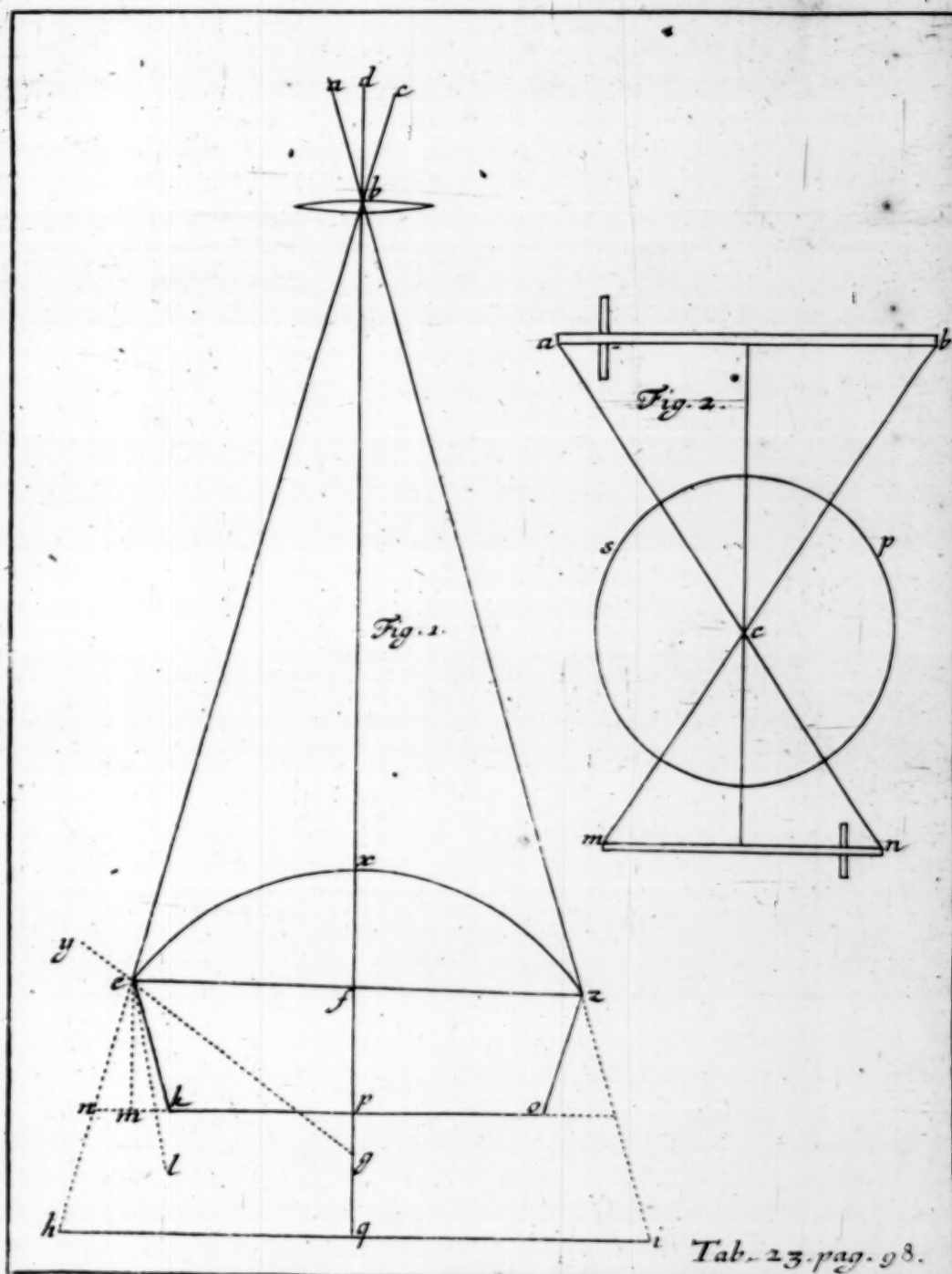
We suppose that the Central Ray *dbxq* is Co-incident with the Axis of the Glasses.

First therefore in the Right Angled Triangle *bbq*, we have the Angle *bbq* (and consequently *bbq = bef*) and *bq* to find *bq*.

(2) Then $bf + fp = bp$. And $bq : bq :: bp : pn :: bf : ef$.

(3) In the Right-Angled Triangle *efg*, *ef* and *eg* are known to find the Angle *feg*.

(4) 180.



(4) $180 - bef - feg = yeb = geb$, Which is the Angle of the Inclination of the Ray be on the Convex surface $e x$.

(5) $300 : 193 :: s. \angle heg : s. \angle hel$, Wherefore the Ray by its first Refraction would proceed in el .

(6) Draw me Perpendicular to the Plane Surface ef , then $ef = mp$. And $me = pf$. And $np - mp = nm$; And the Angle nem is = to qbb , And $hel - nem$ is = to mel , Which is the Inclination of the Ray on the Plane Surface ef .

(7) Then $193 : 300 :: s. \angle mel : s. \angle mek$.

(8) In the Right-angled Triangle mek , me and the Angles are known to find mk .

(9) Lastly $nm + mk = nk$, and $pn - nk = pk = ik$; Which was required to be found.

Corollary 2.

The foregoing Trigonometrical Calculation in the first Corollary I had from my Esteem'd Friend Mr. John Flamsteed *Astr. Reg.* But Instead thereof, or as an Additament thereto, I shall substitute this Problem.

To Determine the Breadth of the Distinct Base Resulting from the Combinations of Glasses expressed in Prop. XVI, XVII, XVIII. There being given (together with the Data in those Propositions) the Breadth of the Object, or the Angle it subtends before the outermost Glass, And if it be a nigh Object, its Distance from the outermost Glass.

We here suppose our Glasses of the least Thickness Imaginable, or that all the Refractions are performed in the Right Line, that passes through the Glasses Breadth at Right Angles to the Glasses Axis, for avoiding Confusion in the Schemes.

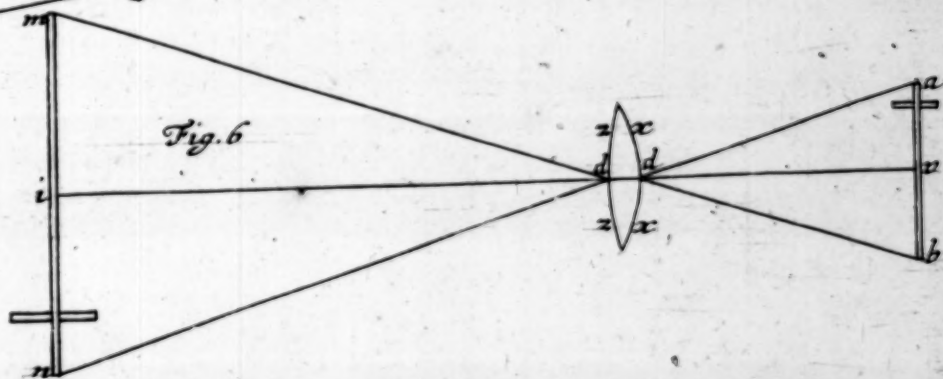
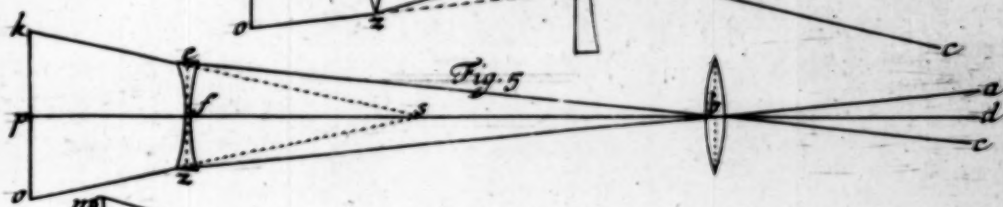
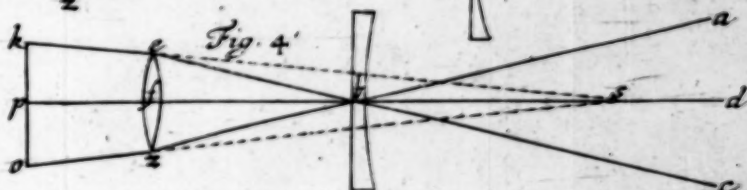
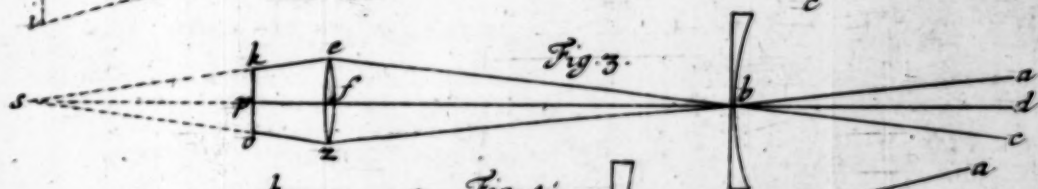
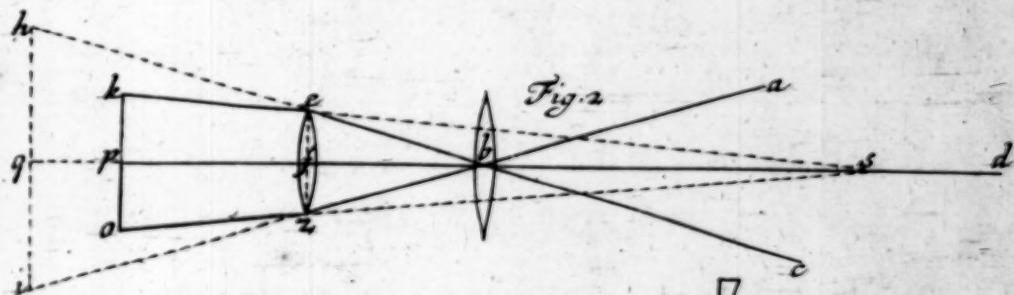
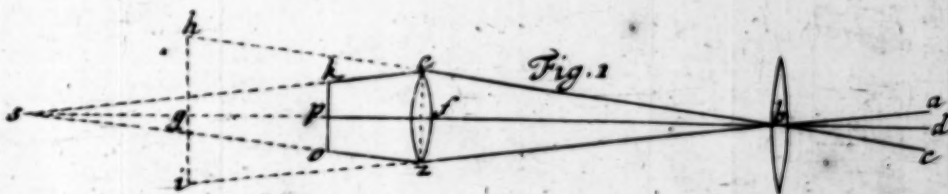
Tab. F. 1,
2, 3, 4, 5.

Wherefore in *Tab. 24. Fig. 1, 2, 3, 4, 5.* *b* is the Glas next the Object, whether Convex (as in *Prop. 16, 17.*) or Concave (as in *Prop. 18.*) We shall take the Sun for our Object, and suppose *a b* a Ray proceeding from its Eastern Limb, *d b* a Ray from its Centre, *c b* a Ray from its Western Limb. Let the single Glas *b*, in the first and second *Figures*, Represent the Image of the Sun in the Distinct Base *h q i*. The Breadth of the Base *h q i* is easily obtain'd from the Data. Let *e f z* be another Glas, so placed behind the Glas *b*, that both these Glasses together may Represent the Suns Image in the Distinct Base *k p o*, 'Tis required to find the Breadth or Diameter *k a* of this Image from these Data; *b q* the Focal Length of the Glas *b*; The Breadth of the Image *h i*, or the Angle $abc = ebz$; The Focal Length of the Glas *e z*; The Distance of the Glasses *f b* and *p f* the Distance of the Distinct Base from the Glas *e z*.

Through *p* at Right-angles to the Axis *d s b f p* of the Glasses draw *k p o* infinitely. First from these Data, let us obtain the Breadth of the Glas *e z*, where the Rays *b e*, *b z*, meet it, or its half breadth *e f*, which is easily had in the Right-angled Triangle *e f b*, the Angle *e b f*, and the Side *f b* being given.

Let us then consider *b* as a radiating Point, either farther from or nigher to the Glas *e z* than its Focus; And by *Prop. V.* and *VIII.* for Convexes, and by *Corol. XV.* for Concaves, let us determine the Respective, Imaginary, or Virtual Focus *s*, whereby we obtain *f s*. From *s* draw *s e*, *s z*, directly; Where these Lines *s e*, *s z*, meet the Distinct Base in *k* and *o*, there the Breadth of the Distinct Base is determin'd; To obtain the Measure whereof, say as $sf : fe :: sp : pk = po = k o$.

And note that in *Fig. 1* and *3*. $sp = sf - pf$. But in *Fig. 2, 4, 5.* $sp = sf + fp$.



Corollary 3.

If the Glas be a Compleat Sphere; Then, as the Distance of the Object from the Centre of the Glas : To the Breadth of the Object :: So the Distance of the Image from the same Centre of the Sphere : To the Breadth of the Image.

This is Manifest from *Tab. 23. Fig. 2.* Whereby it appears, *T23. F22* that a Glas Sphere *s p c* exposed to an Object *a b*; From each point *a, b*, of the Object there proceeds one certain Ray *a c n*, *b c m*, which passes Unrefracted; And these Rays Cross in the Centre *c* of the Sphere, by which means they are Perpendicular to both Sides of the Sphere, at their Ingrefs and Egress, and so pass Unrefracted.

Corollary 4. PROBL.

The Distance of an Object, and its Diameter being given, 'tis required to Represent its Image in the Distinct Base under a given Measure; The Thickness of the Glas being given also.

This is the 52 Problematical Proposition of *Gregorii Opt. Promot.* And is thus solved by our Doctrine.

Tab. 24. Fig. 6. *a v b* is an Object, whose Diameter *a b* *T24. F6.* is given, and its Distance from the Glas *v d* is given also: 'Tis required to Project this Object under a Given Measure *m n*; The Thickness of the Glas *d d* being also given. Let it be made, As *a b : v d :: m n : d i*. Then strike the Arch *x d x* with such a Radius, that the Rays flowing from the Point *v* may thereby be transmitted Parallel within the Body of the Glas (which shall be by the foregoing Doctrine, by making $\frac{1}{2} v d$ the Radius) Afterwards strike the Arch *z d z* with such

a Radius, that it may Unite the Parallel Rays at i , which shall be by making $\frac{1}{2} di$ the Radius: I say the Glas zx shall project the Image of ab under the given Measure mn . This is so manifest from Prop. IX. together with the foregoing Doctrine, that it wants no farther Explication.

Note. I have here consider'd the whole Thickness dd of the Glas in Conformity to *Gregory's* Proposition. But what foregoes concerning the Consideration of the Glasses Thickness may be apply'd here.

PROP. XXVII.

*The Object and its Image in the Distinct Base are Reciprocal.
vid. Prop. VI.*

J.22. F.1. Tab. 22. Fig. 1. abc being an Object, and the Convex Glas km Representing the Image thereof in the Distinct Base efg . Let us now conceive efg an Object, I say the same Glas km the same way posited shall project the Image thereof in the Distinct Base at abc .

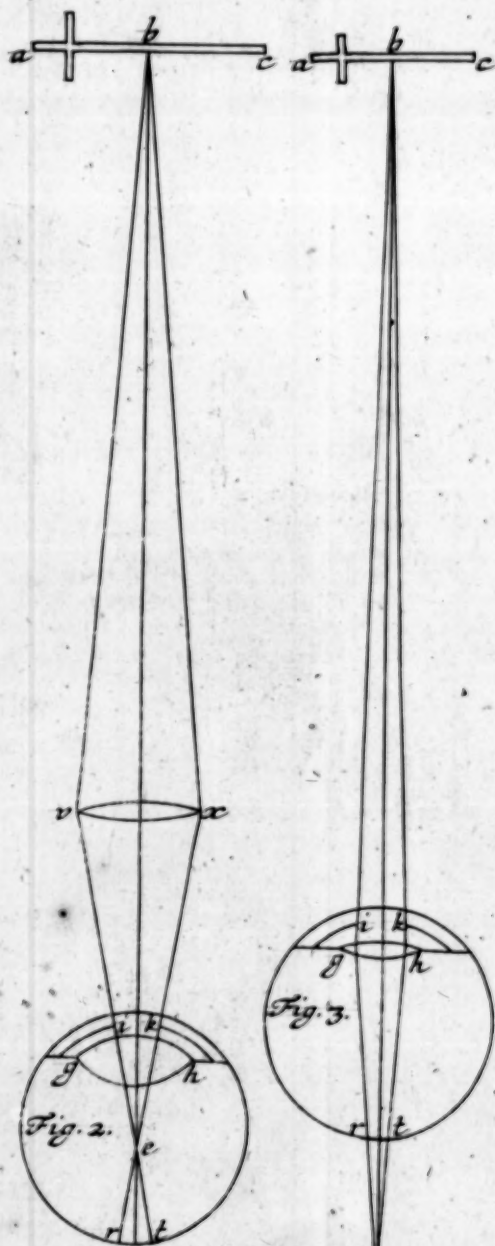
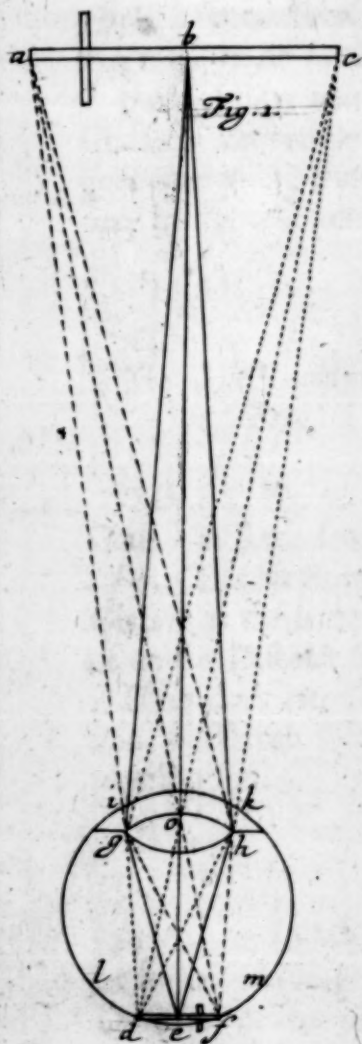
This does necessarily follow from the precedent Proposition, and from Exp. 8. And therefore needs no farther Demonstration.

Corollary.

Hence it follows that the Image in the Distinct Base may be sometimes larger than the Object. On which depends the Doctrine of the double Microscope, as hereafter shall appear more fully.

Let us imagine the Image of the Sun projected in the Focus of a Convex-glas, And let us now conceive the *Converse*, viz. That this Image were now the Real Sun, the Distinct Base
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of this would be as Large and as Remote as is now the Real Sun. So that every Convex-glass may be conceived to have two Foci, or Distinct Bases; one at the Object, t'other at the Image.

PROP. XXVIII.

The manner of Plain Vision with the naked Eye is expounded.

Tab. 25. F. 1. *abc* is an Object, *iklem* is the Globe of the Eye, furnish'd with all its Coats and Humors; But in this Figure we have only expressed the Crystalline Humour *gob*, as being Principally concern'd in Forming the Image on the Fund of the Eye. T25.F.1.

(1) From each Point in the Object we may Conceive Rays flowing on the Pupil of the Eye *ik*; as here from the middle Point *b*, there proceed the Rays *bg*, *bo*, *bb*; These by means of the Coats and Humours of the Eye, and especially by the Chrystalline Humour *gb*, are refracted and brought together on the *Retina* or Fund of the Eye in the Point *e*, and there the Point *b* is represented. For we may conceive the Crystalline Humour *gb* as it were a Convex-glass, in the Hole of a Dark Chamber *ilmk*, and that *def* is the Distinct Base of this Glass. What is here said of the Point *b*, and its Representation at *e*, may be understood of all the other Points in the Object, as of *a* and *c* and their Representations at *f* and *d*.

(2) And as in a dark Chamber, that has a Hole furnish'd with a Convex-glass, if the Paper, that is to receive the Image in the Distinct Base, be either nigher to, or farther from the Glass, than its due distance, the Representation thereon is confus'd; For then the Radius Pencils do not exactly determine with their Apices on the Paper; But those from one Point are mixt and confus'd with those from the Adjacent Points: so in the

the Case of Plain Vision, 'tis requisite that the Pencils should exactly determine their Apices at *d, e, f*, on the Retina, or else Vision is not Distinct.

'Tis therefore contrived by the *Most Wise and Omnipotent Framers of the Eye*, That it should have a Power of adapting it self in some Measure to *Nigh* and *Distant* Objects. For they require different Conformations of the Eye; Because the Rays proceeding from the Luminous Points of *Nigh* Objects do more Diverge, than those from more Remote Objects.

But whether this variety of Conformation consist in the Crystallines approaching nigher to, or removing farther from the Retina; Or in the Crystallines assuming a different Convexity, sometimes greater, sometimes less, according as is requisite, I leave to the scrutiny of others, and particularly of the curious Anatomist. This only I can say, that either of these Methods will serve to explain the various Phænomena of the Eye; And I am apt to believe, that both these may attend each other, *viz.* a Less Convex Crystalline requires an Elongation of the Eye, and a more Convex Crystalline requires a shortning thereof; As a more Flat Convex-Object-glass or of a Larger Sphere requires a Longer Tube, and one more Protuberant, bulging or of a smaller Sphere requires a shorter Tube.

(3) By the foremention'd Scheme we perceive, the Rays from each Point of the Object are all confus'd together on the Pupil in *g h*, so that the Eye is placed in the Point of the Greatest Confusion: But by means of the Humors and Coats thereof each Cone of Rays is separated, and brought by it self to determine in its proper Point on the Retina, there Painting distinctly the Vivid Representation of the Object; Which Representation is there perceived by the *sensitive Soul* (whatever it be) the manner of whose Actions and Passions, He only knows who Created and Preserves it, *Whose Ways are Past*

Past finding out, and by us unsearchable. But of this Moral truth we may be assured, *That He that made the Eye shall see.*

(4) We are likewise to observe, that the Representation of the Object *abc* on the Fund of the Eye *f e d* is *Inverted*. For so likewise it is on the Paper in a dark Room; there being no other way for the Radious Cones to enter the Eye or the dark Chamber, but by their Axes *a o*, *b o*, *c o*, crossing in the Pole *o* of the Crystalline or Glafs. And here it may be enquired; How then comes it to pass that the Eye sees the Object *Erect*? But this Quæry seems to encroach too nigh the enquiry into the manner of the Visive Faculties *Perception*; For 'tis not properly the Eye that *sees*, it is only the Organ or Instrument, 'tis the *Soul* that *sees* by means of the Eye. To enquire then, how it comes to pass, that the Soul perceives the Object *Erect* by means of an *Inverted* Image, is to enquire into the Souls Faculties; which is not the proper subject of this Discourse. But yet that in this Matter we may offer at something, I say, *Erect* and *Inverted* are only Terms of *Relation* to *Up* and *Down*, or *Farther from* and *Nigher* to the Centre of the Earth, in parts of the same thing: And that that is an *Erect* Object, makes an *Inverted* Image in the Eye, and an *Inverted* Object makes an *Erect* Image; That is, that part of the Object which is *farthest from* the Centre of the Earth is Painted on a Part of the Eye *Nigher* the Centre of the Earth, than the other parts of the Image. But the Eye or Visive Faculty takes no Notice of the Internal Posture of its own Parts, but uses them as an Instrument only, contrived by Nature for the Exercise of such a Faculty.

But to come yet a little nigher this difficulty; This enquiry results briefly to no more than this, How comes it to pass, that the Eye receiving the Representation of a Part of an Object on that part of its Fund which is *Lowermost* or *nighest* the Centre of the Earth, perceives that part of the Object as *Upper-*

most or farthest from the Centre of the Earth? And in answer to this, let us imagine, that the Eye in the Point *f* receives an Impulse or Stroke by the Protrusion forwards of the Luminous Axis *aof*, from the Point of the Object *a*; Must not the Visive Faculty be necessarily directed hereby to consider this stroke, as coming from the Top *a*, rather than from the bottom *c*, and consequently should be directed to conclude *f* the Representation of the Top?

Hereof we may be satisfy'd by supposing a Man standing on his Head: For here, tho the Upper Parts of Objects are painted on the Upper Parts of the Eye, yet the Objects are judged to be *Erect*. And from this Posture of a Man, the Reason appears, why we have used the Words *Farthest from*, and *Nighest to the Centre of the Earth*, rather than *Upper* and *Lower*. For in this Posture, because the *Upper* Parts of the Object are painted on that part of the Eye nighest the Earth, (though really the upper Part of the Eye) they are judged to be farthest removed from the Earth.

What is said of *Erect* and *Reverse* may be understood of *Sinister* and *Dexter*: But of these Physical Conjectures enough.

(5) The Image of an *Erect* Object being Represented on the Fund of the Eye *Inverted*, and yet the sensitive Faculty judging the Object *Erect*; it follows that when the Image of an *Erect* Object is Painted on the Fund of the Eye *Erect*, the sense Judges that Object to be *Inverted*.

This is a necessary Conclusion, and is of consequence for explaining some Particulars that follow.

(6) The *Magnitude* of an Object is Estimated by the Angle the Object subtends before the Eye: Thus in the same fore-mentioned *Tab. 25. Fig. 1.* the Length of the Object *ac* is estimated by the Angle $aoc = fod$, this we call *The Optick Angle*.

From hence, & from *Prop. XXVI, XXVII.* it follows, that if the Eye were placed instead of the Glass at *d* (*T. 22. F. 1.*) and *abc* or *T. 22. F. 1. efg* were Objects, the Eye would perceive them of Equal Bigness.

I know very well the Point *o*, which is the Vertex of the Optick Angle, is variously assigned by various Authors; some placing it in the Centre of the Eye; Others in the Vertex of the Crystalline; Others in the Vertex of the outward Coat or Cornea of the Eye: But 'tis a Matter of no great consequence, wherever we place it; for according to the Bigness of this Angle *aoc*, the Image on the Fund of the Eye is Bigger or Less. *Vid. Tacquet Optica. Lib. 1. Def. 4. and Prop. 3.*

I know likewise, that by a curious Experiment in Opticks discover'd by an Ingenious French-Man Monsieur Mariotte, 'tis controverted, whether the *Retina* or *Choroide* be the Seat of Vision, or the Place on which the Pictures of outward Objects are expressed (*vid. Philosoph. Transact. Num. 35. and 59.*) But to our business it matters not which of them we pitch on; and therefore I chuse to speak as commonly 'tis presumed; and mention the *Retina*, or rather the *Fund of the Eye*, as the Place that receives this Picture.

(7) In the same (*Tab. 25. Fig. 1.*) We perceive the Rays ^{T. 25. F. 1.} that flow from the Point *b* do proceed to the Eye *Diverging*, as *bg, bo, bh*; And if the Object *ac* were infinitely distant from the Eye, or so distant from the Eye, that the Breadth of the Pupil *ik* were insensible in Comparison to this Distance, then the Rays *bg, bo, bh*, would proceed as it were Parallel, and so fall on the Eye: In both which Cases, by means of the Refractions in the Eye, they are brought together, and paint the Image of the Point *b* on the Fund of the Eye at *e*. *Vid. Gregorii Opt. Prom. Pr. 30.*

But in *Tab. 25. Fig. 2.* If the Diverging Rays *bv, bx*, that ^{T. 25. F. 2.} flow from the Point *b*, meet the Convex Glass *vx*, and are thereby made to converge as *vi, xk*, and so fall on the Eye,

and there passing through the Crystalline $g b$, are made to Converge yet more as $i e$, $k e$; Here they cross in the Point e , before they reach the *Retina* $r t$, and consequently do paint thereon the Image of the Point b confusedly, for 'tis Painted on the space $r t$; whereas to cause distinct Vision, it should only be painted on a correspondent Point on the *Retina*.

And this is the Fault of their Eyes, who are called *Myopes*, *Purblind*, or *Short-sighted*. For in them the Crystalline is too Convex (as in this *Tab. 25. Fig. 2.* both the Convex Glass and Crystalline joyn'd together make too great a Convexity) uniting the Rays before they arrive at the *Retina*. And therefore they are helped by Concave-glasses, which take off from the too great Convexity of their Crystalline some part of its Refractive Power: Or rather these Concaves make the Rays Diverge so, that their Crystalline shall be sufficient only to bring them again together, so that they be not united, till they arrive at the Fund of the Eye.

Myopes are also helped by holding the Object very near; for then the Rays that fall on their Eye from any single Point do more Diverge, than when the Eye is farther from the Point, and consequently their too Convex Crystalline does but suffice to bring them together on the *Retina*.

(8) On the contrary, the Eyes of Old Men have their Crystalline too Flat (as *Tab. 25. Fig. 3.*) and cannot correct the Divergence of the Rays $b i$, $b k$, to make them meet on the *Retina* $r t$, but beyond the Eye at e . Wherefore for their Help 'tis requisite they add the Adventitious Convexity of a Glass; that both it and the Crystalline together, may be sufficient to unite the Rays just at the *Retina*: And from hence it appears, that Spectacles help Old Men, not by magnifying an Object, but by making its Appearance Distinct; for Old Men cannot read the largest Print without Spectacles, and yet with Spectacles, they read the smallest, though these with Spectacles

cles do not appear so large, as those without Spectacles.

(9) What is said of the *Confused* or *Distinct* Representation of a *Point* in the Object, may be understood of the *Confused* or *Distinct* Representation of the whole Object; at least for those Parts that lye pretty nigh adjacent to that Point that is looked at. For here we do not take a *Point* in the strict sense of the Mathematicians, but in a Physical Sense, for the smallest Part imaginable; or as we have assumed it in the first supposition. And the whole Object consisting of such Points, what is shewn of one Point may be understood of every Point in the Object, that is, of the whole Object.

(10) 'Tis Requisite also (before I proceed farther) to explain, what I mean by *Clear* Vision, and *Faint* Vision; *Distinct* Vision, and *Confused* Vision.

By what foregoes, I suppose these two latter Terms are pretty well understood, viz. *Distinct* Vision is then caused, when the Pencils of Rays from each Point of an Object do accurately determine in Correspondent Points of the Image on the *Retina*: *Confused* Vision on the contrary, when these Pencils do intermix one with another.

But *Clear* Vision is only caused by a Great Quantity of Rays in the same Pencil, illuminating the Correspondent Points of the Image *strongly* and *vigorously*.

Faint Vision is then when a Few Rays make up one Pencil; And tho this may be *Distinct*, yet 'tis *Dark* and *Obscure*, at least not so *Bright* and *Strong*, as if more Rays concurr'd.

Of Single Glasses apply'd to the Eye.

Hitherto I have spoken of Glasses by themselves; And of the Eye by it self: I come now to consider them both together.

But

But first 'tis to be Noted, that when we speak of the *Distinct* or *Confused* Appearance of an Object, 'tis needful only to explain my self concerning some one single Point in the Object; and for this we chuse the middle Point; for if we shew that single Point in the Object to be *Distinctly* or *Confusedly* Represented on the *Retina* (according to the XXVIII. *Prop.*) the same may be understood of the Adjacent Points in the Object.

But when we discourse of the *Erect* or *Inverted* Appearance of an Object: Or of the *Magnify'd* or *Diminish'd* Appearance of an Object; 'tis requisite we consider the whole Object. For a single Point, though Physical, cannot properly be consider'd as *Erect* or *Inverted*, or *Magnify'd* or *Diminish'd*.

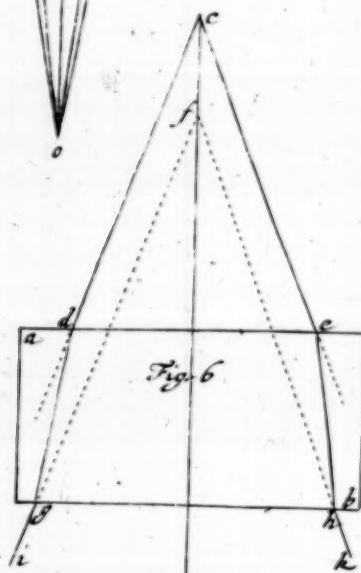
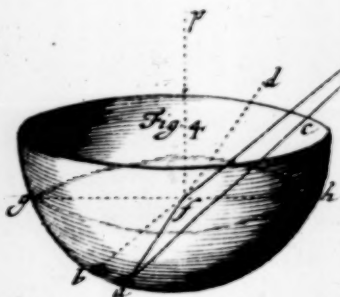
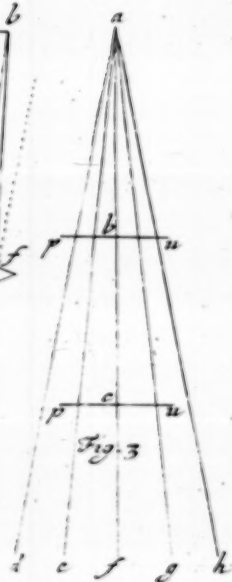
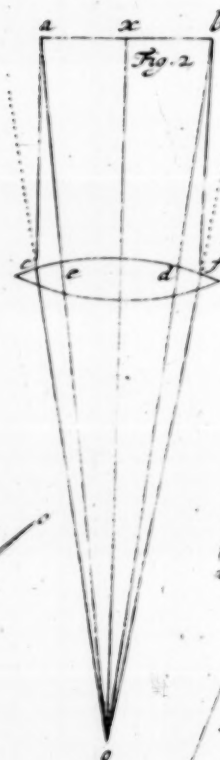
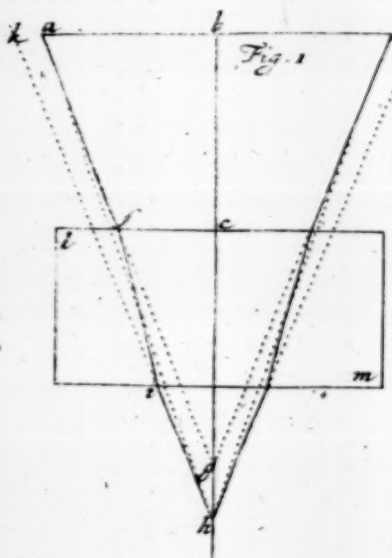
So that we perceive *Distinct* or *Confused* Vision to depend on the Formation of Rays proceeding from each single Point. But *Erect* and *Inverted*, or *Magnify'd* and *Diminish'd* Appearances depend on the Consideration of Rays proceeding from different Points of the Object. I shall always consider those, that proceed from the extremities of the Object.

P R O P. XXIX.

An Object seen through a Plain Glass whose Surfaces are Parallel is Magnify'd thereby.

This Proposition being directly contradictory to what HONORATUS FABER asserts in the XLIII. *Prop. Sec. 2.* of his *Synopsis Optica*, I shall mark my *Tab. 26. Fig. 1.* with the same Letters wherewith he marks his 89th Fig. He acknowledges that the *Apparent Place* of an Object is changed by a Plain Glass; but not that the Visual Angle is alter'd thereby. But I shall shew that the Optick Angle is Magnify'd thus; Let *ml* be a Plain Glass, *ab* an Object, *af* a Ray produced

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duced directly to g , but Refracted into fi ; And at its Emer-
 sion from the Glas at i , Refracted again into ih , parallel to
 afg . (by *Experim.* 1, 2.) Produce ih directly towards k ; let
 the Eye be at h , draw ha . The Angle, under which the Ob-
 ject ab appears through the Glas ml to the Eye at h , is ihb ,
 or khh , which is certainly greater than ahb , which is the na-
 tural Optick Angle. Wherefore the Eye at h through the
 Glas sees the Object ab , under a greater Angle than it would
 do without the Glas, and consequently the Object is magnify'd
 by the Glas. *Which was to be Demonstrated.*

But that which deceived **FABER** is, that because the An-
 gle agb is equal to the Angle khh , therefore (says he) the
 Optick Angle *with* and *without* the Glas are the same; or (as
 he has it) the Eye at g without the Glas would see the Object
 ab under the Angle agb , and through the Glas the Object
 is seen under the Angle khh equal to agb : Which is grant-
 ed him. But this does only prove, that the naked Eye at g , and
 the Eye through the Glas at h , sees the Object under the same
 Angle. Whereas he should have proved (if it were possible)
 that the naked Eye at h , and the Eye through the Glas at h ,
 sees the Objects under the same Angle; And then indeed he
 had rightly proved what he propoed, *viz.* that the Optick
 Angle is not magnify'd by the Interposition of the Glas: For
 'tis not Mathematical, first to place the Eye at g *without* the
 Glas, and then at h *with* the Glas, and proving the Optick
 Angles the same, to conclude that therefore the Plain Glas
 does not magnifie. For at that rate one may shew that even
 a Convex Glas does not magnifie; For at *one* Station an Object
 shall appear under as great an Angle *without* it, as at *another* Sta-
 tion *through* the Glas.

The Reason, why the Common Window Glas, &c. through
 which we look, causes no sensible Alteration of Objects, is be-
 cause 'tis too Thin. *Vid. Barrow Lect. Opt.* 15.

Con:

Concerning the *Apparent Place* of an Object through a Plain Glass, more hereafter.

PROP. XXX.

All Objects seen Erect through Convex Glasses are Magnify'd thereby.

Tab. 26. Fig. 2. Let axb be an Object, o the Eye, draw aeo , bdo , directly strait from the extremities of the Object to the Eye. The Angle comprised by the Direct Rays aeo , bdo , that is the Angle $ao b$, is the *Natural Optick Angle*. Let now the Glass cf be interposed; here because the Rays ae , bd , would *naturally* and by their *Direct* course concur at o , now the Glass is interposed, their Concourse shall be accelerated before they arrive at o . Wherefore the Eye at o shall not perceive the extremities of the Object through the Glass by the Rays ae , bd , but by some *other Rays*. And these *other Rays* must fall either *without* (that is, farther from the Axis ox , than) ae , bd , or they must fall *within* ae , bd : But they cannot fall *within* ae , bd ; for if ae , bd themselves be made by the Glass to concur *before* they arrive at o , much more shall any other Rays that fall *within* ae , bd , be made to concur *before* they arrive there: And consequently they cannot convey the Appearance of the Extreme Points a , b , to the Eye at o . Wherefore it remains, that the Rays that do this must fall *without* ae , bd : Let these Rays be ac , bf , which by the Refractive Power of the Glass are bent from their Direct Course and are made to proceed in co , fo , crossing at the Eye in o ; Here the Optick Angle through the Glass is cof , which is greater than the Natural Optick Angle $ao b$, and consequently the Object is *Magnify'd*. Which was to be Demonstrated.

Prop.

PROP. XXXI.

Concerning the Apparent Place of Objects seen through Convex-glasses.

(1) In Plain Vision the Estimate we make of the Distance of Objects (especially when so far removed, that the Interval between our two Eyes, bears no sensible Proportion thereto; or when look'd upon with one Eye only) is rather the Act of our Judgment, than of Sense; and acquired by Exercise and a Faculty of comparing, rather than Natural. For Distance of it self, is not to be perceived; for 'tis a Line (or a Length) presented to our Eye with its End towards us, which must therefore be only a Point, and that is Invisible. Wherefore Distance is chiefly perceived by means of Interjacent Bodies, as by the Earth, Mountains, Hills, Fields, Trees, Houses, &c. Or by the Estimate we make of the Comparative Magnitude of Bodies, or of their Faint Colours, &c. These I say are the Chief Means of apprehending the Distance of Objects, that are considerably Remote. But as to nigh Objects, to whose Distance the Interval of the Eyes bears a sensible Proportion, their Distance is perceived by the turn of the Eyes, or by the Angle of the Optick Axes. (*Gregorii Opt. Promot. Prop. XXVIII.*) This was the Opinion of the Antients, *Alhazen*, *Vitellio*, &c. And tho the Ingenious Jesuit *Tacquet* (*Opt. Lib.I. Prop.II.*) disapprove thereof, and Objects against it a New Notion of *Gassendus* (of a Man's seeing only with one Eye at a Time one and the same Object) yet this Notion of *Gassendus* being absolutely False (as I could Demonstrate, were it not beside my Present Purpose, but I refer to the 7th Chap. of the 2d Part.) it makes nothing against this Opinion.

(2) Wherefore Distance being only a Line, and not of it self perceivable; if an Object were convey'd to the Eye by

one single Ray only, there were no other means of judging of its Distance, but by some of those hinted before. Therefore when we estimate the Distance of *nigh* Objects, either we take the help of both Eyes, or else we consider the Pupil of one Eye as having *Breadth*, and receiving a *Parcel* of Rays from each Radiating Point. And according to the various Inclination of the Rays from one Point, on the various Parts of the Pupil, we make our Estimate of the Distance of the Object: And therefore (as is said before) by one single Eye we can only Judge of the Distance of such Objects, to whose Distance the Breadth of the Pupil has a sensible Proportion. To illustrate all this by *Tab. 26. Fig. 3.* Let *a* be a Radiating Point, sending forth the Rays *ad, ae, af, ag, ah*, with all the intermediate Rays. Let *pu* be the Breadth of the Pupil, and first placed at *c*, there receiving only *ae, af, ag*: Let *pu* be translated to *b*, where it receives all the Rays; and 'tis manifest that the Rays from *a* are differently inclined on the Pupil in one and t'other Posture, for the Rays, which fall on the Pupil when placed at *b*, diverge very much more than those that fall upon it, when placed at *c*: And therefore the Eye, or Visual Faculty will apprehend the Distance of *a* from *b* and *c* to be *Different*. For it is observed before (*Prop. 29. Sect. 2.* see also, *Gregorii Opt. Promot. Prop. XXIX.*) that for viewing Objects *Remote* and *Nigh*, there are Requisite Various Conformations of the Eye: The Rays from *Nigh* Objects, that fall on the Eye, Diverging more than those from more *Remote* Objects.

(4) If therefore by Refraction through Glasses, that parcel of Rays which falls on the Pupil from each Point in *Nigh* Objects be made to flow as close together as those from *Distant* Objects; or the Rays from *Distant* Objects be made to *Diverge*, as much as if they flow'd from *Nigh* Objects, the Eye through such Glasses shall perceive the *Place of the Object changed*.

(5) But

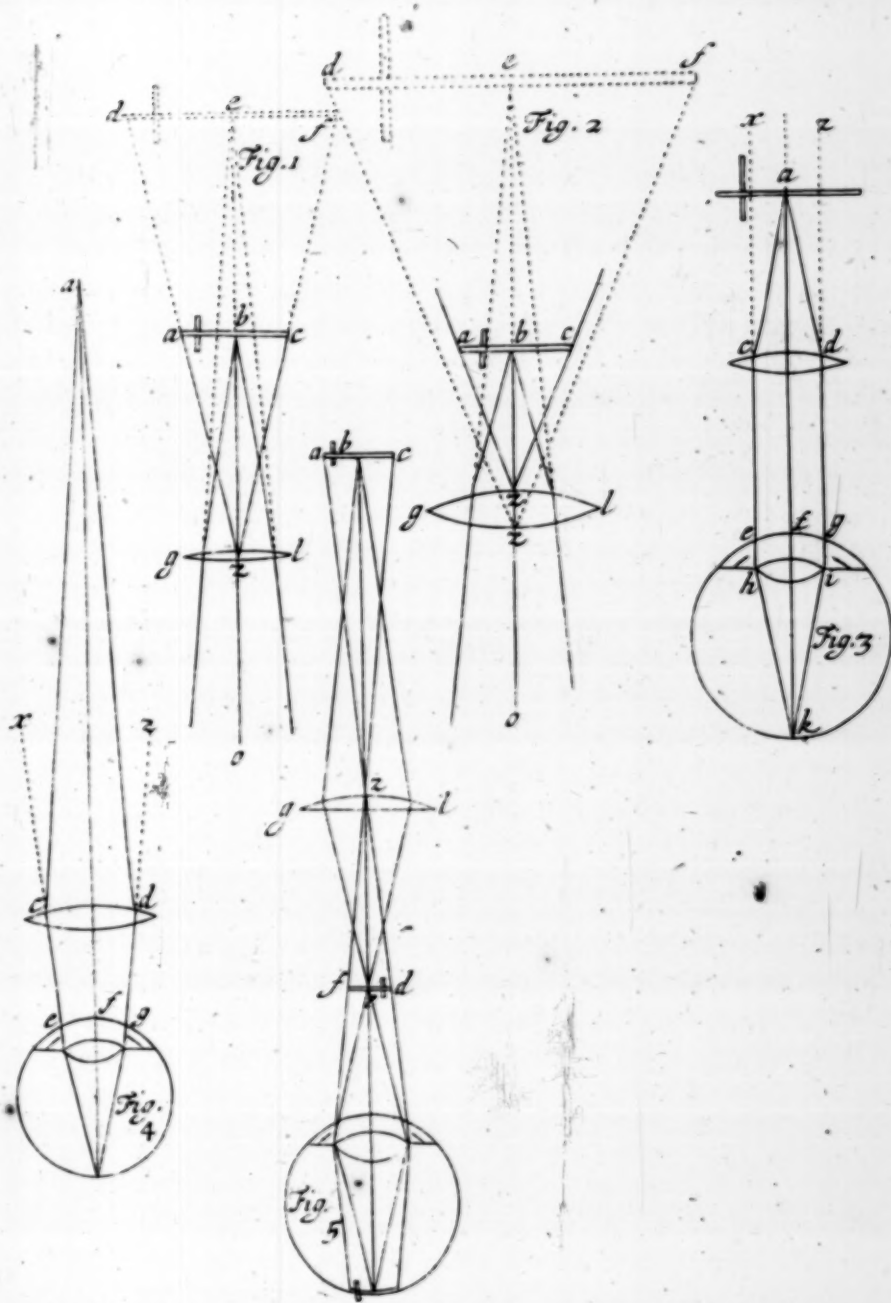
(5) But first for a sensible and common Experiment, to shew the *Change of an Objects Place by Refraction.* *Tab. 26. F. 4.* T.26. F.4. represents a Vessel, on whose Bottom at *a* there is laid a piece of Mony, or any other remarkable Object, so that the Eye at *o* may just perceive the Mony over the Edge *c* of the Vessel. The Mony now appears to the Eye *o* by the direct Line *a c o*. Let now the Vessel be fill'd with Water up to *g f b*; let the Ray *a f* proceed from the Mony, and draw *p f* perpendicular to *g b*; the Ray *a f*, instead of going onwards directly to *d*, emerging from Water, a dense Medium to Air, deflects from the perpendicular *p f*; and becomes (suppose) *f o*. Wherefore the Eye *o*, now the Vessel has Water in it, sees the Silver, not by the direct Ray *a c o* (for that is bent from it, and escapes it) but by the refracted Ray *a f o*. Produce *o f* directly to *b*, the Mony shall appear as if it were at *b*. For the Eye is not sensible of the bending of the Ray, but is affected by it, as if it were directly strait.

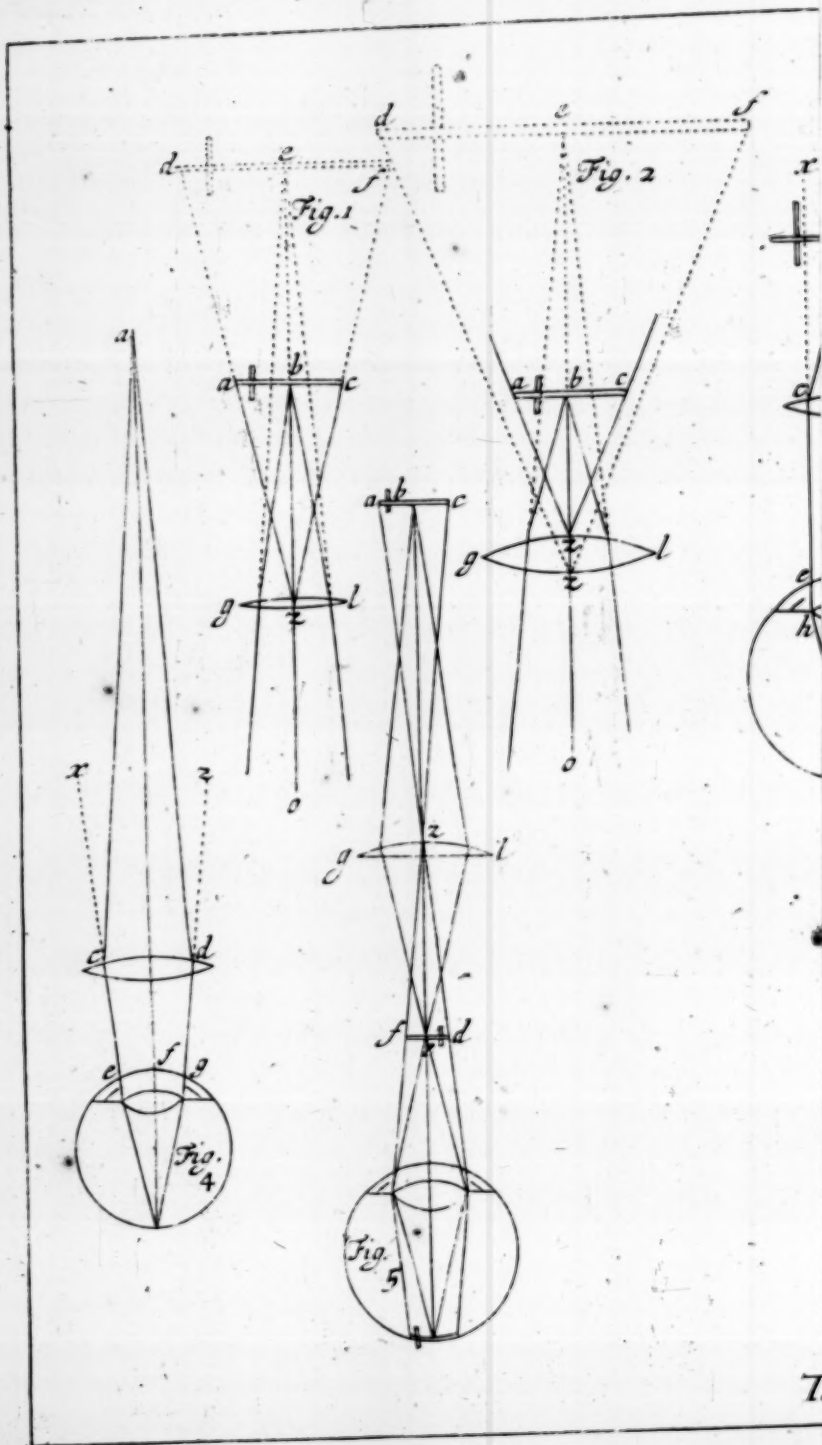
(6) In like manner *Tab. 26. F. 5.* if the Point *c* send its Ray *c e* obliquely on the plain Glass *a b*; and after a double Refraction it arrive at the Eye *o*; This Point *c* is not now seen in its own proper Place, but somewhere in the Ray *o g* produced, as at *f*. For the Eye is not sensible of the outward accidental Refraction, that attends the Ray at its passage through the Glass; but is directed by the next immediate Ray *o g* that falls upon it, and considers it as *strait*, and coming directly from the Point *f*. T.26. F.5.

I say moreover, that by a plain Glass, the place of an Object is changed, and brought nigher the Eye. *Tab. 26. F. 6.* T.26. F.6. *c* is an Object sending its Rays *c d*, *c e*, upon the plain Glass *a b*; these after a double Refraction proceed (by supposition) in *g i*, *h k*. Produce these directly towards *f*; and suppose the Pupil of the Eye large enough to receive the Rays *g i*, *h k*; the Point *c* shall appear to the Eye as at *f*.

(7) To determine the Locus Apparens of an Object placed nigher a Convex-glass than its Focus.

To perform this, we are to have, the Power of the Glass, the Distance of the Object from the Glass, and the Length of the Object given. *Tab. 27. Fig. 1.* abc is an Object, whose Distance from the Glass bz is given. Let us suppose the Glass of the least thickness imaginable, that we may not be at the trouble of considering the Optick Angle, both at the Immersion and Emerision. Let the middle Point b Radiate upon the Glass, and after the Rays have passed the Glass, let them be so Refracted, as if they came directly from the Point e . From the foremention'd Data, this Point e , being the Imaginary Focus, is easily determin'd by *Prop. VIII.* hereof: Through e draw the Infinite Right Line def Parallel to abc . Wherefore as the Locus Apparens of the Point b is at e , so the Loci of the Collateral Points a, c , shall be somewhere in the Line def (unless perhaps Convexes on very small Spheres, will Represent the Object Crooked or Bowed, but of this we shall take no notice) to determine which, from the Vertex of the Glass z draw the Lines za, zc (or
T. 27. F. 2. if the Glass have any Thickness, as in *Tab. 27. Fig. 2.* from the Vertex of Immersion, or outward Vertex, draw za, zc ; And from the Vertex of Emerision or Inward Vertex, draw zd, zf , Parallel to za, zc , and then the Thickness of the Glass must be one of the Data. *Vid. Prop. 47. Gregorii Opt. Promot.*) the Points d and f are the Loci Apparentes of the Points a and c . For certainly were the Eye behind the Glass just at z , the Object abc would appear under the Angle azc , and the Point b would appear as at e , and therefore the Points a and c would appear somewhere, so as to make the Object keep the same Angle; But that can only be by the Points a and c appearing at d and f (supposing the Object to appear in its own Natural strait shape) Wherefore d and f are





are the *Loci* of the Points a and c , and def is the *Locus* of the Object abc .

Hence it appears, that a Convex-Glass Represents Objects as farther off than really they are: And this is the Reason, why Pieces of Perspective (as of Churches and Long Porticoes) appear very Natural and strong through Convex-Glasses duly apply'd. For these Glasses making Objects appear further off than really they are, must consequently make the Parts of the Perspective seem really *Hollow'd* or sunk in, the French term it *Renfoncè*.

Hence also 'tis Manifest, why Convex-Glasses help the Eyes of those, that see only *Distant* Objects, as *Old Men*, for these make *Nigh* Objects appear as *Distant*. *Vid. Prop. XXVIII. Sec. 8.*

We may perceive also that the Distance between the Object and Glass continuing the same, the *Locus Apparens* is never alter'd, though the Eye be removed to and from the Glass. *Gregorii Opt. Promot. Corol. Prop. XLVII.* For the Distance between the Object and Glass continuing the same, the Imaginary *Focus* e shall always be the same.

(8) The *Locus* of an Object expos'd to a Convex-Glass in its *Focus* is not to be determin'd.

When the Radiating Point a . *Tab. 27. Fig. 3.* is placed T. 27. F. 3. in the *Focus* of the Convex-Glass cd , the Rays ce , dg , after passing the Glass, run Parallel, and being produced to ex , dz , they never Intersect. Wherefore in this Case there is no Rule whereby to determine the *Locus* of the Object. And Barrow tells us only, *Quod Remotissime Positum Æstimatur. Lect. 8. ad Finem.*

(9) The *Locus* of an Object, beyond the *Focus* of a Convex-Glass, to the Eye between the Glass and *Distinct Base*, cannot be determin'd.

T. 27. F. 4. *Tab. 27. Fig. 4.* Let *a* be a Radiating Point placed more distant from the Glass *cd* than its Focus; the Rays *ce*, *dg*, after passing the Glass, do *Converge* towards the Distinct Base; let the Eye *efg* be placed between the Glass and Distinct Base, it shall then receive the Rays *ce*, *dg*, *Converging*; and they being produced towards *x* and *z* separate the further.

In this and the last Section lies the great Difficulty, which the *Incomparable* and *most profoundly Learned* BARROW (*Lect. Opt. 18. Sect. 13.*) confessedly passes over as insuperable, and not to be explained by whatever Theories we have yet of Vision. For seeing that the Object which applies to the Eye by *Less-diverging* Rays, is judged the *more remote*; And that which applies to the Eye by *Parallel* Rays, is reputed *most remote*; it should seem reasonably to follow, that what is seen by *Converging* Rays, should appear yet *most remote* of all: And yet Experience contradicts this, and testifies, that the Point *a*, *Tab. 27. Fig. 4.* appears variously *distant*, according to the various Situations of the Eye between the Glass and Distinct Base; and that it does almost never (if ever) appear *more distant* than the Point *a* it self to the naked Sight, and sometimes it appears *much nigher*: Or rather, by how much the Rays, which fall through the Glass on the Eye, do *more Converge*, by so much the *nigher* does the Object appear to approach, insomuch, that if the Eye approach the Glass *very nigh*, the Object *a* appears in its *natural Place*: The Eye being a little farther removed from the Glass towards the Distinct Base, the Point *a* seems yet to approach. The Eye being yet farther, the Point seems yet *nigher*; and so by degrees, till at last the Eye being placed at a certain station, as at the *Distinct Base*, the Point *a* appears *very nigh*, so that it begins to vanish away in mere *Confusion*. All which (continues the candid BARROW) seems repugnant, or at least not so well to agree to what we have laid down. And so he leaves

leaves this Difficulty to the solution of others, which I (after so great an Example) shall do likewise; but with the resolution of the same admirable Author, of not quitting the *evident Doctrine*, which we have before laid down, for determining the *Locus Objecti*, on the account of being pressed by one Difficulty, which seems inexplicable, till a more intimate Knowledge of the *Visive Faculty* be obtained by Mortals. In the mean time, I propose it to the consideration of the ingenious, whether the *Locus Apparens* of an Object placed as in this 9th. Section, be not as much *before* the Eye, as the *Distinct Base* is *behind* the Eye. *Vid. Corol. 1. Prop. LVII.*

(10) If an Object be more distant from a Convex-Glass than its Focus, and the Eye beyond the *Distinct Base*, the *Locus Apparens* of the Object is in the *Distinct Base*. *Vid. Prop. XXXIX. Sect. 5. item Schol. Prop. L.*

Tab. 27. Fig. 5. The Object *abc* is projected by the Glass *gzl* in the *Distinct Base def*. The *Locus* of each Point in the Object is in the correspondent Point of the Image in the *Distinct Base*: Thus *a* appears at *d*, *b* at *e*, *c* at *f*. *Gregorii Opt. Prom. Prop. XLVI.* T27.F.5

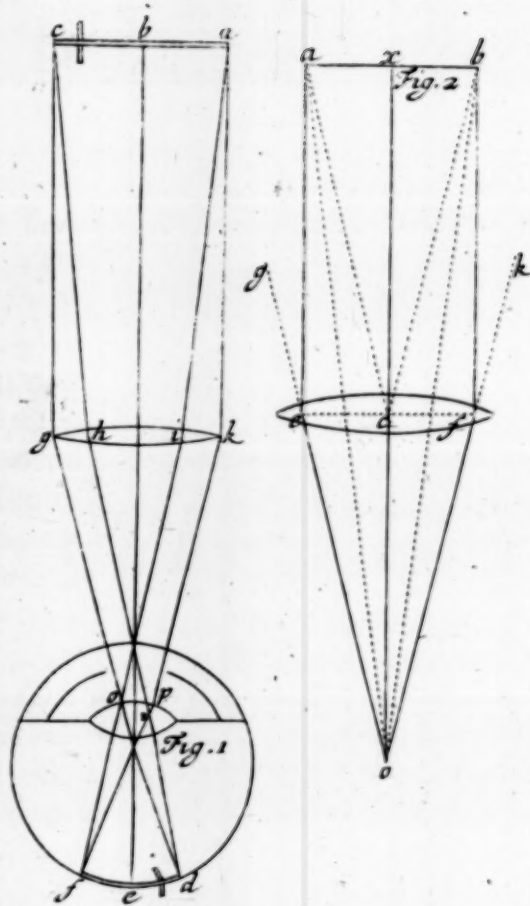
Dechales (*Dioptr. Lib. II. Prop. XI.*) remarks a Matter of some moment in this Business of the *Locus Objecti*. The reason (says he) that the Appearances through Glasses of the Change of the Objects Place, do not so strongly strike the Sense, as the Doctrine here laid down seems to intimate, proceeds from hence; that Optick-Glasses are seldom or never made so large, as to be look'd through by both Eyes at once; for if they were, he asserts, That the *Locus apparens Objecti* would be much more plainly and sensibly determin'd to the sight: In this particular certainly he is much in the right; for we see at all times, that two Eyes make a more exact estimate of the Position of an Object, than one single Eye. And we have a sensible Experiment hereof in *Catop-tricks*,

tricks, by the large concave Miroirs, where an Hand and Dagger presented beyond the Focus, seem to strike far without the *Speculum*, at him that presents it: But not so strongly if the *Speculum* be but small, so that the Image can be seen but by one Eye at once.

Scholium.

To this Property of the Change of the *Apparent Place* of an Object, may we attribute that common Effect of Objects seeming to *Dance* and *Move*, being seen through Spheric-Glasses, whether Convex or Concave, nimbly shaken between the Eye and the Object. 'Tis a noted Experiment, that to know, whether a Glas be Plain or Spherical (which is not to be found by Inspection of the Glas it self, or by Touch, if the Glasses be formed on large Spheres, and be but small portions thereof), the common Tryal is, to shake them something nimbly between the Eye and an Object; and if the Object seem to move by the motion of the Glas, the Glas is *not Plain*: The reason hereof is this; That toward the *Extremities* of a Glas, the Glas refracts *more* than towards the middle (for the very middle Ray, that is perpendicular, is not refracted at all) and consequently the *Apparent Place* of an Object is more changed by the Refraction in the *Extreme* parts of the Glas, than in the *middle* of the Glas: So that the Object, by the Motion of the Glas, appearing sometimes through the middle of the Glas, sometimes through the extremities of the Glas; the *Apparent Place* thereof is *varied* likewise, and the Object seems therefore to *change its Place*, or to *move*. But in a plain Glas the Case is otherwise, for the Refraction thereof is equally prevalent throughout the whole Glas, and neither *stronger* nor *weaker* in the *middle*, than in the *extremities*; so that the shaking of it between the Eye and

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Tab. 28. pag. 122.

and Object, makes no other difference in the *Apparent Place*, than if the Glas were look'd through fixt and immoveable; and therefore the Object seems not through it to shake and move. And tho the *Locus* be changed by a Spherick Glas's being removed to and from the Eye, yet in Glasses of large Spheres, the *Locus* is not so sensibly changed, as by being shaken before the Eye. The same Reason holds for the apparent Dancing of Objects seen through rising Smoaks or Vapors.

PROP. XXXII.

An Object being placed in the Focus of a Convex-Glass, and the Eye on tither side the Glas, sees this Object distinctly and erect.

That an Object so posited, is seen distinctly, is evident, because the Rays from each particular Point, after passing the Glas, become parallel (by Prop. VI.) and so fall on the Eye. And therefore (by Prop. XXVIII. Sec. 7.) they are fit to cause Distinct Vision. *Tab. 27. Fig. 3.* The Point *a* sendeth forth its Diverging Rays *ac*, *ad*, on the Convex-Glas *cd*; these are transmitted by the Glas parallel in *ce*, *dg*, and so fall on the Eye; by whose Coats and Humors, especially by the Crystalline *hi*, they are refracted and brought together in the Point *k* on the *Retina*, there representing distinctly the Point *a* of the Object. T27. F.3.

Or thus, *Tab. 28. Fig. 1.* The Eye *op*, being in the place where some of the Rays from every Point in the Object are mixt together, is in the place of the greatest Confusion; and therefore by Prop. XXVIII. Sec. 3. the Vision is distinct. T28. F.1.

I say also the Object is seen erect. In the same Figure *gk* is a Convex-Glas, *abc* an Object; we may imagin that each Point of this Object sends from it a Cone of Rays, fall-

R

ing

ing on the whole Surface of the Glas; but for avoiding Confusion in the Scheme, I only express and consider the Rays cg , cb , from the Point c ; and ak , ai from the Point a . op is the Crystalline of the Eye. The Rays cg , cb , after passing the Glas become parallel as go , bp , and fall so on the Crystalline, by whose Refractions they are again brought together in the Point d on the Retina, there painting the lively Representation of the Point c ; and the same may be shewn of the Representation of the Point a at f on the Retina. Wherefore the *sinister* Point c is represented on the *dexter* part of the Fund of the Eye d ; and the *dexter* part of the Object a is painted on the *sinister* part of the Eye f . And consequently the Image on the Retina being inverted, the Object is seen erect (by Prop. XXVIII. Sec. 4, 5.). Which was to be Demonstrated.

PROP. XXXIII.

An Object being placed in the Focus of a Convex Glas, and the Eye being placed in the Focus, on t^other side the Glas, sees this Object under the same Angle, as were the naked Eye placed at the station of the Glas.

Tab. 28. Fig. 2. axb is an Object placed in the Focus of the Glas ef (which we suppose of the least thickness imaginable). Let the Rays ae , bf , run parallel to the Axis xco , these Rays are united in the Focus at o , at which Point we suppose the Eye. Produce oe and of directly to g and k , and draw ecf (which by supposition is parallel to axb , and then draw ac , bc . The Optick-Angle through the Glas is eof , which we are to prove equal to acb . Thus, ax and ec being parallel, and ae , xc , being so likewise; ax is equal to ec , and ae to xc , being the opposite Sides of a Parallelogram. But xc is equal to co (the Focal Length or one side

side and t'other). Wherefore the Right-angled Triangle eco is equal and similar to the Right-angled Triangle axc . Wherefore the Angle $ecoc$ is equal to the Angle acx , or eof is equal to acb . And consequently the Eye at e sees the Object through the Glass under the same Angle, as the naked Eye being placed at c , would see it. Which was to be proved. Vid. *Gregorii Opt. Prom. Prop. XLIV.*

Scholium.

This is the Posture of both Eye and Object for causing the most perfect Vision possible through a single Convex Glass; unless perhaps we say, That seeing all Objects, which we perceive most distinctly are pretty nigh our Eyes; therefore we may conceive the Rays flowing from each Point of them, as falling on the Pupil *Diverging*. And consequently, that to see an Object most perfectly through a Convex Glass, 'tis best that the Object be placed a very little nigher the Glass than its Focus: For then the Rays, after passing the Glass, do something *Diverge*, and fall so on the Pupil; and this being the most natural and usual occurrence of Objects, it suits best, and is best adapted and most agreeable to the Eye; which by its refractive Coats and Humors easily collects each Cone of these *Diverging* Rays, and brings them together in a Point on the Fund of the Eye.

Corollary 1.

The Eye and Object being so placed as above; the Eye sees this Object *magnify'd* under an Angle almost *double* to that under which the Object would appear to the Eye, were the Glass removed; that is, the Angle $ecoc$ or acx is almost double the Angle aoc (ao and bo being drawn directly) for the Angle acx is equal to $aoc + cao$ (32. 1.) but cao

is almost equal to aoc . Therefore the Angle acx is almost double aoc . Which was to be Demonstrated. That cao is almost equal to aoc , is manifest; for the Triangle oca is almost Ilosceles, $oc (=cx)$ being almost equal to ca .

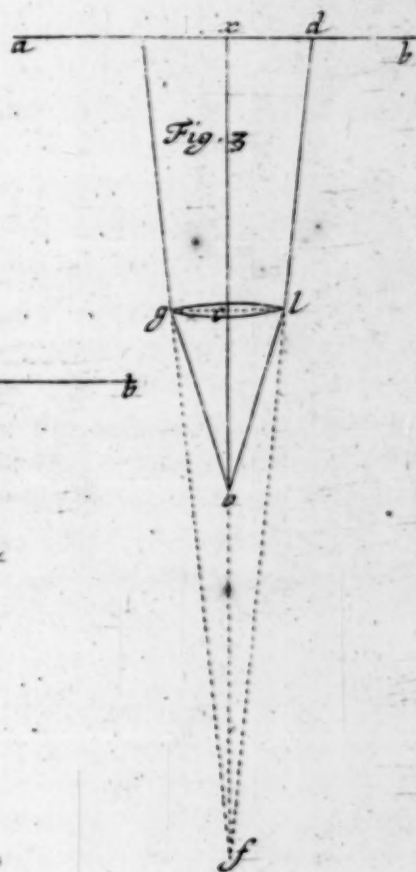
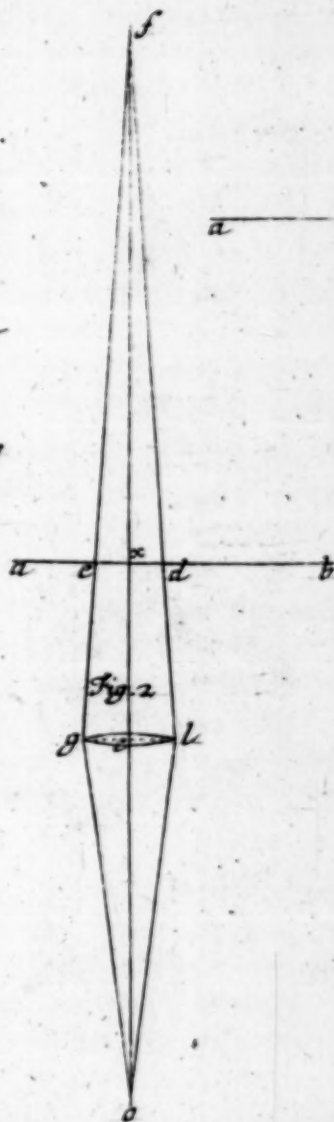
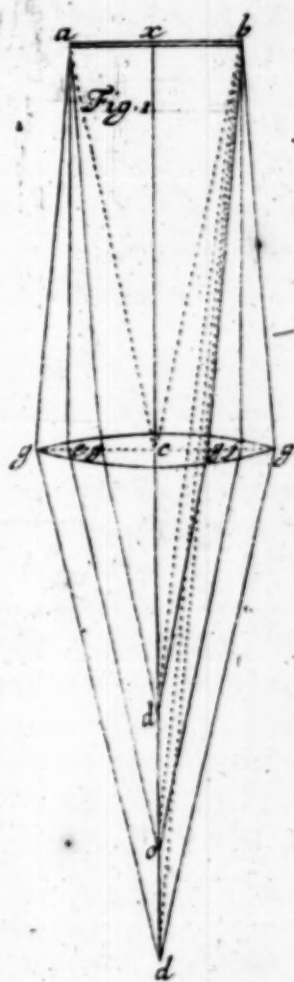
If now we suppose the Eye to continue in the Focus at o , and the Object axb to be brought nigher the Glas ef ; still the Optick Angle through the Glas shall be eof . By which (tho it continue the same, yet) the Object would not be magnified by the Glas in this second Posture, as much as in the former. For the Angle eof in this latter Posture would not so much exceed the natural Optick Angle aob . For at last axb being brought so near ecf , as to be coincident therewith, the Angles eof and aob would be coincident also.

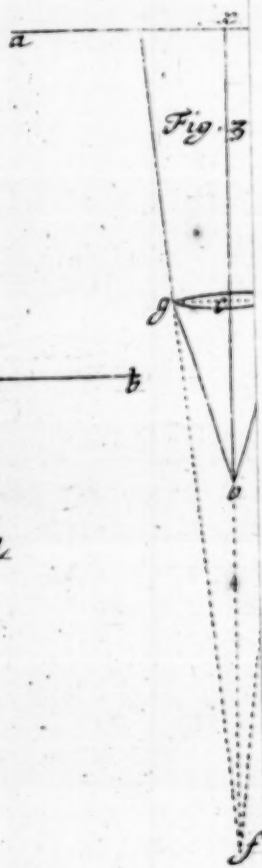
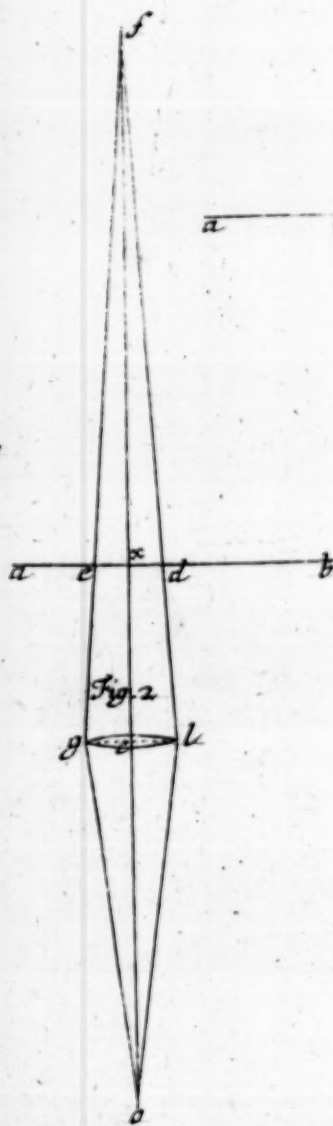
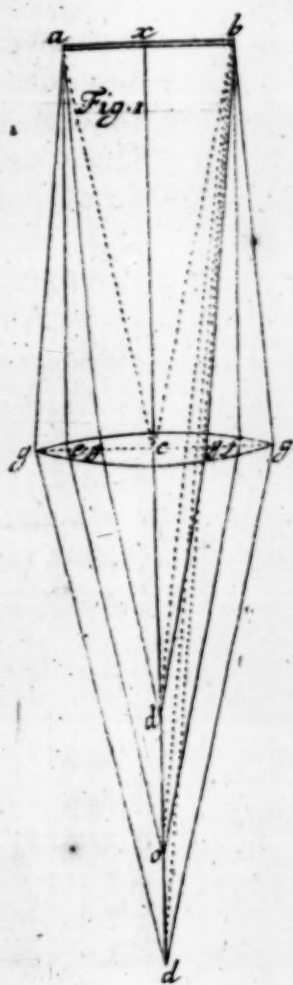
The same may be shewn, if we suppose the Object to continue in the Focus, and the Eye o to approach the Point c . For at last the Eye arriving at c , perceives the Object through the Glas under its own natural Angle acb .

Corollary 2.
Hence it appears, that the Eye being in the Focus of a Convex Glas, and the Object in the Focus also, the Eye can perceive no greater an Area or Space of the Object, than the Breadth of the Glas, or the Breadth of that Portion thereof, which the Eye makes use of; for ab is equal to ef . And the reason is, because those Rays ae , bf , that come from the Extremities of the Object a and b , and concur at the Eye in the Focus o , must necessarily fall on the Glas parallel to the Axis xc .

Hence it appears, that the Eye being in the Focus of a Convex Glas, and the Object in the Focus also, the Eye can perceive no greater an Area or Space of the Object, than the Breadth of the Glas, or the Breadth of that Portion thereof, which the Eye makes use of; for ab is equal to ef . And the reason is, because those Rays ae , bf , that come from the Extremities of the Object a and b , and concur at the Eye in the Focus o , must necessarily fall on the Glas parallel to the Axis xc .

PROP.
If the Angle acx be almost equal to aoc , then the Angle acx is almost double aoc .





PROP. XXXIV. PROBL.

To determine the Optick Angle, or apparent Magnitude of an Object in the Focus of a Convex-Glass, to the Eye at any other Station; from these Data, the Glasses Focal Length, or the Distance of the Object from the Glass xc , Tab. 29. F. 1. T. 29. F. 1.
 The Distance of the Eye from the Glass cd , and the Breadth of the Object ab .

The Thickness of the Glass I take no notice of, because I would not perplex the Scheme. And therefore the Glass is supposed of the least Thickness imaginable; or all the Refractions are supposed performed at the Line in the middle of the Glass gec .

Wherefore let axb be an Object, and let the Rays ae , bf , fall parallel to the Axis xco ; these are united in the other Focus at o ; where if the Eye be placed, it sees the Object under the Angle $eof = acb$, by the preceding Prop. Wherefore having bx the half Object, and xc ; in the Right-angled Triangle bxc , we may find the Semi-optick Angle bcx . Draw the prickt Line bo directly; the Angle box is the natural Optick Angle of the Object bx , which is easily obtained, in the Right-angled Triangle box , having bx and xo .

Let now the Eye be out of the Focus at d ; the Rays ag , after passing the Glass, become parallel to eo (Prop. VI.) And therefore the Angle gdx is equal to box or acx , which we have found before. Draw the prickt Line bd directly; the Angle bdx is the natural Optick Angle of the Object bx , were the Glass removed; and is easily had in the Right-angled Triangle $bx d$, having bx and $xd (= xc + cd)$. So the Problem is satisfied.

PROP.

PROP. XXXV. PROBL.

To determine the Visible Area of an Object in the Focus of a Convex Glass; from these Data, the distance of the Object from the Glass or the Glasses Focal Length, the distance of the Eye from the Glass, and the Breadth of the Glass.

It is shewn before in the 2d. Corollary to Prop. XXXIII. That the Visible Area of an Object in the Focus of a Convex Glass, to the Eye placed in t'other Focus, is equal to the Breadth of the Glass. Wherefore I shall only here consider the two other Cases.

T. 29. F. 2. And first, Let the Eye at o , Tab. 29. Fig. 2. be placed further from the Glass gl than its Focus, the Object $a \times b$ is in the Focus, gl the Glasses Breadth, xc the Glasses Focus or distance of the Object, and co the distance of the Eye from the Glass are given. Draw go , lo , and produce the Axis ocf infinitely. Let us now imagin the Point o a nigh Radiating Point or Object, sending its Rays og , oc , ol , on the Glass. By Prop. V. Let us determin the Distinct Base or Respective Focus of this Point projected by this Glass, which by the foregoing Data is easily performed. Suppose this respective Focus to be cf . Draw gf , lf ; where these Lines intersect the Object in e and d , the Visible Area of the Object is determined by ed .

Demonstration. For by Exper. 8. the Progress of a Ray through different Mediums is Reciprocal. Wherefore if the Rays og , ol , be refracted into ge , ld ; it will necessarily follow, that eg , dl , being consider'd as two Rays flowing from the Points e and d , they shall be refracted into go , lo ; wherefore this Analogy will hold, $fc : cl :: fx (=fc - xc) : xd$. Which xd is half the Visible Area to the Eye at o .

Secondly,

Secondly, *Tab. 29. F. 3.* Let the Eye at *o* be placed *nigher* ^{*T29. F. 3.*} the Glass than its Focus. Draw *go, lo.* And produce the Axis *oc* infinitely backwards towards *f*. Then (as before) supposing the Point *o* a Radiating Point or Object; by the *Data*, and by *Prop. VIII.* we may easily determin the *Imaginary* Focus thereof, which let be *cf*. From *f* draw *fg* directly to *e*, and *fl* to *d*. *ed*, by the foregoing Demonstration, is the Visible Area of the Object *ab*. And $cf : cl :: f x (=fc + cx) : xd$. Which is half the Visible Area to the Eye at *o*.

Scholium.

The same Rules may be used, when the Object is *nigher* to or *further* from the Glass than its Focus. So that this Proposition is universal to all Cases in *Erect* Vision through Convex Glasses. But whether this Area be *distinctly* visible or not, depends on other Considerations, and will be obvious enough to those that consider what *has* been before, and what *shall* be hereafter laid down concerning the *distinct* and *confused* appearance of Objects.

PROP. XXXVI.

An Object being placed more distant from a Convex Glass than its Focus, and the Eye placed on t'other side this Glass nigher to the Glass than the Distinct Base to the Glass, sees this Object erect and confused.

I have shewn before (*Prop. V. XXVI, XXVII.*) that if an Object be placed more distant from a Glass than its Focus, this Object is projected in a distinct Base somewhere on t'other side the Glass. And that this Distinct Base, may be sometimes *Less*, sometimes *equal* to, and sometimes *Greater* than the Object it self. *Tab.*

Tab. 30. Fig. 1. abc is an Object exposed to the Glass gl , and projected in the Distinct Base $f e d$. Each Cone of Rays from the Object is made to converge, as $b l g$ is formed into $g e l$. Let the Eye be placed at k . I say, first the Object appears erect. For the Rays from a the upper part of the Object tend to, and determin in the lower part of the Fund of the Eye. And the Rays from c the lower part of the Object tend towards the upper part of the Retina; and therefore the Appearance is erect, (by Prop. XXVIII. Sec. 4, 5.)

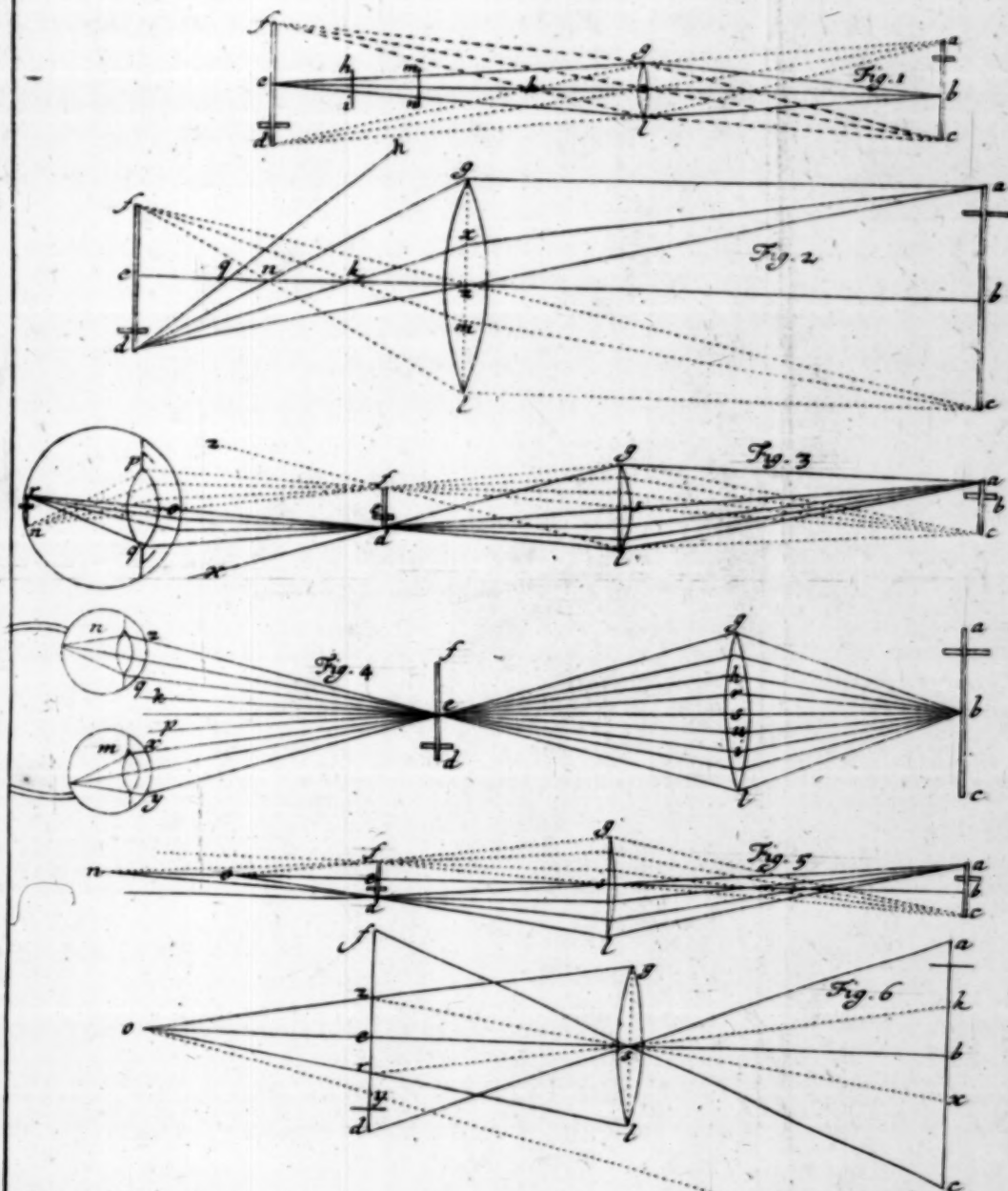
Moreover, the Eye being placed any where between the Glass gl , and the Distinct Base $f e d$, does not receive the Species of the Object inverted; for that only happens just in the Distinct Base it self. And on this account, 'tis manifest, that the Appearance to the Eye in this posture shall be Erect.

I say likewise, that the Appearance of the Object is *confused*. Let us conceive the Pupil of the Eye at $m n$; here it receives the Rays from the Point b converging; and therefore by Prop. XXVIII. the Representation of the Point b on the Fund of the Eye shall be *confused*. And so of the other Points in the Object, that is, of the whole Object. And if the Eye recede from the Glass, as to $h i$, the Vision is *more confused*, because the Rays are *nigher* converged.

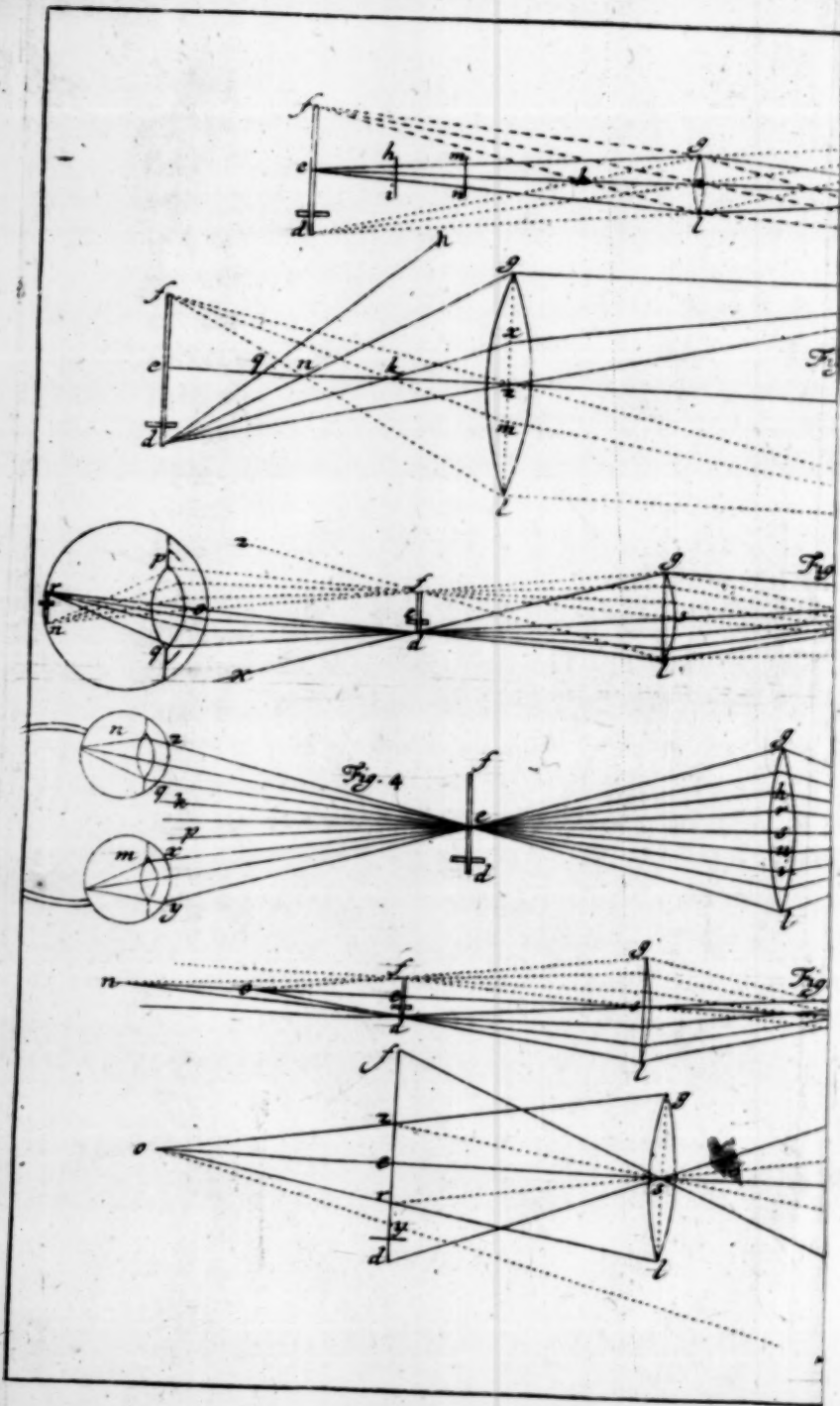
PROP. XXXVII. PROBL.

To determin the Optick Angle, or Apparent Magnitude, and the Visible Area of an Object placed as in the last; from these Data, The Breadth of the Object abc (Tab. 30. Fig. 2.), Its Distance from the Glass zb , The Glasses Power, The Glasses Breadth gl , And the Distance of the Eye from the Glass kz .

(1.) From



Tab. 30. pag. 128.



(1.) From these Data by the foregoing Doctrine of Convex Glasses, 'tis easie to determine the Breadth of the Distinct Base. For by Prop. XXVI, XXVII. *As the Distance of the Object from a Convex Glass : To the Breadth of the Object :: So the Distance of the Image in the Distinct Base : (which is easily obtained from the foregoing Data) To the Breadth of the said Image.* That is (in the forelaide Figure) $bz : ac :: ez : df$. And As $ab : bz :: de : ez$. And consequently the Angle ezd is equal to the Angle azb ; the Triangles ezd and azb being similar. And in the Right-angled Triangle abz , we have ab and bz , to find the Angle $azb = ezd$, which is the Angle, under which the Object would appear to the Eye placed just touching the Glas at z ; supposing the Glas to be there of the least Thickness imaginable.

(2.) Let us now suppose the Eye at k . I say if the Rays from the single Points a, c , in an Object be converged by the Convex Glas gl to any other Points d, f ; the Object shall appear under the Optick Angle xkm , as do the Apices of the Pencils from the Extreme Points of the Object, which is $dkf = xkm$. Vid. *Gregori Opt. Promot. Prop. XLV.*

(3.) Wherefore in the Right-angled Triangle ekd , we have ek and ed , To find the Angle $ekd = xkx$, which is the Semi-Optick Angle to the Eye at k . In like manner may we find the Optick Angle at any other given Section of the Eye, as at n , $end = gnz$ being equal to half fnd .

(4.) As to the Visible Area of the Object, 'tis determined as before, Prop. XXXV. and Schol. I shall only add, That if the Eye be at q , and the Line dq being drawn, and produced towards b , it does not fall upon the Glas gl ; then the Eye at q shall not see through the Glas the Point a , the Apex of whose Pencil is in the Point d . And from hence we may perceive, that at k , we may see through the Glas a greater Space than the Area of the Object abc . At n we can but just see

the whole Object *abc*; and at *q* we cannot see the extreme Point *a*. By which the 81 and 82 Prop. of Kepler's Dioptricks are manifest.

(5.) If the Glass have any Thickness considerable, see Prop. XXXI. concerning the *Locus Objecti*, Sec. 7.

Scholium.

If we first determine the *Visible Area* by Prop. XXXV. the Breadth of the Object need not be one of the *Data* for determining the *Optick-Angle*. For the Breadth of the Object is found by obtaining the *Visible Area*.

PROP. XXXVIII.

An Object being more distant from a Glass than its Focus, and the Eye placed in the Distinct Base, the Object appears most confused.

By Prop. XXXVI. 'tis manifest, that the nigher the Eye approaches the Distinct Base, the more *confused* the Object appears; because the Rays from each particular Point, do fall on the Eye *closer Converged*, and more orderly separated by the Glass from the Rays of adjacent Points. But in the Distinct Base, the Rays are most of all Converged, and most orderly united in their proper Points, each with the Rays flowing from the same Point in the Object. Wherefore the Eye is there in the greatest Confusion.

Moreover (by Prop. XXVIII. Sec. 3.) 'tis requisite to Distinct Vision, that the Rays from the several Points in the Object, fall on the Eye altogether *confused* thereon (as they do on the Glass in the Hole of a dark Room), that the Coats and Humors of the Eye may refract them, and bring them together,

ther, distinctly painting the Image on the *Retina*. But when the Eye is in the *Distinct Base*, all this is frustrated.

PROP. XXXIX.

The Object being more Distant from the Glass than the Focus ; and the Eye further from the Glass than the Distinct Base, begins to perceive the Object inverted ; and, at a proper distance, Distinct.

(1.) This is manifest by *Tab. 30. Fig. 3.* wherein *abc* is T. 30. F. 3. an Object, *gsl* the Glass projecting the *Distinct Base fed*, *o* is the Eye. And because half the Pencils of Rays that flow from each of the extreme Points of the Object *a* and *c*, proceeding forwards from the Points *d* and *f* in the *Distinct Base*, after they cross in the aforesaid Points *d* and *f*, by reason of their too great Divergence, do escape the Eye. Therefore, for avoiding Perplexity in the Scheme, I have only expressed the Half Cones *sal*, *sdl*, and *scg*, *sf g*, which incur the Eye.

(2.) Wherefore we may observe, that the Diverging Rays *pfo* and *odq*, falling diverging and confused on the Crystalline *pq*, are thereby collected, and made to concur on the *Retina* in the Points *n* and *r* ; and there paint the upper Point *a* of the Object on the upper part *r* of the Eyes Fund ; and the lower Point *c* of the Object on the lower part *n* of the *Retina*. Therefore by *Prop. XXVIII. Sect. 4.* and *5.* the Object appears inverted, because the Image represented on the *Retina* is *Erect*.

(3.) Or more plainly thus, We may now conceive the *Distinct Base* it self *fd* to be as it were an inverted Object (as here the Cross turned with its head downwards). Therefore the appearance thereof to the Eye, shall seem inverted. For if the Eye perceive an erect Object, erect, it must needs perceive the *Distinct Base*, being an inverted Object, inverted.

And this may be laid down as a General Rule, That where an Object, or the Image, or Representation of an Object, which is next the Eye, is *inverted*, there the Object shall appear *inverted*. And where the Object, or Image next before the Eye, is *erect*, the Object appears *erect*.

(4.) I say also, *At a proper distance the Inverted Appearance is Distinct*. For when the Eye is so far removed from the Distinct Base, as to be able to correct the Divergence of the Rays, that come to it from each Point in the Distinct Base, and to form them into correspondent Cones determining their *Apices* on the *Retina*, the Vision is Distinct, otherwise not.

Moreover there are two other *Phænomena* of this Posture of Object, Glass, and Eye; which I shall here explain as follows.

(5.) If the Eye be moved *upwards* towards z , the Object shall appear to move *downwards*. If the Eye be moved downwards towards x , the Object seems to move *upwards*. To explain this, let us consider the Point c ; and how the appearance thereof is brought to and formed in the Eye. And we shall find, that in the Posture of the Scheme, the Point c is expressed in the Eye, only by that parcel of its Rays that fall on the half Glass sg , for these are brought together in $sf g$, and flowing forward, become $pf a$: But the other parcel of Rays $sc l$, that fall on the half Glass sl , and are brought together in $sf l$, proceeding forwards, escape the Eye; for they go on in $pf z$. Wherefore the Point c is now represented through the half Glass sg , or seen somewhere between g and s . But the Eye moving upwards towards z , so that it miss the Rays $pf a$, and receive only the Rays $pf z$; the Point c shall then be represented through the half Glass sl , or seen somewhere between s and l , and consequently shall seem to move *downwards*. What is shewn of the Point c , may be conceived of the other Points in the Object, that is, of the whole Object.

And

And this *Phænomenon* is the strongest Confirmation imaginable, that the *Locus Apparens Objecti* in this Case, is in the *Distinct Base* (as is laid before, *Prop. XXXI. Sec. 10.*). For by raising the Eye, we depress the Point *f*; and by depressing the Eye, we raise the Point *f*. Wherefore the appearance of the Point *c* is at *f*. For the Point *c* in the Object seems to move contrary to the motion of the Eye.

From the foregoing Explication it follows, that if the Glass be moved upwards, the Object appears to move upwards. And if the Glass be moved downwards, the Object appears to move downwards; for the moving of the Glass upwards is the same as moving the Eye downwards; but moving the Eye downwards, makes the Object appear to move upwards; therefore the moving of the Glass upwards makes the Object appear to move upwards.

Or thus, Because moving the Glass upwards, the *Distinct Base* is moved upwards (for it follows the Glass), and the *Distinct Base* is in this case as it were the Object. (*Prop. XXXI. Sec. 10.*) Therefore by the Glasses motion upwards, the Eye shall perceive the Object moved upwards.

(6.) If with both Eyes we look at the inverted Appearance of an Object through a Convex-Glass, we shall perceive the Object double. Supposing the Eyes not very far removed from the *Distinct Base*. *Tab. 30. Fig. 4.* *a b c* is an Object projected by the Glass *g l* in the *Distinct Base* *f e d*; the Point *e* being correspondent to the Point *b*. Let the Cone of Rays *g e l* flow forwards from *e*, and become *z e y*. Let the two Eyes *n, m*, meet the Cone of Rays *z e y*. I say the Eye *n* perceives the Point *e* (the Representation of the Point *b*) by the Rays *z e q*, which flow forward from *e*; and consequently the Eye *n* perceives the Point *b* or *e*, as it were, somewhere between *s* and *l*. In like manner, the Eye *m* perceives the Point *b* or *e* somewhere between *s* and *g*. Wherefore the two Eyes seeing the Point *b* or *e*, as it were, in two places, see it Double.

Double. What is said of the Point *b*, may be understood of any other Point in the Object, and consequently of the whole Object. And if we shut the Right Eye *m*, the Left Appearance of *b* vanishes; if the Left Eye *n*, then the Right Appearance of the Object vanishes.

But if the Eyes be removed very far from the Distinct Base *f e d*, their Interval continues the same; and the Rays *ek*, *ep*, Diverging continually, they shall at last fall on the Eyes; that is, *ek* on the Eye *n*, and *ep* on the Eye *m*. By which the Eye *n* shall perceive *e* as it were at *u* (*k e u* being one Right Line), and the Eye *m* shall perceive the same *e* as it were at *r* (*p e r* being one Right Line), whereby the Difference of the Places *r* and *u* becomes less than before, till at last the Eye be got at such a Distance from the Distinct Base, that this Difference of the Apparent Places becomes so small, that 'tis insensible to the sight.

Quere. How Dr. Briggs would explain this Phenomenon of Double Vision by his Theory of sight. *Philosoph. Collect. Numb. 6.*

PROP. XL.

To determine the Optick-Angle or Apparent Magnitude, and the Visible Area of an Object, placed as in the last, from these Data; The Breadth of the Object *a b c*, (Tab. 30. F. 5.) Its Distance from the Glass *s b*, The Glasses Power or Focal Length, The Glasses Breadth *g l*, And the Distance of the Eye from the Glass *o s*.

(1.) It is shewn before (Prop. XXXVII.) how we may from these Data obtain the Breadth of the Distinct Base *f e d*, and its Distance from the Glass *es*. Wherefore $os - es = oe$. Then in the Right-angled Triangle *o e f*, we have *oe* and *ef* to find the Angle *foe*, which is the Semi-Optick Angle, under

der which the Eye perceives the Inverted Appearance of the Object. In like manner may we find the Optick Angle to any other station as at *n*.

(2.) The Learned GREGORY in his *Opt. Promot. Prop. 46.* has a curious Theorem to this purpose, 'tis this, *The Rays from each single Point in an Object a b c, being formed by the Glass g s l into the Distinct Base f e d; and the Eye o being placed behind the Points of Concourse f e d; the Image of each Point in the Object shall appear in the Apex of its Pencil (That is, The Image of the Point a shall appear in the Apex of its Pencil at d, &c.), And the Object shall appear Inverted; And under that Optick-Angle, as the Apices of the Pencils of the Extreme Points of the Object; That is, under the Angle f o d.*

(3.) The Visible Area is thus determined, *Tab. 30. Fig. 6. T30.F6.* The Object *a b c* is projected by the Glass *g s l* in the Distinct Base *f e d*. Let the Eye be at *o*; draw *o g, o l*; the Eye at *o* shall see no more of the Object, than what is represented between *z* and *r*, where the Lines *o g, o l*, intersect the Distinct Base. To determine the Points in the Object, that answer the Points *z, r*, in the Distinct Base. From the Centre of the Glass *s* (for I suppose it of no Thickness), draw *z s x, r s k*; the Point *k* answers *r*, and *x* answers *z*. Lastly, from the foregoing Data to determine the Measure of *z r* lay, As *so:sg::eo:ez=1zr*; but the three first are known, therefore the fourth is known also, *Which was to be found.*

(4.) If the Line *o y* produced do not fall on the Glass, then that Point in the Object, whose Apex of its Pencil is at *y*, or which is projected in the Distinct Base at *y*, shall not be seen through the Glass by the Eye at *o*. Vid. *Schol Prop. XLVI. Gregorij Opt. Promot.*

Corollary. 7.

Corollary.

Hence it follows, That the further the Eye is removed from the Distinct Base, it perceives the greater Area of the Object; but can never see more than what is projected in the Breadth of the Glas; that is, zr can never be greater than gl , nor ever equal thereto, unless the Eye o be at an infinite Distance.

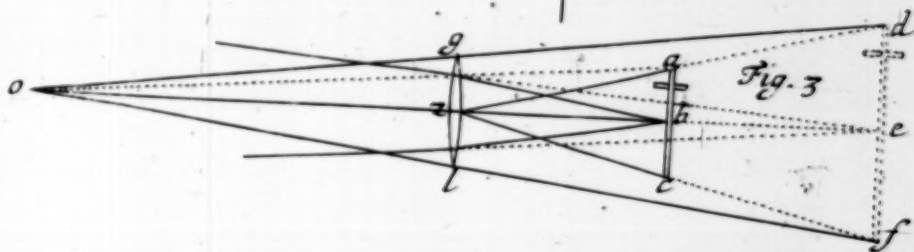
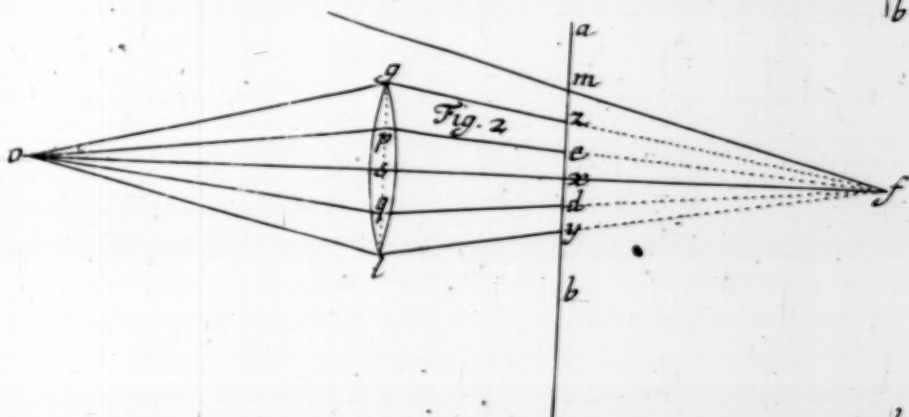
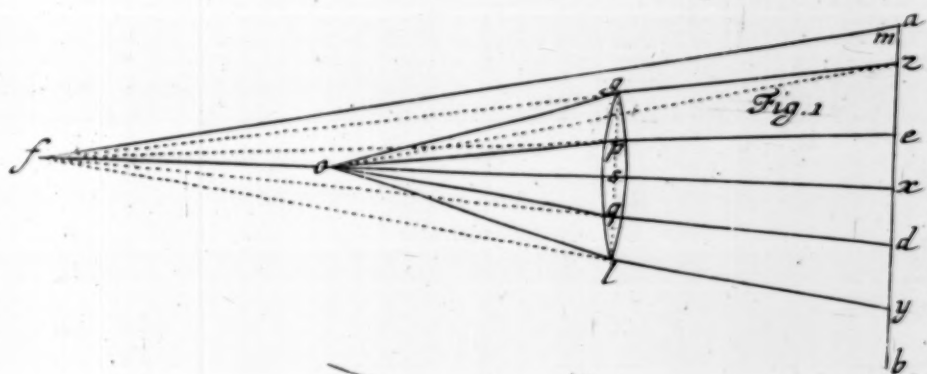
PROP. XLI.

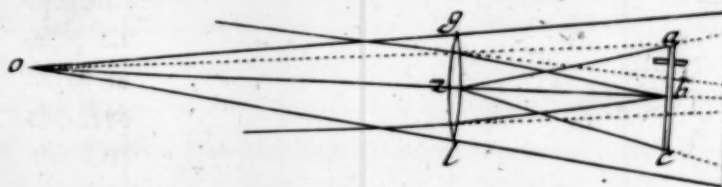
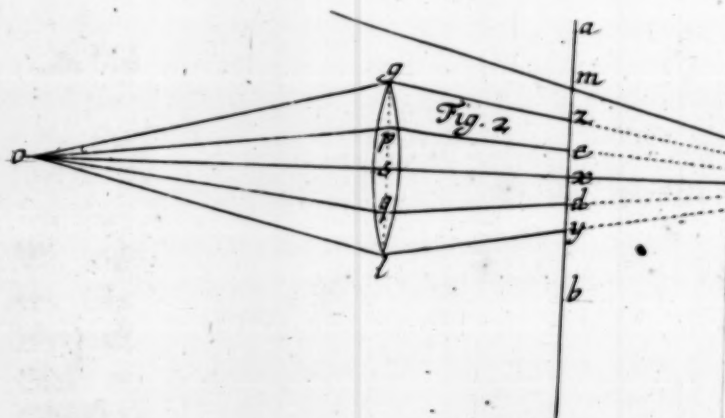
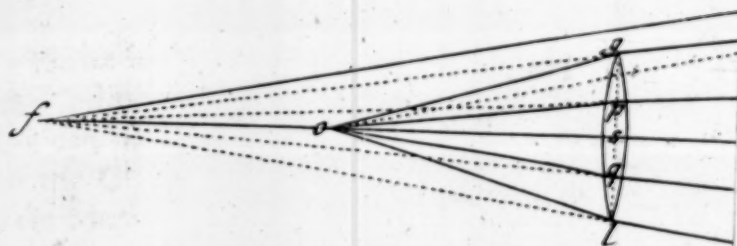
An Object being placed nigher a Convex Glas than its Focus, and the Eye on tother side the Glas, at any Distance within the Eyes power, sees this Object Distinct and Erect.

I say, *Within the Eye's Power*; for 'tis shewn before in Prop. VII. and VIII. That when an Object is placed nigher a Convex Glas than its Focus, the Rays from each Point thereof, after passing the Glas, do Diverge, but not so much as if the Glas were away. Wherefore, tho the Eye looking at an Object within the Focus of this Glas, may be nigher the Object, than if the Glas were away, and yet see it distinctly; yet there is a mean to be observed; for even with the Glas it self the Eye may be too nigh the Object, and not able to correct the Divergence of the Rays it receives.

But that an Object thus posited appears *Distinct* to the Eye at a convenient Distance, is manifest; for the Rays from each Point proceed moderately *Diverging*, and so fall on the Eye. Prop. XXVIII. Sec. 7.

That an Object thus posited is seen *Erect*, is evident from Prop. XXXII. and XXXIX.





PROP. XLII.

To determine the Visible Area, and the Optick-Angle or Apparent Magnitude of an Object placed as in the last, from these Data, The Power or Focal Length of the Glass, The Distance of the Object from the Glass, The Distance of the Eye from the Glass, and the Breadth of the Glass.

Tab. 31. Fig. 1. and 2. ab is an Object exposed to the Glass ^{Tab. 32.} gl , nigher to it than its Focus. o is the Eye, which in Fig. 1. ^{Fig. 1, 2.} is supposed nigher the Glass than its Focus; and this is the First Case. But in Fig. 2. the Eye o is supposed further from the Glass than its Focus; and this is the Second Case. 'Tis shewn before (Prop. V. VIII.), how the Respective and Imaginary Foci f are determined in one and t'other Case.

First let the given Breadth of the Glass be gl . By Prop. XXXV. we determine the Visible Area zy . Let us then suppose the given Breadth of the Glass to be pq . By the same Prop. XXXV. we determine the Visible Area ed .

As to the Optick-Angles gol (supposing the Object zy) or poq (supposing the Object ed) they are determined from the Data, by Plain Trigonometry of Right-angled Triangles, as before in Prop. XXXIV.

And we shall find by Calculation (as indeed 'tis evident by the very Inspection and Consideration of the Schemes), that the Collateral Parts ze , dy , of the Object zy , are much more magnified (in respect of their Natural Appearance) by Broad Glasses formed on small Spheres, than the Middle Parts ex , dx ; for the Angle gop is the Optick-Angle, through the Glass, of the Part ze ; and the Angle pos is the Optick-Angle, through the Glass, of the Part ex ; but the former exceeds the Natural Optick-Angle much more than the latter.

T

As

As to the Natural Optick-Angle of the Object zx , were the Glass away; draw zo directly (Fig. 1.), then in the Right-angled Triangle zxo , we have zx and xo to find the Angle zox , which is the Natural Optick-Angle of the Object zx .

From hence it is, that by Broad Glasses formed on small Spheres, the Extreme Parts of *strait* Objects, seem to be *in-curved* and *bent*; as is manifest in the Case of the *Micrometer*, or Lattice of fine Hairs, strained before the Eye Glass in a Telescope, for Measuring the Diameters of Objects. As *Pere Cherubin* complains in his *Dioptrique Oculaire*. Part. III. Sec. 7. Chap. 1. pag. 239. but understood not the reason. Of this we may make Experiment, by looking with a very Convex Glass at two Parallel Lines drawn pretty close on a Paper.

Schol. 1.

If instead of the Glasses Breadth, we have the Breadth of the Object ed given, we may easily determine the Breadth of the Glass, through which this Portion of the Object is seen: For from the Point f , draw fe , fd , these intersect the Glass in p , q . I say this is the Portion of the Glass, through which ed is visible. And if the Line fm , being produced, do not fall on the Glass, the Point m in the Object is not visible through the Glass gl to the Eye at o .

Schol. 2.

This XLII. Proposition, as it relates to the Optick-Angle, may be solved another way by Determining the *Locus Objecti*. But then, to the former *Data* in that Proposition, instead of the Glasses Breadth, we must add the Breadth of the Object.

T.32. F.1. Wherefore, Tab. 31. Fig. 3. Let the *Locus* of the Object abc be determin'd def , by Prop. XXXI. Sec. 7. Let the
Eye

Eye be at o , draw do , fo directly; if the Right Line do do not fall on the Glass, the Eye at o cannot perceive through the Glass the Point of the Object a (Vid. Prop. XXXVII. Sec. 4. Prop. XL. Sec. 4. & Schol. Prop. XLVI. Gregorii Opt. Promot.). But if do , fo , fall on the Glass; I say the Angle dof is the Angle under which the Object abc appears to the Eye at o through the Glass. For the Object abc is seen through the Glass under the same Angle, as the Image def would be seen without the Glass (by Prop. XXXI. Sec. 7.). Now because we have ab and bz given, and ez found, we may find also ed ; for $bz:ba::ez:ed$. Then in the Right-angled Triangle doe , we have de and $eo (=ez+zo)$ to find the Angle $doe=;dof$. Draw then ao directly; the Angle aob is the Natural Optick-Angle, under which the Object ab would appear to the Eye without the Glass. Now having ab and $bo (=bz+zo)$ we may find the Angle aob ; the Difference between which and the Angle dof (that is dof) shews how much the Object is magnified by the Glass.

I. Example of the First Case computed by the Method in Prop. XLII. Tab. 31. Fig. 1.

T31. F.1.

Glasses Focus = — — — — — 4637
 Dist. of the Eye from the Glass = $so = 2500$
 Dist. of the Obj. from the Glass = $sx = 2721$
 Breadth of the Glass = $gl = sg = 145$

} Data.

Visible Area = — — — — — $zx = ?$
 Optick-Angle = — — — — — $\angle gos = ?$

} Quæsit.

First supposing the Point o a Radiating Point,
 By Prop. VIII. As Focus Glass — so : to Foc. Glass :: $so : fs$
 That is in Numbers — — — 2137 : 4637 :: 2500 : 5425
 T 2 Then

Then — — — — — $fs : sg :: fx (=fs + sx) : zx$

That is $5425 : 145 :: 8146 : 217 = x$ the Visible Area in this Case according to the Data.

Secondly, To find the Optick-Angle gos , under which this $zx = 217$, appears; say,

$$so : \text{Rad.} :: sg : \text{Tang. } \angle gos = 3^\circ 19' 10''.$$

The foregoing Example calculated according to Schol. 2.
T31. F3. Prop. XLII. But the Letters refer to Tab. 31. Fig. 3.

Glasses Focus = — — — — — 4637

Dist. of the Eye from the Gl. = $zo = 2500$

Dist. of the Obj. from the Gl. = $zb = 2721$

½ Breadth of the Object = — — — — — $ab = 217$

} Data.

Distance of the Imaginary Focus $ze = ?$

Breadth of the Imaginary Focus = $de = ?$

Optick-Angle — — — — — $\angle doe = ?$

} Quæsitæ.

First, Supposing the Point b a Radiating Point, to determine the Imaginary Focus at e , by Prop. VIII. we must say,

As Focus Glas — zb : Focus Glas :: zb : ze

That is in Numbers $1916 : 4637 :: 2721 : 6585$

Then by Pr. 2. 6. Eucl. zb : ba :: ze : ed

That is in Numb. $2721 : 217 :: 6585 : 525$

Lastly, As $eo (=ze + zo) : \text{Rad.} :: de : \text{Tang. } \angle doe = 3^\circ 18' 30''$, which wants only $40''$ Seconds of the former Calculation, by reason of the neglect of the Fractions.

II. Ex-

II. Example of the Second Case calculated according to

Prop. XLII. Tab. 31. Fig. 2.

T₃₁. F₂

$$\begin{array}{lcl}
 \text{Glasses Focus} = & - & - & - & - & - & 4637 \\
 \text{Dist. of the Eye from the Glass} = & s o = & 5000 \\
 \text{Dist. of the Obj. from the Glass} = & s x = & 2721 \\
 \frac{1}{2} \text{ Breadth of the Glass} = & \frac{1}{2} g l = & s g = 145 \\
 \frac{1}{2} \text{ Visible Area} = & - & - & - & - & - & 2 x = ? \\
 \frac{1}{2} \text{ Optick-Angle} = & - & - & - & - & - & \angle g o s = ?
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Data.} \\ \\ \\ \text{Quæstia.} \end{array}$$

First, Supposing the point *o* a Radiating Point, Then by Prop. V.

As *s o* : Foc. of the Gl. : Foc. of the Gl. :: *s o* : *f s*

That is in Numbers 363 : 4637 :: 5000 : 63870

Then — — — — — *f s* : *s g* :: *f x* (= *f s* — *s x*) : *2 x*

That is in Numbers 63870 : 145 :: 61149 : 138, being the Visible Area in this Case, according to the Data.

Secondly, To find the Optick-Angle *g o s*, under which this *2 x* = 138 appears; say,

As *s o* : Rad. :: *s g* : Tang. $\angle g o s = 1^\circ 39' 40''$

The foregoing II. Example calculated according to the Solution in Schol. 2. Pr. XLII. The Letters relate to Tab. 31. F. 3. T₃₁. F. 3.

$$\begin{array}{lcl}
 \text{Glasses Focus} = & - & - & - & - & - & 4637 \\
 \text{Dist. of the Eye from the Glass} = & 2 o = & 5000 \\
 \text{Dist. of the Obj. from the Glass} = & 2 b = & 2721 \\
 \text{Breadth of the Object} = & - & - & - & - & - & a b = 138 \\
 \text{Distance of the Imaginary Focus} = & 2 e \\
 \frac{1}{2} \text{ Breadth of the Imaginary Focus} = & d e \\
 \text{Optick-Angle through the Glass} = & \angle d o e
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Data.} \\ \\ \\ \text{Quæstia.} \end{array}$$

Let

Let us suppose the Point b a Radiating Point. To determine the Imaginary Focus at e , by *Prop. VIII.* say,

As Foc. of the Glas— zb : Foc. of the Gl. :: zb : ze

That is in Numbers $1916 : 4637 :: 2721 : 6585$

Then— — — — — $zb : ba :: ze : ed$

That is in Numbers $2721 : 138 :: 6585 : 334$

Lastly, As eo ($=ze+zo$) : Rad. :: de : Tang. $\angle doe = 1^\circ 39' 0''$, which by reason of the neglect of Fractions wants $40''$ Seconds of the foregoing Calculation.

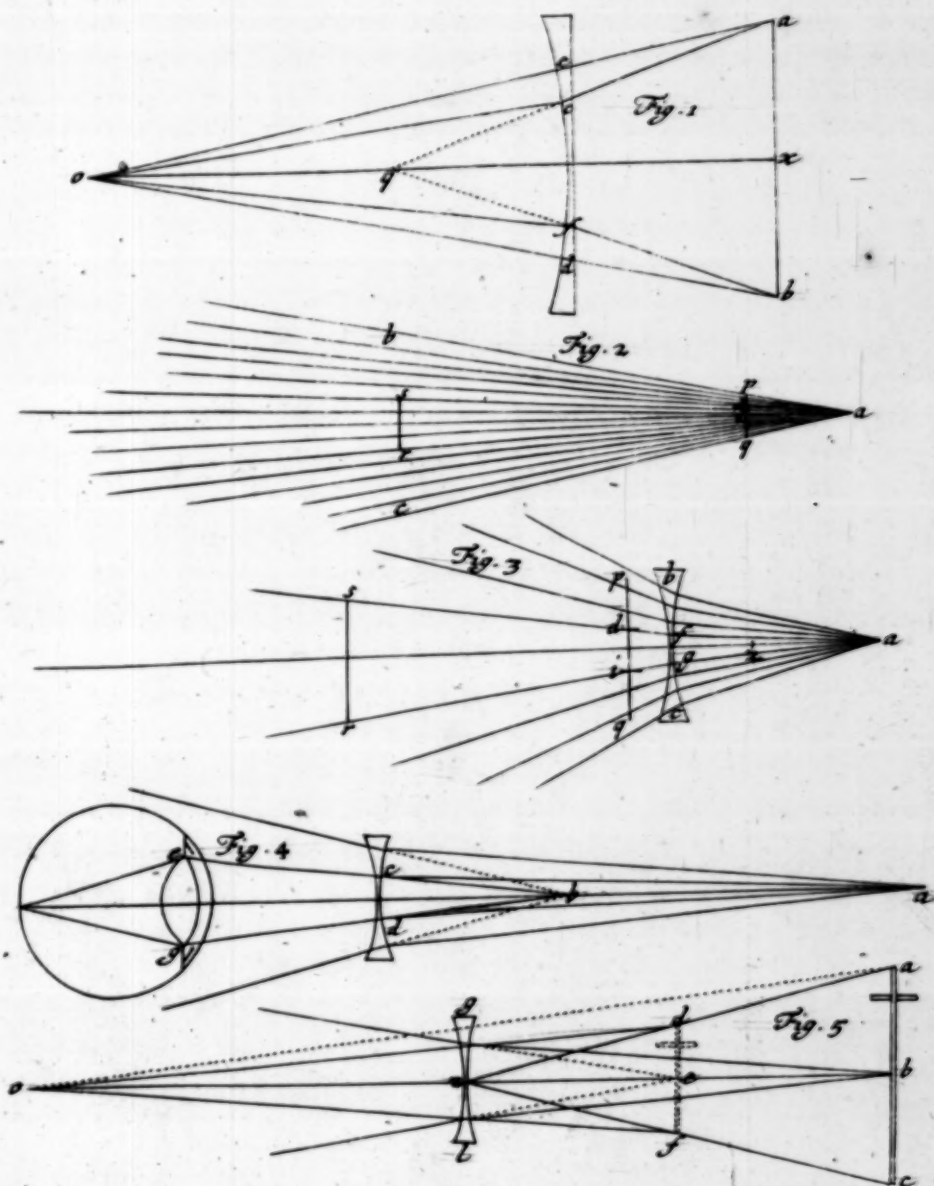
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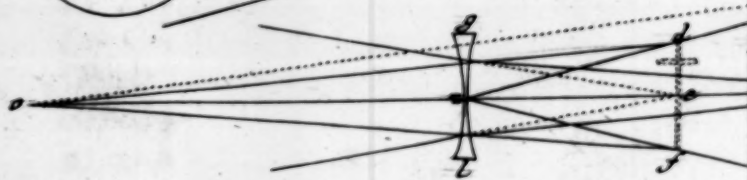
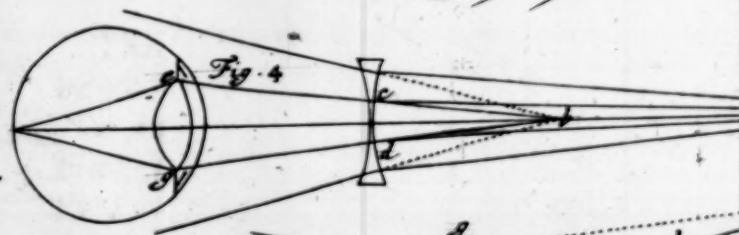
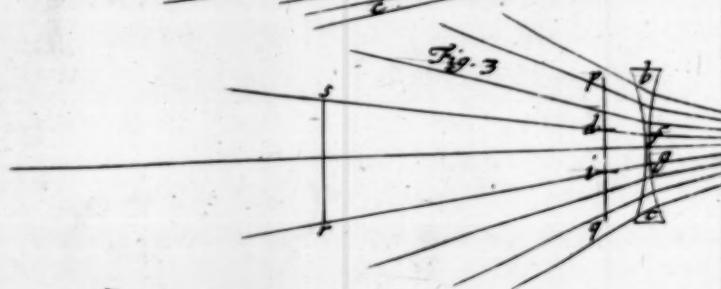
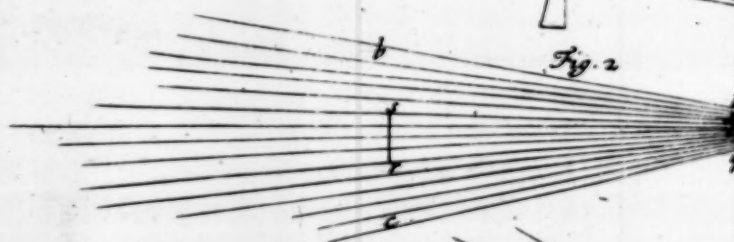
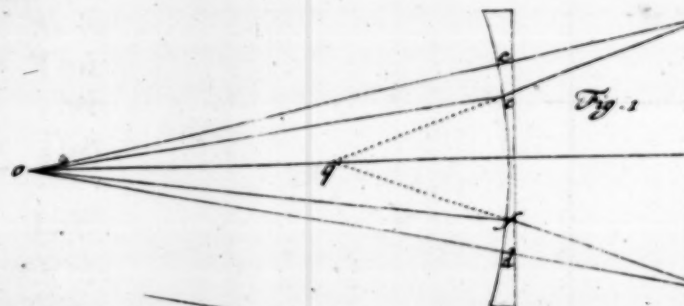
I Now proceed to the Consideration of Vision through Concave Glasses.

PROP. XLIII.

All Objects seen through Concave Glasses appear Erect and Diminish'd.

That the Object shall appear *Erect* is manifest; for 'tis before (*Prop. XXXIX. Sec. 3.*) laid down as a General Rule, That where the Object, or Image that is next before the Eye, is *Erect*, the Eye shall perceive the Object *Erect*; and where *Inverted*, the Eye sees it *Inverted*. But Concaves have no *Distinct Base* (as Convexes have) wherein they represent the Image of the Object *Inverted*, and consequently, wherever the Eye is placed behind a Concave Glas, it shall perceive the Object through it, in its Natural Posture. That Concaves have no *Real Distinct Base*, is most plain from the Doctrine that foregoes relating to them: For a *Distinct Base* is caused by the Collection





lection of the Rays proceeding from a single Point in the Object, into a single Point in the Representation; but Concave-Glasses do not *unite*, but scatter and dissipate the Rays. 'Tis true indeed, a Convex-Glass may cause a Distinct Base, notwithstanding a Concave placed behind it or before it (as we see in *Prop. XV. XVII, XVIII.*) ; but then this Distinct Base proceeds not from the Concave, but from the Convex : For the Concave exerts its scattering power even in this Case; and it protracts the Uniting of the Rays to a greater Distance from the Convex, as is manifest from the fore-cited *Propositions*.

I say also, *The Appearance of Objects through Concaves is diminished: Tab. 32. Fig. 1.* *a x b* is an Object, *o* the Eye. Draw *a e o*, *b d o*, directly strait from the Extremities of the Object to the Eye. The Angle comprised by the direct Rays *a e o*, *b d o*, that is, the Angle *a o b* is the Natural Optick-Angle. Let now the Concave-Glass *c f* be interposed; here, because the Rays *a e*, *b d*, would naturally concur at *o*, now the Concave-Glass is interposed, their Concourse shall be protracted beyond *o*; wherefore the Eye at *o* shall not perceive the Extremities *a*, *b*, of the Object through the Glass by the Rays *a e*, *b d*, but by some other Rays, and these other Rays must either fall *without* (that is farther from the Axis *o x*, than) *a e*, *b d*, or they must fall *within* *a e*, *b d*: But they cannot fall *without* *a e*, *b d*; for if *a e*, *b d*, themselves be made by the Glass to concur beyond the Eye at *o*, much more shall the Concourse of any other Rays that fall *without* *a e*, *b d*, be protracted beyond *o*; and consequently they cannot convey the Appearance of the Extreme Points *a*, *b*, to the Eye at *o*. Wherefore it remains, that the Rays that do this, must fall *within* *a e*, *b d*. Let us suppose these Rays to be *a c*, *b f*, which in their Natural direct Course, would concur at *q*, but by the Refractive Power of the Concave, are bent and made to proceed in *c o*, *f o*, their Concourse being protracted till they arrive
at

at the Eye in o . Here the Optick-Angle through the Glass is cof , which is less than the Natural Optick-Angle $ao b$, and consequently the Appearance of the Object is diminished. Which was to be proved.

PROP. XLIV.

Concerning the Distinct and Confused Appearance of Objects through Concaves; as also of their Faint or Obscure Appearance.

We are here to remember what before is laid down, concerning the *Virtual Focus* of a Concave exposed to Parallel or to Diverging Rays (in Prop. XI, XII, XIII. & Corol. Prop. XV.): We are likewise to take notice, that in Plain Vision, when the Rays from any single Point in an Object do not Diverge more than what the Refractive Power of the Coats and Humors of the Eye can correct; so that these Rays may be brought together in a Correspondent Point on the *Retina*; then the Appearance of that Point is *Distinct*. For instance, *Tab. 32. F. 2.* Let the Point a diffuse the Rays ab , as , ar , ac . Suppose the Breadth of the Pupil were pq , and the Eye there placed; perhaps the Refractive Power of the Eye is not sufficient to correct the Divergence of the Rays apb , aqc . But if the Pupil (continuing of the same Breadth) recede to rs , then only the Rays as , ar , fall into it; and these perhaps may be reduced by the Eye to determine in a Point on the *Retina*; because they do not Diverge so much as the former. If therefore we suppose the Pupil in its former station at pq , but now only to be of the Breadth id , so as to admit only the Rays ads , air , then the Point a may be seen as distinctly as at rs . This is manifest even by Experiment; for apply a Minute Object so near the Eye, that it appears very confused; then place before the Eye a very small Hole made with

a Pins end in a Paper; The Object shall now appear *Distinct*. For the Hole in the Paper serves to make the Pupil more *Narrow*. Which evidently proves what we have laid down.

The same may be shewn concerning Vision through a Concave Glass *Tab. 32. f. 3.* a is a Radiating Point, whose Rays $a b, a c$, fall on the Concave $b c$. These after passing the Glass Diverge more than before. Let $p q$ be the Breadth of the Pupil, receiving the refracted Rays $a p, a q$, These Diverge *so much*, that 'tis not in the power of the Eye to reduce them, and form them into an inward Cone, determining its *Apex* on the Fund of the Eye. But let the Pupil continue of the same Breadth, and recede to $r s$, where it may only receive the Rays $a d s, a i r$; and then perhaps the Eye may prevail to reduce these; because they do not Diverge *so much* as the other Rays that fell on the Pupil at $p q$. Or otherwise, let the Pupil continue at $p q$, But let its Breadth be contracted to $d i$; so that it may receive no more Rays in this Posture, than (continuing of its natural Breadth) at $r s$. Here likewise the Appearance of the Point a may be *Distinct*. Of this likewise we may make a most convincing Experiment: For take a Concave of a small Sphere, and place it very near the Eye, and the Appearance of distant Objects through it is *Confused*. Remove it farther from the Eye, and the Appearance shall be more *Distinct*. But even in a Posture where the Appearance is *Confused*, contract the Pupil by placing before it a small Hole prick'd in a Paper, And the Object shall appear *more Distinct*, though *Obscure*, which upon the removal of the Hole, shall be again *Confused*, though more *Lightsome*. *Vid. Kepleri Dioptr. Prop. C.*

Wherefore if we have the Distance of a Radiating Point from a Concave, and the virtual *Focus* of the Concave, and the Distance of the Eye behind the Glass; we may easily by the foregoing Rules (*viz. Corol. Prop. XV.*) find the virtual *Focus* of these Rays. As suppose in the same *Fig.* the Rays flowing

V

from

from *a*, after passing the Glass, Diverge as if they came directly from *z*. And consequently, if in Plain Vision the Eye at *r s* be able to see distinctly the Point *a*, if it were removed nigher, as at *z*, then it shall be able to see the Point *a* distinctly through the Glass. But whether any Particular Eye be able to do this, is impossible to be known by Rule; The *Strength* and *Weakness* of Mens Eyes being *infinitely various*. And therefore one Man may see a Point at a certain Distance *distinctly* through a Glass, which Glass to another Man would render it *Confused*; As 'tis Plain in the Case of *Myopes*, or *Short-sighted* Persons.

T.32. F3. As to the *Strong* or *Faint* Appearance of Objects through Concaves; Because the Concave (*Tab. 32. f. 3.*) scatters the Rays flowing from the Point *a*, insomuch that now the Rays *a b*, *a c*, scape the Pupil *p q*, the Point *a* shall appear *more Faint* through the Glass than naturally. For *Strong* or very *Luminous* Vision proceeds from a *greater* Quantity of Rays or Light entering the Pupil; And *Faint* or *Obsure* Vision from a *less* Quantity of Rays.

Scholium.

We may here consider the *different* Effects of *Convex* and *Concave* Glasses. For when a Point appears *Confused* through a *Convex*, 'tis by Reason of the *Too great Convergence* of the Rays that fall on the Eye. And therefore, Because the farther from the Glass towards the Point of Concourse the Rays that fall on the Eye Converge the *more*; the Appearance is through Them still the *more Confused*.

But in *Concaves*; Because the *Confused* Appearance of a Point proceeds from the *Too great Divergence* of those Rays that fall on the Eye; and because the farther the Eye is from the Glass, the Rays that fall on the Eye Diverge *less* than those that fell upon it when it was nigher the Glass; Therefore through *Concaves*,

caves, the farther the Eye is from the Glafs, the more Distinct is the Appearance, but still more Faint:

P R O B. XLV.

Concerning the Apparent Place of Objects seen through Concave Glasses.

Tab. 32. f. 4. If the Point *a* Radiate on the Concave Glafs T.32. F.4. *c d*, and the Rays after passing the Glafs Diverge as if they came directly from the Point *b*. And the Eye *e g* receive all or part of the Rays, and thereby perceive the Radiating Point *a*, the Point *b* is the *Locus Apparens* of the Point *a*. That is, the Point *a* is seen by the Eye, as if it were at *b*.

By which we may Observe, that the *Apparent Place* of Objects seen through Concaves is brought *nigher* the Eye. And hence 'tis manifest, why they help their Eyes, who are short-sighted, or can only see *nigh* Objects. For these Glasses make *distant* Objects seem *nigh*. *Vid. Dechaes Dioptr. Lib. 2. Prop. XXXIX.*

Here also by the way we shall Note, That suppose a *Purblind* Person, that can Read distinctly or see Objects at the distance of a Foot from his naked Eye: A Concave Glafs, whose virtual *Focus* is a Foot distant from it, makes such a Person see *distant* Objects *distinctly*. Wherefore knowing the Distance at which a *Purblind* Person Reads distinctly, 'tis easie to assign him a proper Glafs for his Eye, to see *distant* Objects. *Vid. second Part. C. 3.*

Tab. 32. f. 5. Is in all things Correspondent to the Doctrine T.32. F.5. laid down in *Prop. XXXI. Sec. 7.* and marked with the same Letters; so that the very Words of that Section may be applied to the Concave, as well as to the Convex. Only the *Imaginary Focus e* for the Concave is to be determin'd by *Prop. XI, XII, XIII. XV. and Corol. Prop. XV. &c.* I shall not therefore Repeat, but refer to that Section.

PROP. XLVI. PROBL.

To determine the visible Area, and the Optick Angle or Apparent Magnitude of an Object seen through a Concave from these Data; The Power or Focal Length of the Glass; The Distance of the Object from the Glass; The Distance of the Eye from the Glass, and the Breadth of the Glass.

T.33. F.1. Tab. 33. f. 1. gzl is a Concave Glass, whose virtual Focus is given. Likewise gz the half Breadth of the Glass is given also. abc is an Object, whose Distance from the Glass bz is given. And the Distance of the Eye o from the Glass, oz is also given.

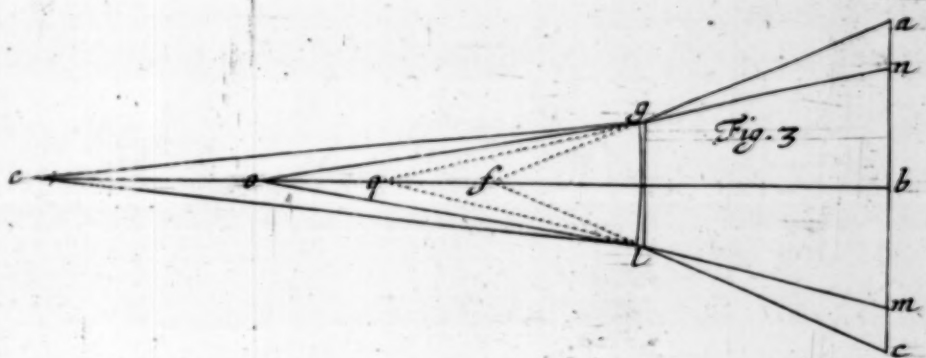
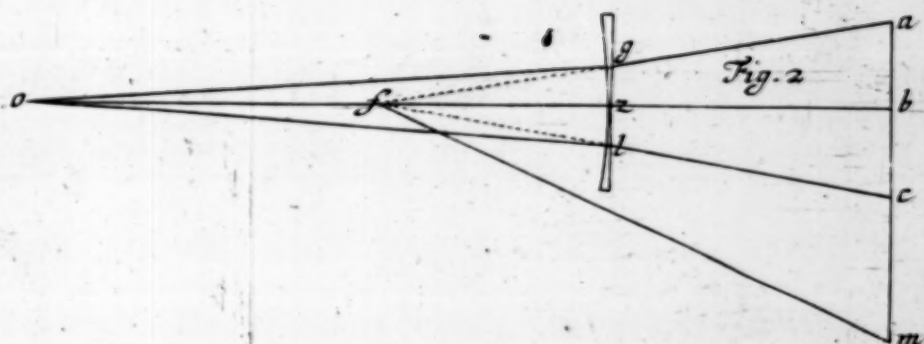
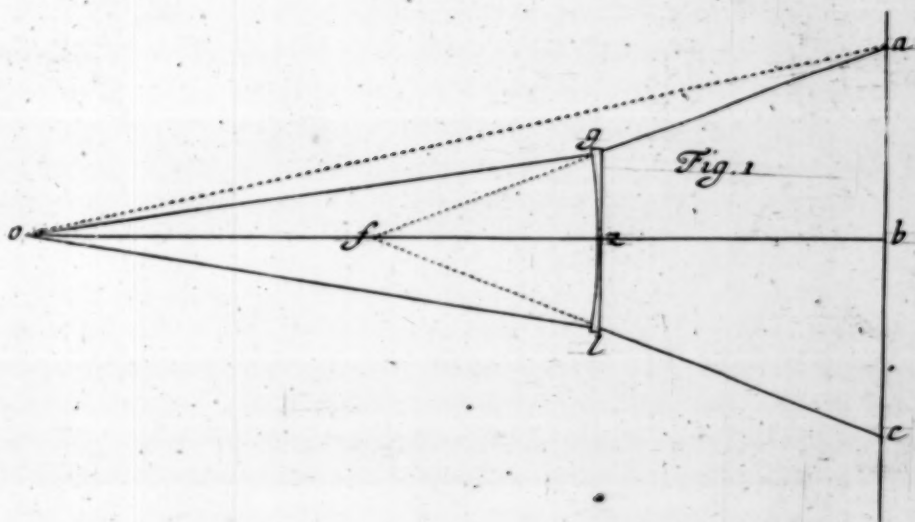
Let us now conceive the Point o an Object or nigh Radiating Point, sending its Rays og , oz , ol , on the Concave Glass; These (by the foregoing Doctrine of Concaves) after passing the Glass Diverge more than before their Entrance, and proceed in ga , lc , as if they came directly from a certain Point f . This Point f or the Line fz , is easily determin'd by Prop. XI, XII, XIII. XV. and Corol. thereof. Wherefore having fz , we may say, As $fz :: zg :: fb (= fz + zb)$ ba . Which is half the Visible Area.

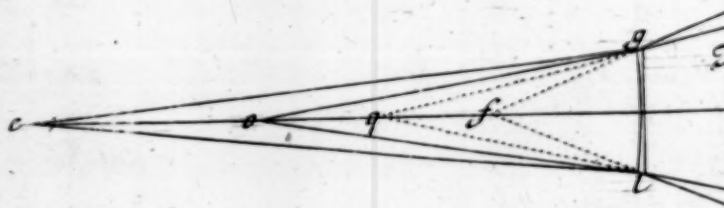
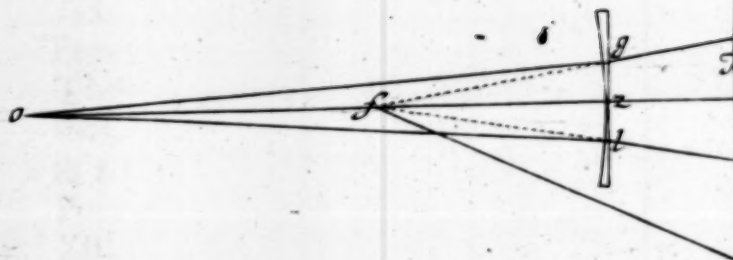
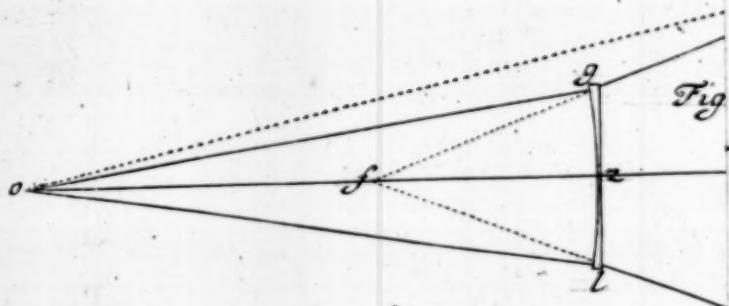
As to the Optick Angle through the Glass, goz , 'tis easily obtain'd in the right-angled Triangle goz , having gz and zo .

Draw ao directly, The Angle aob is the natural Optick Angle, under which the Object would appear to the Eye, were the Glass removed; This we obtain in the right-angled Triangle abo by having ab and $bo = bz + zo$.

Scholium 1.

If instead of the Glasses Breadth, we have given the Breadth of the Object abc (Tab. 33. f. 2.) We may easily determine the





the Portion of the Glas through which this Object is seen. For from the Point f , draw fa , fc , These Intersect the Glas in gl ; I say gl is the Portion of the Glas, through which the Object ac is visible.

And if the Line fm being drawn do not fall on the Glas, the Point m in the Object is not visible through the Glas to the Eye at o .

Scholium 2.

The foregoing Problem, as it relates to the Optick Angle, may be solved another way by determining the *Locus Objecti*. But then to the former *Data*, instead of the *Breadth of the Glas*, we cannot add the *Breadth of the Object*.

This is Evident from *Tab. 32. f. 5.* To which we may apply the Words of the Second Scholium, *Prop. XLII.* without further Repetition.

Example of a Calculation according to *Prop. XLVI.* apply'd to *Tab. 33. f. 1.*

Focus of the Glas = — — — 4573.	}	<i>Data.</i>
$\frac{1}{2}$ Breadth of the Glas = $gz = 500$		
Distance of the Object = $bz = 10000$		
Distance of the Eye = $zo = 20000$		
Half the Visible Area = $ab = ?$	}	<i>Quæsit.</i>
Half Optick Angle = $goz = ?$		

First by *Corol. Prop. XV.* $zo + \text{Fo. Glas} : zo :: \text{Fo. Gl} : zf$

That is in Numbers — — 24573 : 20000 :: 4573 : 3722.

Then — — — — — $zf : gz :: bf (= bz + zf) : ab$

That is in Numbers — — — 3722 : 500 :: 13722 : 1843.

Lastly in the right-angled Triangle goz , as zo : Rad. :: gz : Tang. $\angle goz = 1^\circ 25' 50''$.

Example Calculated according to the Method Propos'd in this Second *Schol. Prop. XLVI.* but the Letters refer to *Tab. 32. T. 32. F. 5. f. 5.*

Focus

Focus of the Glafs = — — — — — 4573.
 Distance of the Object = bz = — — — — — 10000
 Distance of the Eye = zo = — — — — — 20000
 Half Breadth of the Object as }
 Found in the foregoing Example } = ab = 1843 } *Data.*

First by *Corol. Prop. XV.* $bz + Gl.$ Focus : $bz :: Gl.$ Fo : ze
 That is in Numbers 14573 : 10000 :: 4573 : 3138
 Then — — — — — — — — — — — zb : $ba :: ze : ed$
 That is in Numbers — — 10000 : 1843 :: 3138 : 578.

Lastly in the right-angled Triangle deo , as oe (= $oz + ze$ = 23138) : Rad :: ed : Tang. $\angle doe = 1^\circ 25' 50''$. agreeable to what was found by the immediately preceding Calculation.

I shall now shew, how by this Method of Demonstrating the Properties, Effects, and Appearances of Glasses; Some of the noted Propositions in Dioptrick Writers may be easily proved. In which the Authors have been very *Operose*, and in some very *Obscure*. And because I will not interrupt the Series of my Propositions, I shall give them in the following Order.

P R O P. XLVII.

The further the Eye is removed from the Concave Glafs, the Object appears the less. Zahn Telescop: Fund. 2. Synt. 2. Cap. 5. Prop. XXIII. Dechaes Dioptr. Lib. 2. Prop. LII.

T. 32. F. 5. This is manifest from *Tab. 32. f. 5.* For we are to Consider the Image def , in the virtual Focus, as the Object, and as look'd at without the Glafs. For the Lines do , fo , which determine the Optick Angle, are drawn directly to the Eye at o . Wherefore if we conceive the Eye o removed farther from the Glafs; the Angle dof must needs decrease, and consequently the Object appear *less*. What is here said of the Concave may be apply'd to the Convex in *Tab. 31. f. 3.*

P R O P.

PROP. XLVIII.

If a Concave Glass be removed from the Eye, so large an Area or space of the Object cannot be seen through it.

This is the 97th Prop. of *Kepler's Dioptricks*. Wherein he seems to make no Distinction between removing the Glass from the Eye, and removing the Eye from the Glass. Whereas there is a very great Difference between both Motions to be consider'd in *Dioptricks*. For in moving the Glass from the Eye, the Glass Approaches the Object, and the *Locus Apparens Objecti* is changed. But in moving the Eye to or from the Glass, the *Locus Apparens Objecti* never alters. Prop. XXXI. Sec. 7. And from his Proof of this Proposition 'tis manifest, that he should have expressed it thus,

If the Eye be removed from a Concave, so large a space of the Object cannot be seen through it.

Wherefore instead of the foregoing 48th. Proposition, I substitute this; Which is Evident by our foregoing Method from Tab. 33. f. 3. Wherein abc is an Object, gl a Concave Glass, $T33.F3.$ o the Eye nigh the Glass, e the Eye more Distant from the Glass. Let the Point of Divergence, answering to the Station at o , be f (determin'd by the foregoing Doctrine): draw fga , flc , directly; the space visible at o is ac . Now, when the Eye is removed from the Glass to e , the Point of Divergence shall also be removed from the Glass, and determin'd by what foregoes, as suppose at q (till the Eye be at an infinite Distance, and then the Point of Divergence is as far from the Glass as 'tis possible, viz. in the virtual Focus): draw qgn , qlm , directly; The space visible at e is nm less than ac . Which was to be Demonstrated.

Or.

T43 F. 1. Or otherwise. *Tab. 34. f. 1. def* is the *Locus Apparens Objecti* abc through the Glass gl ; the Eye is at o ; og , ol produced directly meet the Image in the Points d , f . Wherefore exactly the whole Object abc , and no more, is seen through the Glass gl by the Eye at o . Let the Eye be removed to q ; If we draw qd , qf , these fall not on the Glass, and consequently the Extremities of the Object a and c shall not be perceived by the Eye at q through the Glass. (*Schol. 1. Prop. XLVI.*) Draw therefore qg , ql , and produce them directly to n and m ; the Points in the Object y , x , answerable to the Points n , m , are the utmost Extremities visible through the Glass gl to the Eye at q . Which Points y , x , are easily determinable by what is laid down *Prop. XL. Sec. 3.* For from the Center of the Glass z , draw zny , zmx , directly; y , x , are hereby determin'd.

As to this 48th. *Prop. viz. That if a Concave Glass be removed from the Eye, so large an Area or space of the Object cannot be seen through it.* 'Tis needless to enlarge thereon, after our 46th. *Prop.*

PROP. XLIX.

The farther a Concave Glass is removed from the Eye, The Objects are thereby the more diminish'd, as long as the Glass continues nigher to the Eye than to the Object.

This is the 98th. *Prop. of Kepler's Dioptricks.* We may find it also, In *Cherubin's La Dioptrique Oculaire* pag. 77. *Dechales Dioptrica Lib. 2. Prop. XXXVIII. Herigonii Dioptr. Prop. XXXI.* But in all of them obscurely and loosely proved.

The Proposition is universally True, as well in Convex as Concave Glasses, only with this Restriction in the Convex, That the Object is to be nigher the Glass, than the Glasses Focal Length. And then we may express the Proposition universally thus,

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T43-F. 1. Or otherwise. *Tab. 34. f. 1.* *def* is the *Locus Apparens Objecti abc* through the Glass *gl*; the Eye is at *o*; *og*, *ol* produced directly meet the Image in the Points *d*, *f*. Wherefore exactly the whole Object *abc*, and no more, is seen through the Glass *gl* by the Eye at *o*. Let the Eye be removed to *q*; If we draw *qd*, *qf*, these fall not on the Glass, and consequently the Extremities of the Object *a* and *c* shall not be perceived by the Eye at *q* through the Glass. (*Schol. 1. Prop. XLVI.*) Draw therefore *qg*, *ql*, and produce them directly to *n* and *m*; the Points in the Object *y*, *x*, answerable to the Points *n*, *m*, are the utmost Extremities visible through the Glass *gl* to the Eye at *q*. Which Points *y*, *x*, are easily determinable by what is laid down *Prop. XL. Sec. 3.* For from the Center of the Glass *z*, draw *zn*, *zm*, directly; *y*, *x*, are hereby determin'd.

As to this 48th. *Prop. viz. That if a Concave Glass be removed from the Eye, so large an Area or space of the Object cannot be seen through it.* 'Tis needless to enlarge thereon, after our 46th. *Prop.*

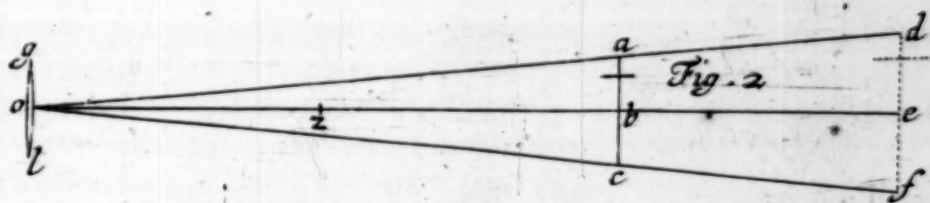
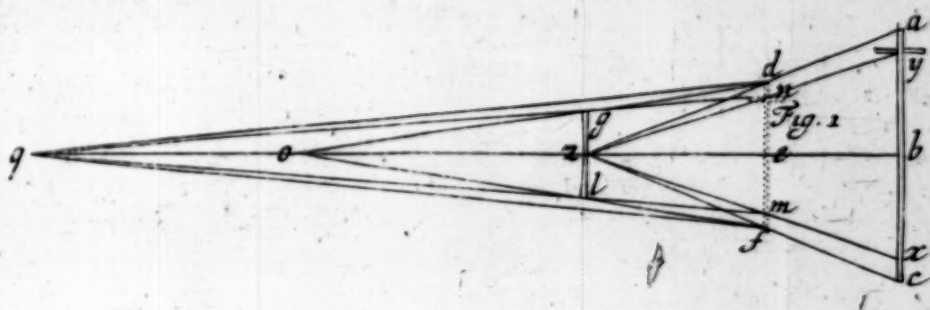
PROP. XLIX.

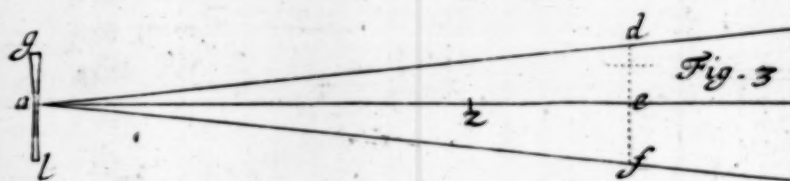
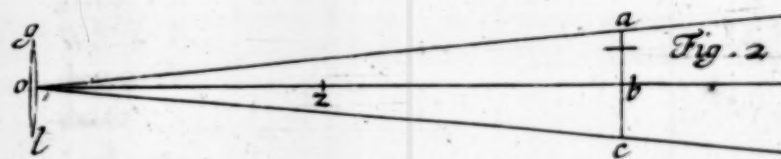
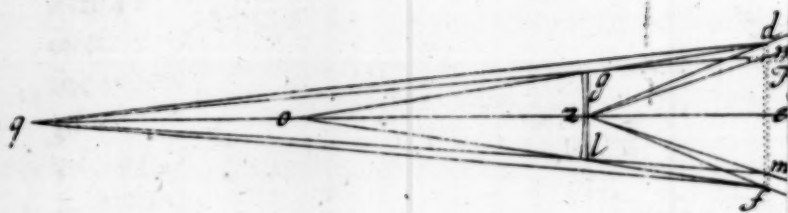
The farther a Concave Glass is removed from the Eye, The Objects are thereby the more diminish'd, as long as the Glass continues nigher to the Eye than to the Object.

This is the 98th. *Prop. of Kepler's Dioptricks.* We may find it also, In *Cherubin's La Dioptrique Oculaire* pag. 77. *Dechales Dioptrica* Lib. 2. *Prop. XXXVIII.* *Herigonii Dioptr. Prop. XXXI.* But in all of them obscurely and loosely proved.

The Proposition is universally True, as well in Convex as Concave Glasses, only with this Restriction in the Convex, That the Object is to be nigher the Glass, than the Glasses Focal Length. And then we may express the Proposition universally thus,

A Con-





Tab

A Convex-Glass being equally Distant from the Eye and from the Object, renders the Appearance the most Magnified; and a Concave the most Diminished, that That Distance of Eye and Object will allow.

For the Proof hereof I need offer no more than the following Calculations (instituted according to the Doctrine Precedent) wherein we shall find (*Tab. 31. f. 3. and Tab. 32. f. 5.*) <sup>T.31. F.5.
T.32. F.5.</sup> the Angle *doe*, (which is the Angle under which the Object appears through the Glass) in the second Case (wherein the Glass is equally Distant from the Eye and Object) to be greater for the *Convex*, and less for the *Concave*, than the same Angle in either of the other Cases.

And by the same Calculations we may likewise observe in the first and third Cases, That when the Glass is *equally* removed from the *middle* towards the Eye, or towards the Object, the Appearance is *equally* magnified by the *Convex*, and diminish'd by the *Concave*; for we find in the first and third Cases, the Angles *doe* equal.

I say, this may be sufficient for the Proof of the foregoing Position. But to this I shall add also this farther Consideration. That by the preceding Doctrine, the magnify'd Appearances of Objects through *Convexes*, and their diminish'd Appearances through *Concaves*, being deduced from the *Loci Apparentes* of Objects through those Glasses. We may easily conceive, That supposing the *Convex-Glass gl* (*Tab. 34. f. 2.*) or *Concave gl* (*Tab. 34. f. 3.*) to touch the Eye *o*, and that the Apparent Place of the Object *abc* is *def*. <sup>T.34. F.2.
T.34. F.3.</sup> The Eye perceives the Object under its own Natural Optick-Angle, neither magnify'd or diminish'd by one or t'other Glass.

Likewise if, in the same Figures, we conceive the Glasses (which are still supposed of the least thickness imaginable) to touch the Objects, the Object and *Locus Apparens* thereof are the same; and consequently, the Optick-Angle in this Posture can neither be *increased* or *diminish'd* by either of the Glasses.

X

Where-

Wherefore it remains, that seeing the Glasses do *not* at all Exert their Effects in either of the *Extremes*, that is, *either touching the Eye, or touching the Object*: And seeing they Exert their Effects *equally* being *equally* removed from the middle z between the Eye and the Object; It will follow, that they Exert their Effects *most powerfully* being placed just in the middle z between the Eye and Object, that is, the Convex by *Magnifying*, and the Concave by *Diminishing* the Appearance.

And thus much shall suffice concerning *single Glasses apply'd to the Eye*.

Here follow the Calculations.

T.31. F.3.

Calculation for the CONVEX Tab. 31. F. 3.

Focus of the Glas = 10,000 } Given the same
Breadth of the Object = $ab = 1,000$ } in all the Cases.

First Case.

Wherein the Distance between the Eye and the Glas oz is given greater than the Distance between the Object and Glas bz

$$\text{Data } \begin{cases} bz = 3,000 \\ oz = 9,000 \end{cases}$$

$$ze = 4,286$$

$$ed = 1,429$$

$$oe = oz + ze = 13,286$$

$$L. doe = 6^{\circ} 8' 15''$$

Second Case.

Wherein the Glas is equally distant from the Eye and Object, that is,

$$\text{Data } \begin{cases} bz = 6,000 \\ oz = 6,000 \end{cases}$$

$$ze = 15,000$$

$$ed = 2,500$$

$$oe = oz + ze = 21,000$$

$$L. doe = 6^{\circ} 47' 26''$$

Third Case.

Wherein the Distance between the Eye and the Glas oz is given less than the Distance between the Object and the Glas bz

$$\text{Data } \begin{cases} bz = 9,000 \\ oz = 3,000 \end{cases}$$

$$ze = 90,000$$

$$ed = 10,000$$

$$oe = oz + ze = 93,000$$

$$L. doe = 6^{\circ} 8' 15''$$

Calculation

Calculation for the *CONCAVE* Tab. 32. F. 5. T. 32. F. 5.

Focus of the Glafs = 10,000 } Given the same
Breadth of the Object = $ab = 1,000$ } in all the Cases.

First Case.	Second Case.	Third Case.
Wherein the Distance between the Eye and the Glafs oz is given greater than the Distance between the Object and the Glafs bz .	Wherein the Glafs is equally Distant from the Eye and Object, that is, $bz = zo$	Wherein the Distance between the Eye and the Glafs oz is given less than the Distance between the Object and Glafs bz .
Data $\begin{cases} bz = 3,000 \\ oz = 9,000 \end{cases}$	Data $\begin{cases} bz = 6,000 \\ oz = 6,000 \end{cases}$	Data $\begin{cases} bz = 9,000 \\ oz = 3,000 \end{cases}$
$ze = 2,308$ $ed = 0,769$	$ze = 3,750$ $ed = 0,625$	$ze = 4,737$ $ed = 0,526$
$oe = oz + ze = 11,308$ $L.doe = 3^{\circ} 53' 20''$	$oe = oz + ze = 9,750$ $L.doe = 3^{\circ} 40' 0''$	$oe = oz + ze = 7,737$ $L.doe = 3^{\circ} 53' 20''$

Hitherto we have Treated of *single* Convexes and Concaves, apply'd to the Eye. I proceed now to the *Combination* of Convexes with Convexes, and Convexes with Concaves; Wherein the Properties, Effects, and Appearances of *Telescopes*, and *Microscopes*, of all kinds shall be declared.

Definitions.

The *Object-Glass* is the Glass next the Object.

The *Eye-Glass*, simply so called, is the Glass *immediately* next the Eye. But if there be more than one, the *first Eye-Glass* is that next the Object-Glass; the *second* is that next the first, &c.

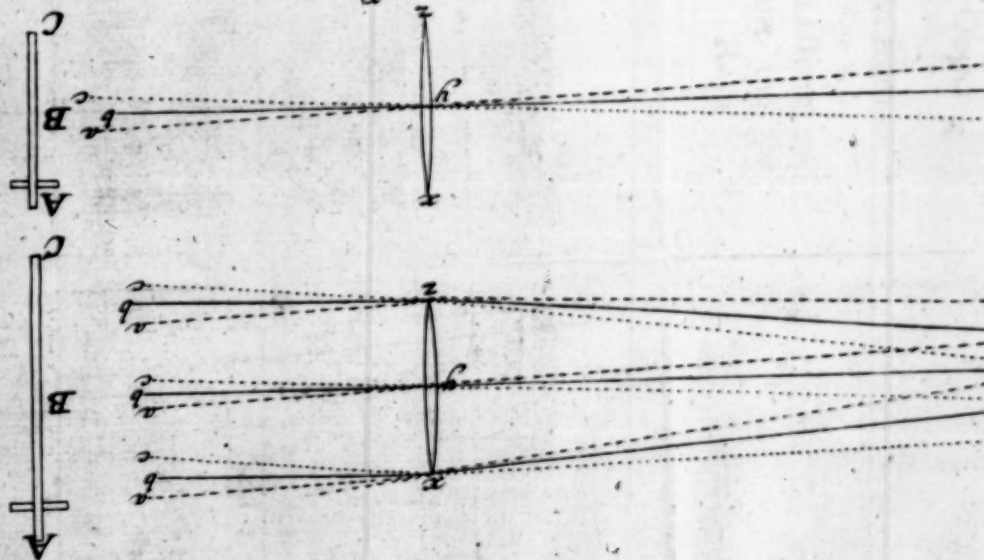
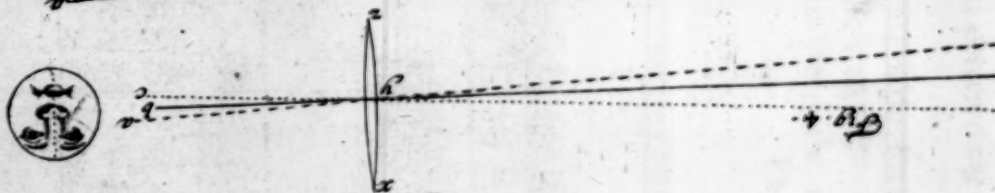
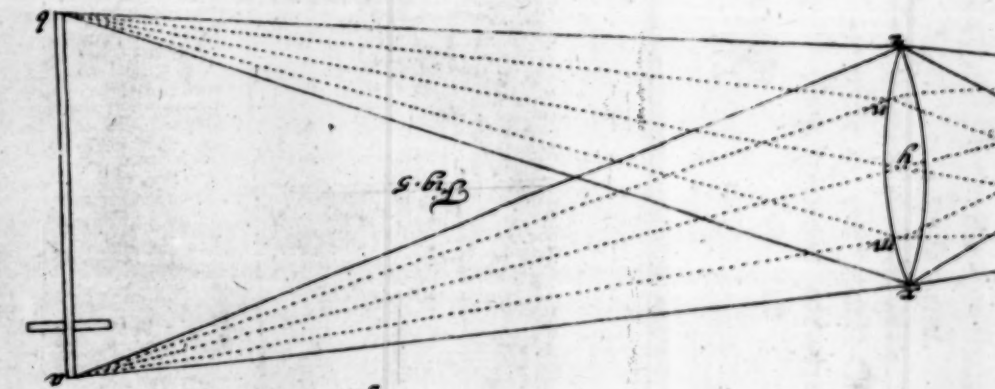
PROP. L.

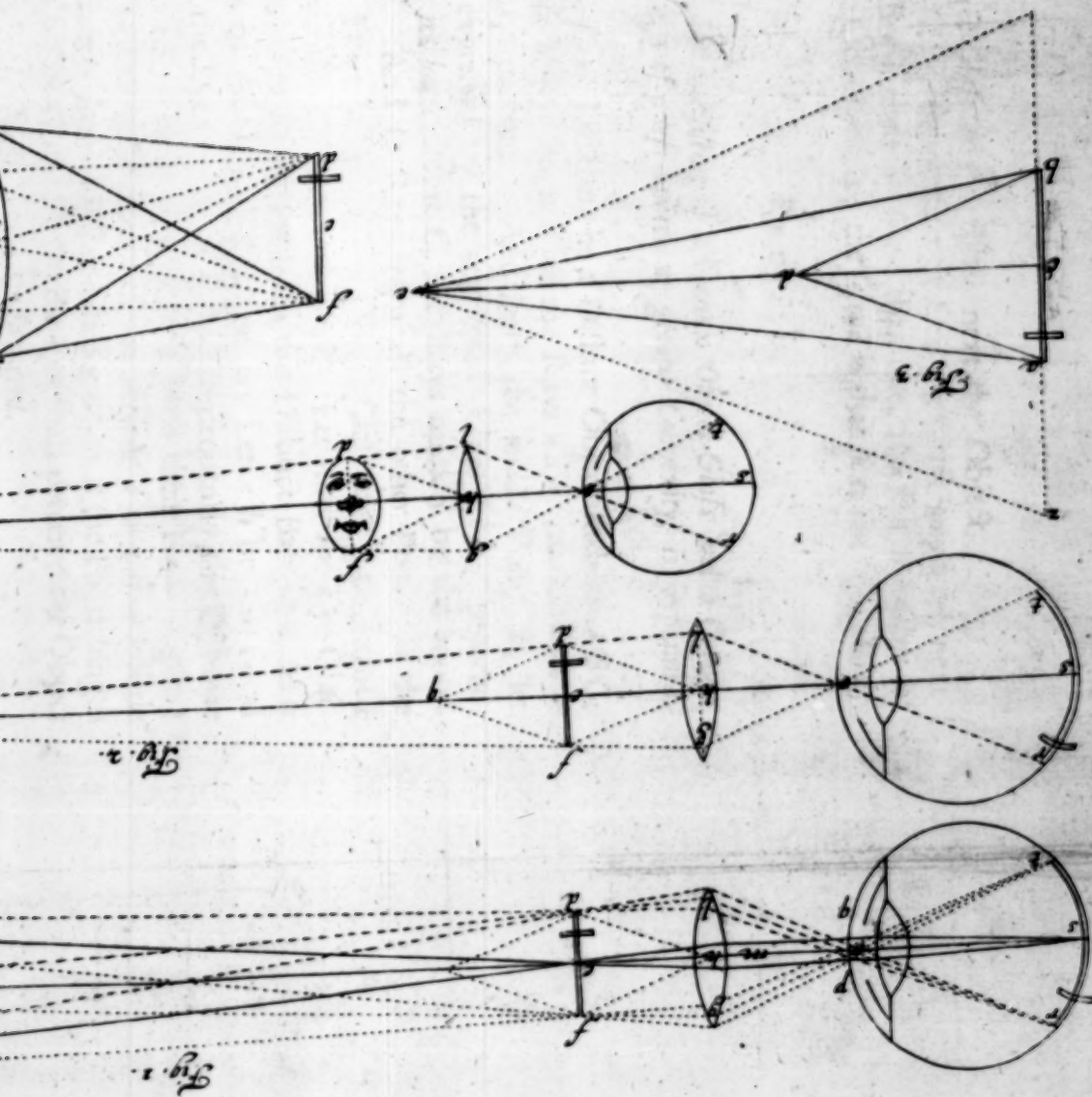
The Telescope, consisting of a Convex Object-Glass and a Convex Eye-Glass of a less Sphere or greater Convexity, is explained.

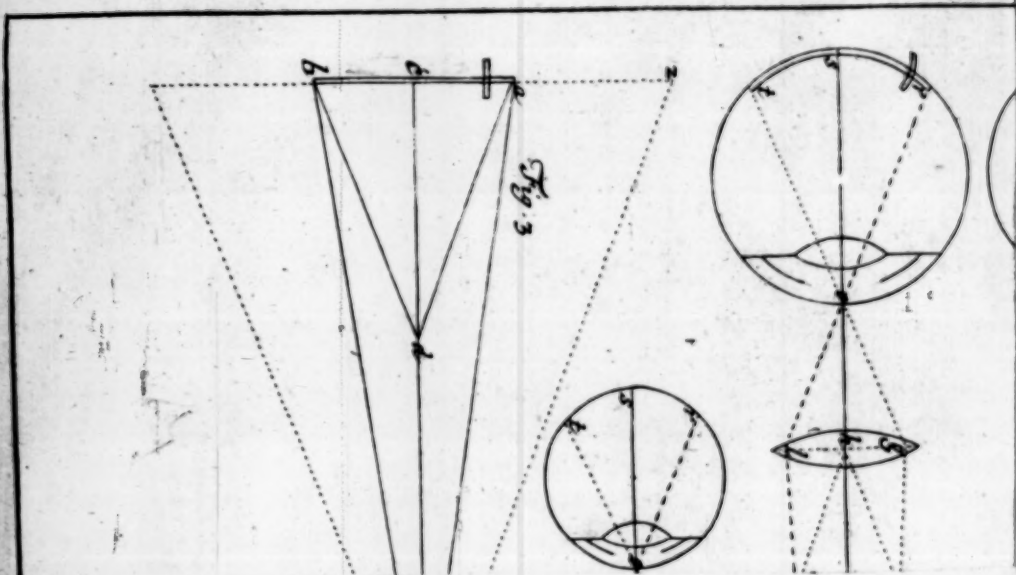
T.35. F.1. Tab. 35. f. 1. Let there be a Distant Object such as ABC ; From whose *highest* Point A let the Rays aaa , mark'd by the long Pricks, proceed. And from the *middle* Point B , let the Rays expressed by the continued Lines bbb , proceed. And from its *lower* Point C , the Rays mark'd by the round Pricks ccc . And so Rays from all the other Points in the Object. These falling on the Object-Glass xyz , are formed thereby into the Distinct Base fed . Let now the Eye-Glass ghl be placed as far distant from this Distinct Base fed , as is the Focus of this Eye-Glass, that is; Let eb be the Focal length of the Eye-Glass ghl (And consequently the Distance of the Glasses yh is the Aggregate of both their Focal lengths) And let the Eye o be placed as far distant (or rather a little more distant) from the Eye-Glass ghl as is the Focal length of the same Eye-Glass. I say the Eye shall perceive the Object ABC , Distinct, Magnified, and Inverted.

First, I say the Object is seen *distinctly*. For the Rays from each Point, being made by the Object-Glass to Converge towards the Distinct Base, proceed forward from the Distinct Base Diverging, and so fall on the Eye-Glass. Thus the
Rays

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Rays bx , by , bz , from the middle Point of the Object B , are made to Converge into xe , ye , ze ; And crossing at e , they flow forward, and fall on the Eye-Glass about b Diverging. Wherefore the Point e being now in the Focus of the Eye-Glass, the Rays that flow from it upon the Eye-Glass, after passing the Eye-Glass become parallel (*Prop. VI.*) and fall so on the Eye at o ; By whose Coats and Humors they are refracted and brought together on the Point s on the *Retina* (*Prop. XXVIII. Sec. 7.*) there painting the lively Image of the Point B in the Object.

Secondly, In like manner may we conceive the Representation of the Collateral Points: thus the Rays ax , ay , az , proceeding from the upper Point A of the Object, are by the Object-Glass xyz made to Converge towards the Distinct Base in d ; From whence flowing forward they fall Diverging on the Eye-Glass at l ; by which they are made to run parallel amongst themselves, and are bent towards the Focus at o ; where falling on the Pupil parallel, they are by the Eye refracted and brought together in the Point r on the *Retina*. There painting the Representation of the Point A in the Object.

Thirdly, That the Rays flowing *Diverging* from each particular Point in the Distinct Base $f e d$ are brought by the Eye-Glass to a *Parallelism* amongst themselves, is manifest from *Prop. VI.* And that the Rays from f , from e , and from d , are mixt and confus'd by the Eye-Glass in its Focus at o (or thereabouts) is manifest from hence, that we may conceive one Ray in the Cone of Rays gf , or in the Cone of Rays zfx , that runs parallel to the Axis $yehos$, or at least that would run so parallel, if the Breadth of the Object-Glass in respect of the Breadth of the Eye-Glass will permit. This Ray, I say, by the known Properties of the Convex Eye-Glass, shall be refracted into its Focus at o ; and all the other Rays of the same Cone fg , after passing the Eye-Glass, shall
be

be refracted and made to proceed parallel to that single Ray, that is bent into the Focus o , because the Point f is supposed in t'other Focus of the Eye-Glass. Wherefore the Rays from all the Points in the Distinct Base are confounded together about the Focus of the Eye-Glass.

Fourthly, Or otherwise I shall explain this matter thus; If we conceive y , a Radiating Point, sending forth the Rays yg , yh , yl : And the difference between the Focal length of the Object-Glass and Focal length of the Eye-Glass to be very great (as suppose the Object-Glass to be twelve Foot, and the Eye-Glass three Inches) we shall find by Prop. V. the Point o , where the Rays go , lo , cross the Axis or Perpendicular Ray bo , to be very nigh the Focus of the Eye-Glass (*viz.* in our Supposition bo shall be 3,063 Inches) and let the Focal lengths of the Object-Glass and Eye-Glass bear whatever Proportion, the Point o , where go , lo , shall cross bo , may be determin'd by Prop. V. But then indeed the Rays that fall on the Eye *without* go , or *within* it, do cross the Perpendicular Ray bo , *farther* from the Eye-Glass, or *nigher* to the Eye-Glass than go it self; For they do not proceed from the same Point y ; And go is only the Refraction of the Ray yg . And therefore, unless the difference between the Focal lengths of the Object-Glass and Eye-Glass be *Considerable*, the Eye may move considerably *nigher* to the Eye-Glass, or *farther* from the Eye-Glass than the Point o , and not perceive any Alteration in the Appearance of the Object. But if the said Difference be *Considerable*, (as it always is in Telescopes) the Eye can move but very little either *farther* from or *nigher* to the Eye-Glass than o , but it shall perceive a great Alteration in the Appearance (that is, in the Visible Area) of the Object. As shall be manifested more plainly hereafter.

From all which it appears, That the Pupil of the Eye at o , being in the place of the *greatest Confusion*, where the Rays
from

from all Points in the Object are mixt together, and the Rays from each single Point fall parallel amongst themselves; The Appearance of the Object must needs be *Distinct*. By *Prop. XXVIII. Sec. 3.*

Fifthly, I say likewise the Object is *magnified*. If the naked Eye were in the place of the Object-Glass at *y* (*Tab. 35. f. 2.*) T.35. F.2. the Object would appear to it under the Angle $ayc = fyd$. That is, the Object would appear to the Eye at *y*, under the same Angle as the *Distinct Base* fcd , were it an Object, would appear to the same Eye at *y*. Wherefore let us now consider the *Distinct Base* fcd as the Object: This appears to the Eye at *o* through the Eye-Glass under the same Angle gol , as were the naked Eye viewing it at *b*, by *Prop. XXXIII.* Make og equal to ob , and draw bf , bd , and gf , gd . The Angle fbd by *Prop. XXXIV.* is equal to gol , the Optick-Angle through the Telescope, and the same Angle fbd is equal to fqd . But fqd is much greater than $fyd = ayc$ the Natural Optick-Angle to the Eye supposed at *y*. and yet much greater than the Natural Optick-Angle would be to the Eye at *o*. Because *o* is yet further from the outward real Object than *y* by the whole length of the Telescope. Wherefore the Object is *Magnified*.

(Lastly, I say the Object appears *Inverted*. This is manifest from the very Scheme (*Tab. 35. f. 1.*) without farther Ex- T.35. F.1. plication. For the Object is *Inverted* in the *Distinct Base*, and the Eye-Glass does not return it again before it arrive at the Eye, but 'tis painted on the *Retina* *rst Erect*; wherefore by *Prop. XXVIII. Sec. 4, 5.* the Object appears *Inverted*. See also *Prop. XXXIX. Sec. 4, 5.*

Scholium.

The *Locus Apparens* of an Object through this Glass is the *Distinct Base* fcd (*Prop. XXXI.*) as is manifest from this Ex-

Experiment. Stretch an Hair exactly in this Distinct Bale, it shall appear as it were fixt to the very Object. *Vid. Chap. 5. Sec. 3. of the Second Part.*

PROP. LI. LEMMA.

If the Eye directly approach to or recede from an Object; it shall be, as the Tangent of the Semiopstick-Angle of one Station: to the Tangent of the Semiopstick-Angle of t^h other Station: So (reciprocally) the Distance of the Eye from the Object in this latter Station: to the Distance of the Eye from the Object in the Former Station.

T. 32. F. 3. Tab. 35. f. 3. Let abc be an Object, from whose middle Point c erect ce Perpendicular to ab . Let the first Station of the Eye be at e , and its second Station at d ; and draw ad , ae ; I say therefore, As ce : to cd :: so Tangent of the Angle adc : to the Tangent of the Angle aec .

Demonstration.

Produce ca infinitely towards z , draw ez parallel to ad . The Angle zec is equal to the Angle adc (29. 1. *Eucl.*) then ec being put Radius, zc is the Tangent of the Angle $zec = adc$; And ac is the Tangent of the Angle aec ; And it shall be, As ce : to cd :: so zc : to ac : (2. 6. *Eucl.*) which was to be Demonstrated.

This is the 31 *Prop. of Gregorii Opt. Promot.* but there otherwise Demonstrated.

PROP. LII. LEMMA 2.

If the Eye directly approach to, or recede from an Object, its apparent Bigness increases or diminishes, as the Tangents of the Semiopstick-Angles at one and t other Station.

This is manifest, for the Eye at d (Tab. 35. f. 3.) sees ac T. 35. F. 3. as big as the Eye at e would see zc , by Prop. XXVIII. Sec. 6. Because the Angles adc , zec are equal; and *Quæ sub equali Apparent Angulo, Æqualia videntur.* So that the Eye advancing from e to d , sees ac as much bigger, as if, continuing at e , the Object ac had increased to cz .

Corollary.

From this and the last Proposition it follows, that if the Eye directly approach to or recede from an Object, its apparent Magnitude increases or diminisheth, as the Distances of one and t other Station reciprocally, that is, the apparent Magnitude of ac to the Eye at d : is to the apparent Magnitude of ac to the Eye at e :: $asce$: to cd .

PROP. LIII.

The apparent Diametral Magnitude of an Object viewed through the Telescope of Prop. L. Is to the apparent Diametral Magnitude of the Object viewed by the naked Eye at the Station of the Object-Glafs :: As the Focal length of the Object-Glafs : to the Focal length of the Eye-Glafs.

This is the great Proposition asserted by most Dioptrick Writers, but hitherto proved by none (for as much as I know)
Y
they

they offer indeed Experiments and Methods of Tryal to confirm the Truth thereof, but proceed no farther.

Vid. *Cherubin Diop. Oculaire Part. II. Prop. XXI. LIX. LX. LXII. Kepleri Dioptr. Prop. CXXIV. Galilei Nuncius Sidereus pag. 12. Edit. Lond. 1653. 8vo.*

Honoratus Faber in his *Synopsis Optica* *Prop. XLIV.* for the Telescope consisting of a Convex Object Glass and Concave Eye-Glass; and in *Prop. XLV.* for the Telescope consisting of a Convex Object-Glass and Convex Eye-Glass, indeavours at something, which he calls a *Demonstration* of this Property. But whether that which he there offers will amount to clear Satisfaction, I leave to their Judgments, who shall Read him.

Dechales in *Prop. LIV. Lib. II. Dioptr.* thinks this Proposition so far from *Demonstrable*, that he takes it to be *False*; and says, He never met with any *Demonstration* thereof, that did not include manifest *Paralogisms*. Perhaps he may be right in this latter part of his Assertion; but the Reason he gives for his concluding it a False Proposition is manifestly Weak and Erroneous: And that on the account of an inartificial kind of Notion and Method, that he takes for Explicating the Magnifying of Telescopes; especially of the Telescope furnish'd with a Concave Eye-Glass, which he explains after his manner in *Prop. LIII.*

It were needless to enlarge in this matter, I shall therefore pass it over, and hasten to the *Demonstration* of this Proposition.

T.35. F.2. *Tab. 35. f. 2.* Let the Object-Glass xyz Project the Image of the Object ABC in the Distinct Base fed , ghl is the Eye-Glass, he the Focal length of the Eye-Glass, to which let eq be made equal. Draw bf , bd , and qf , qd . And let the Eye at o be placed in the Exterior Focus of the Eye-Glass. The Rays yfg , ydl , are refracted by the Eye-Glass, and cross at o . Wherefore gol is the Angle under which the Object appears through the Telescope. But the Angle gol is equal to the

the Angle fbd (as well by what foregoes in *Prop. L. Sec. 3.* & 5. as by *Prop. XXXIV.*) And the Angle fbd is equal to the Angle fqd .

Wherefore were the naked Eye at y the Station of the Object-Glass, it would perceive the Object under the Angle $fyd = ayc$. But now being armed with the Telescope, it sees the Object under the Angle fqd . Let us take their halves fye , fqe ; and consider fe , half the Distinct Base, as the Object viewed by the naked Eye at the Stations y and q . I say (by *Lemma 2.*) The apparent bigness of the Object fe at the Station q : Is to its apparent bigness at the Station y : As the Tangent of the Angle fqe : To the Tangent of the Angle fye . But (by *Lemma 1.*) the Tangent of the Angle fqe : Is to the Tangent of the Angle fye : As ey : To eq . Therefore the apparent bigness of the Object fe at the Station q : Is to its apparent bigness at the Station y : As ey : To eq : that is, As the Focal length of the Object-Glass: To the Focal length of the Eye-Glass. But the Angle gob , under which the Eye sees half the Object through the Telescope, is equal to the Angle fhe or fqe . Therefore the apparent Diametral bigness of an Object viewed through a Telescope: Is to the apparent Diametral Magnitude of the Object viewed by the naked Eye at the Station of the Object-Glass: As the Focal length of the Object-Glass: To the Focal length of the Eye-Glass. Which was to be *Demonstrated*.

The same may be declared otherwise. Thus; *Tab. 35. T.35.F.2.*
f. 2. Let us suppose the naked Eye at b to view the Object Inverted by means of the Distinct Base fed ; The Inverted Object shall appear under the Angle fbd (by *Prop. XL.*) But the Eye at o through the Glass perceives the Inverted Image of the Object under the Angle gol equal to fbd , (by *Prop. XXXIII.*) and fbd is equal to fqd , and consequently (as in the foregoing Demonstration) the Proposition is manifest.

I shall now mention the common Method for trying the Truth of this Proposition by Experiment. Having the Focal length of an Object-Glass (for Instance) 144 Inches, and the Focal length of an Eye-Glass three Inches. A Telescope composed of these, shall make the apparent Diametral Magnitude of an Object: To the apparent Magnitude of the same Object viewed by the naked Eye :: As 144: To 3 :: or 48: To 1. Wherefore, such a Glass is said to Magnifie 48 times in the Diameter of the Object, and 2304 (= square of 48) in the Surface of the Object. The Superficies of like Figures being to each other, as the Squares of their Diameters, or Homologous Sides.

Wherefore from a convenient Scale take one part, and therewith describe a Circle; And from the same Scale take 48 parts, and describe another Circle. Let these two Circles be cut out in Paper, or other Conspicuous Material, and placed at three or four Foot from each other, on a Wall at such a Distance as will require the length between the Glasses in the Telescope but just 147 (= 144 + 3) Inches, to shew these Objects distinctly; Then with one Eye through the Telescope observe the smaller Circle, and at the same time with t'other Eye naked look upon the greater Circle; these two Circles shall appear equal to both Eyes.

Perhaps it may be objected, That the Comparison is not fair between both Appearances. For the Proposition supposes the naked Eye at the Station of the Object-Glass; But this Experiment sets the naked Eye Distant from the Object-Glass the whole length of the Telescope. This would be a material Objection against this Method of Tryal, were not the Distance of the two Circles from the Eyes vastly greater than the length of the Telescope; so that the Telescopes length may not bear any sensible Proportion thereto. And such we suppose it in this Experiment, by advertising that this Distance is

to be so great, that the Distance between the Glasses may be no longer than for viewing a Distant Object, *viz.* the just Aggregate of the Focal lengths of the Glasses, that is, $144 + 3 = 147$ Inches, that is, $ye + eb = yb$.

I shall now give an Example of a Calculation according to this Proposition. Wherefore in *Tab. 35. f. 4.* let us take the *T. 35. F. 4.* Moon *ABC* for our Distant Object; and let us suppose its Diameter to subtend an Arch of a great Circle of Heaven of 30' Minutes. Let the Ray *ayd* proceed from its upper Limb, *b ye* from its Centre, *cyf* from its lower Limb. These cross in the Vertex *y*, or middle Point of the Object-Glass *xy z*; making the Angle $fyd = aye$, equal to 30' Minutes. Let the Focal length of the Object-Glass *ey* be given twelve Feet = 144 Inches, or 144.00 Parts: And the Focus of the Eye-Glass *be* be given three Inches, or 3.00 such Parts. Let the Distinct Base, wherein the Image of the Moon is Projected by the Object-Glass be *fed*, and draw *fb, db*. It is shewn before that the Angle *gob* is equal to the Angle *fbe*.

Wherefore in the Right-angled Triangle *fey*, we have *ey* = 144.00, and the Angle *fye* = 15', to find *fe* = 0.63.

Then in the Right-angled Triangle *feb*, we have *be* = 3.00, and *fe* = 0.63, to find the Angle *fbe* = 11° 51' 40".

Wherefore the Semidiameter of the Moon, which by the naked Eye would be seen under the Angle of 15' Minutes, is seen through this Telescope under an Angle of 11° 51' 40".

Let us now enquire, whether the Object appearing under an Angle of 15' Minutes, and being afterwards made to appear under an Angle of 11° 51' 40", doth not thereby appear 48 times bigger than naturally. (for so much, by what foregoes, does this Glass Magnifie).

And for shewing this, let us imagine *fe* increased 48 times its length 0.63; And then inquire what Angle *fye* would be. Wherefore *fe* is now supposed = $30.24 = 48$ times 0.63; Then
in

in the Right-angled Triangle fey , we have $fe = 30.24$, and $ey = 144.00$, to find the Angle $fye = 11^{\circ} 51' 40''$. Which shews that the Semidiameter of the Moon, being made by the Glass to appear under an Angle of $11^{\circ} 51' 40''$, is seen by the Eye as big, as if the Semidiameter of the Moon it self were really increased 48 times, and viewed by the naked Eye. Which is the proposed Design of this Calculation.

Corollary 1.

From hence it follows, That the same Object-Glass being at one time combined with an Eye-Glass whose Focus is 1. And at another time with an Eye-Glass whose Focus is 2. The first Telescope Magnifies twice as much as the latter.

Corollary 2.

Supposing two Telescopes of different lengths; If the Focus of the Eye-Glass of the shorter bears the same Proportion to the Focus of its Object-Glass, as the Focus of the Eye-Glass of the longer bears to its Object-Glass: These two Telescopes Magnifie *equally*.

And hereupon perhaps it may be enquired, To what end then is all the Pains and Trouble in forming and managing Telescopes of 30. 40. 50. 100. 200. 300, &c. Feet; When Objects may be Magnified as much by smaller Object-Glasses, or Object-Glasses of shorter Focal lengths, combined with Proportional Eye-Glasses?

I answer First, That Object-Glasses of a shorter Focus will not bear proportionably Eye-Glasses of such short *Foci*, without coloring the Object and rendring it dark, as Object-Glasses of longer *Foci*. For Instance, let us suppose that an excellent Object-Glass of twelve Foot Focus will receive an Eye-Glass

Glas of no shorter a Focus than three Inches with Clearness and Distinctness. I say an Object Glas of 24 Foot Focus of the same Perfection shall receive an Eye-Glas of less than six Inches Focus with equal Clearness and Distinctness. And perhaps it may take an Eye-Glas of five or four Inches Focus. And then an Object-Glas of twelve Foot with an Eye-Glas of three Inches Magnifies but 48 times. But an Object-Glas of 24 Foot with an Eye-Glas of four Inches Magnifies 72 times, viz. $\frac{1}{2}$ more than the former, which is a great Difference, and of vast Advantage, when it may be obtained with the same Clearness and Distinctness. I confess the longest Telescopes do generally render the Objects more Dark and Obscure, yet when shorter Glasses have proportionably as short Eye-Glasses, and as close Apertures, they are more Obscure, than the longer Telescopes.

I answer Secondly, That the Image of the Moon or other Object in the Distinct Base of an Object-Glas of 24 Foot is twice as long as the Image in the Distinct Base of an Object-Glas of twelve Foot. And consequently we shall not wonder, that the Picture in the former, should be much more Distinct and Perfect, than in the latter; As 'tis much more easie to represent every Feature and Line of a Face in a large Piece, than in a small Piece of Miniature.

Corollary 3.

And if the Object-Glas be formed on a less Sphere than the Eye-Glas (as suppose the Object-Glas formed on a Sphere of six Inches Radius, and the Eye-Glas on a Sphere of twelve Inches Radius) hereby the Appearance of the Object shall be Diminished. And the Appearance through the Glas shall be to the naked Appearance as six to twelve, or $\frac{1}{2}$ the Natural Appearance.

Scholium.

Scholium.

From hence it is manifest, how requisite it is in relating any *Phænomena* observed by the Telescope (or even by the Microscope) to mention not only the length of the Tube in general; But to specify the particular Focus of the Eye-Glass, as well as of the Object-Glass; as also the Aperture of the Object-Glass. For by this means, they that intend to observe the same *Phænomena*, may understand how to adapt their Telescopes proper for the Observation. This the *Learned and Ingenious Monsieur Hugen* in his *Systema Saturnium* puts down exactly, pag. 4. Where also we find this Passage. *Illud in Dioptriciis Nostriis Demonstratum invenietur, Speciei per Tubum visæ ad eam quæ Nudo Oculo percipitur, hanc secundum Diametrum esse Rationem, quæ Distantiæ Foci in Exteriori vitro (Objectivo Scilicet) ad illam quæ in Interiori sive Oculari vitro est Foci Distantiam.* But hitherto we are so unhappy as to want that excellent Persons Dioptricks. In the mean time, let that which I have given in the foregoing *Prop. LIII.* serve till a better be offered.

PROP. LIV. PROBL.

To Determine the Angle received by a Telescope of the foregoing Combination. The Rule is, as the Distance between the Object-Glass and Eye-Glass: To half the Breadth of the Eye-Glass :: So Radius: To the Tangent of half the Angle received.

T. 35. F. 2. *Tab. 35. f. 2.* The Distance of the Glasses is hy . Let half the Breadth of the Eye-Glass be gh . Then, as hy : To gh :: So Radius: To the Tangent of the Angle gyh , which is half the Angle gyl , the Angle received. That is, the Eye at o shall perceive no more of the Object, than subtends this Angle before the Object-Glass.

Schol.

Scholium 1.

But if the Eye Approach *nigher to*, or recede *further from* the Eye-Glass $g\ h\ l$ (*Tab. 35. f. 1.*) than its Focus at o , it *T35. F.1.* shall perceive a lesser *Area* of the Object, though what it sees shall be as *Distinct* as at o . For let us suppose the Pupil of the Eye at m ; The Rays $g\ o$, $l\ o$, do not enter the Eye, and consequently the Points in the Object answerable to f , d , in the *Distinct Base*, shall not be visible. The same may be conceived if the Eye recede farther from the Eye-Glass than o ; because all the Rays from the several Points in the Object are mixt together, and intersect at o , in the Focus of the Eye Glass, and thence flowing forward they separate and Diverge. But then the Eye at m receives the Rays, that do enter it, *Parallel* or at least a very little *Diverging*; and consequently the Vision is *Distinct*.

Scholium 2.

From hence also 'tis manifest, that the Angle received, or Visible *Area* of the Object, is not increased or diminished, by the greater or lesser Aperture of the Object-Glass. For the Angle $g\ y\ l$ continues the same, though the Object-Glass were all covered to the very middle Point y . All that is effected by this greater or lesser Aperture is the more *Bright* or *Obscure* Appearance of the Object. But of this more fully in the next Proposition.

PROP. LV.

Concerning the Apertures of Object Glasses.

By the *Aperture* of a Glass I mean, that part of the Glass which is left open and uncovered. And this ought to be va-

rious according as we would have more or less Light admitted. It also varies according to the various Focal lengths of the Object-Glasses. For a ten Foot Object-Glass shall bear a greater Aperture than an Object-Glass of one Foot; and a twenty Foot Glass yet greater than a ten Foot Glass.

But at what Rate or Proportion the Apertures of Glasses alter in respect of their lengths, is not yet well settled.

Monsieur Auzout, (*Phil. Transact. N. 4. P. 55.*) Tells us, that he finds, That the Apertures, which Glasses can bear with Distinctness, are in (about) a Subduplicate Ratio to their lengths: Or as the Square Roots of their lengths. Whereof he intends to give the Reason and Demonstration in his *Dioptrica* (which we yet want.) But this Ingenious Person should have told us, when he speaks of the Apertures of Glasses, whether he designs them for Objects on the Earth or in the Heavens. And if in this latter, whether for the *Moon*, *Mars*, *Jupiter*, or *Venus*. For each of these Objects will require a different Aperture of the same Glass. Because the Strength of their Light is different. For to view *Venus* there is requisite a much smaller Aperture, than to view the *Moon*, *Saturn* or *Jupiter*.

However till some better Rule can be found for settling the Apertures of Object-Glasses (which at present I shall not pretend to) I shall here Present you with Mr. *Auzout's* Table, as 'tis to be found in the fore-cited *Philosophical Transaction*, Numb. 4. Noting only, that his Feet are *Parisian Feet* (which is to the *London* Foot as 1068: to 1000) and each Inch (which is the $\frac{1}{4}$ part of his Foot) is subdivided into twelve Lines. For it had not been worth our Pains to have reduced the whole Table to our *English* Measure. *Vid. Tab. 36.*

I have said before (*Schol. 2. Prop. LIV.*) That the Angle received, or Visible Area of an Object, is not Increased or Diminished by the greater or lesser Aperture of the Object Glass; all that is effected thereby is the Admittance of more or less Rays;

A TABLE of the Apertures of Object-Glasses.
The Points put to some of these Numbers denote Fractions.

Length of Glasses.	For Excellent ones.	For good ones.	For ordinary ones.	Length of Glasses.	For excellent ones.	For Good ones.	For ordinary ones.
Feet. Inches.	Inch. Lines.	Inch. Lines.	Inch. Lines.	Feet. Inches.	Inch. Lines.	Inch. Lines.	Inch. Lines.
4	4	4	3	25	3	42	10 2 4
6	5	5	4	30	3	83	22 7
9	7	6	5	35	4	03	4.2 10
0	8	7	6	40	4	33	73
1 6	9	8	7	45	4	63	103 2
2 0	11	10	8	50	4	94	03 4
2 6 1	0	11	9	55	5	04	33 6
3 0 1	1 1	0	10	60	5	24	63 8
3 6 1	2 1	1	11	65	5	44	83 10
4 0 1	4 1	2 1	0	70	5	74	103
4 6 1	5 1	3 1	.	75	5	95	04 2
5 0 1	6 1	4 1	1.	80	5	115	24 5
6	1	7 1	5 1 2	90	6	45	64 7
7	1	9 1	6 1 3	100	6	85	94 10
8	1	10 1	8 1 4	120	7	56	55 3
9	1	11. 1	9 1 5	150	8	07	05 11
10	2	1 1	10 1 6	200	9	68	06 9
12	2	42	0 1 8	250	10	69	27 8
14	2	62	2 1 9	300	11	610	08 5
16	2	82	4 1 11	350	12	6110	90 0
18	2	102	62 1	400	13	411	69 8
20	3	02	72 2				

*The feet here expres'd are Paris-feet, and a Line is the
 12 part thereof. The Paris-Foot is to the London-foot
 as 1068 to 1000*

A TABLE of the Apertures of Obli
The Points put to some of these Numbers de

Length of Glasses.	For Excellent ones.	For good ones.	For ordinary ones.	Length of Glasses.	For excellent ones.
Feet Inches	Inch Lines	Inch Lines	Inch Lines	Feet Inches	Inch Lines
4	4	4	3	25	3
6	5	5	4	30	3
9	7	6	5	35	4
0	8	7	6	40	4
1 6	9	8	7	45	4
2 0	11	10	8	50	4
2 6 1	0	11	9	55	5
3 0 1	1 1	0	10	60	5
3 6 1	2 1	1	11	65	5
4 0 1	4 1	2 1	0	70	5
4 6 1	5 1	3 1	.	75	5
5 0 1	6 1	4 1	1.	80	5
6	1	7 1	5 1	90	6
7	1	9 1	6 1	100	6
8	1	10 1	8 1	120	7
9	1	11. 1	9 1	150	8
10	2	1 1	10 1	200	9
12	2	4 2	0 1	250	10
14	2	6 2	2 1	300	11
16	2	8 2	4 1	350	12
18	2	10 2	6 2	400	13
20	3	0 2	7 2		

*The feet here expres'd are Paris-feet, and
 $\frac{1}{12}$ part thereof. The Paris-Foot is to the
as 1068 to 1000*

Rays; and consequently the more Bright or Obscure Appearance of the Object. *Tab. 35. f. 5.* Let the greater Aperture ^{T. 35. F. 5.} of the Object-Glass xyz be xz ; And the lesser Aperture mn . ab is a Remote Object Projected in the Distinct Base def . The Cone of Rays xaz is Projected in the Cone of Rays xdz ; And consequently the Cone of Rays man (as being a part of the former xaz) shall be Collected at d in the Cone mdn . But then by this latter Aperture mn , all the Rays that fall on the outward Ring of the Glass, here expressed by xam , zan , are excluded, and consequently the Point d shall not be illustrated with so much light as were the Aperture as wide as zx . And therefore (supposing an Eye-Glass behind this Object-Glass, so as to constitute a Telescope) such a vigorous Light from each Radiating Point in the Object will not be brought into the Eye.

We have the exact Natural Resemblance hereof in the Eye it self: whose Pupil is contracted and dilated, according as the Light of an Object is more or less Intense.

Another Particular, wherein this Contraction or Dilatation of a Glasses Aperture is requisite, is this: An Object may be so nigh a Glass that the Rays from each single Point, falling upon the whole Breadth of the Glass, may Diverge so much that the Glass is not able to Correct the Divergence of those Rays that fall towards its outward Borders, so as to reduce them to Determine or Unite in the Distinct Base with those Rays, that fall nigher the middle of the Glass (as before is noted after *Prop. III.*) And then 'tis requisite to contract the Aperture of the Glass, so as to exclude these Exorbitant Rays. A notable Experiment of this we may make by holding a Minute Object very nigh the Pupil of the Eye, the Object shall appear very *Confused*. But by applying a Paper with a small Pin-hole before the Pupil, it shall reduce the Appearance to much more Distinctness than before.

P R O P. LVI.

The Telescope Consisting of a Convex Object-Glass, and Three Convex Eye-Glasses is Explained.

T.35 F.1. I have shewn in *Prop. L.* &c. the Nature and Properties of the Telescope consisting of a Convex Object-Glass, and Convex Eye-Glass. I have shewn how the Image of the Object being formed in the Distinct Base of the Object-Glass xyz (Tab. 35. f. 1.) by the Rays from each single Point of the Object there uniting, and flowing forward on the Eye-Glass, are thereby all collected together and confounded in its outward Focus at o

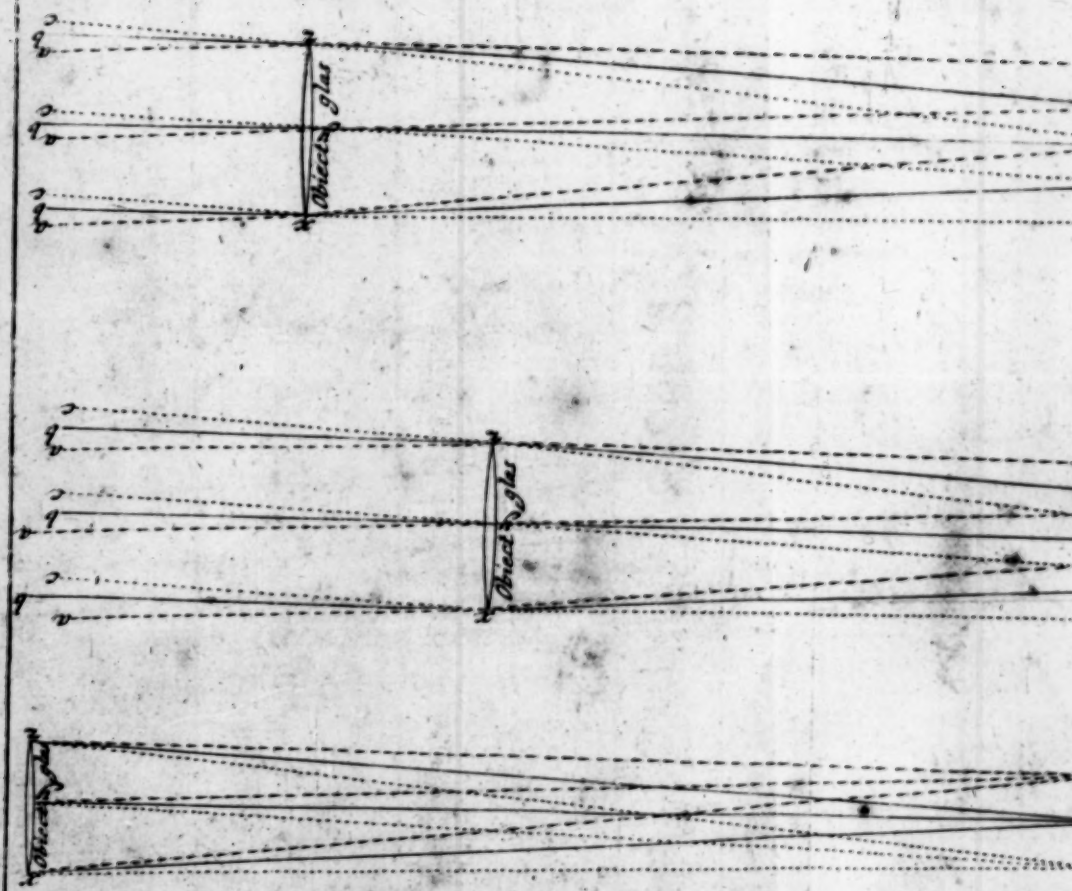
T.37. F.1. Now (in Tab. 37. Fig. 1.) Let us Combine two other Eye-Glasses k, l , with the said Telescope of *Prop. L.* And place them so, that the Distance between the first Eye-Glass b , and the second Eye-Glass k , may be the sum of their Foci. Also that the Distance between the second Eye-Glass k , and the third Eye-Glass l , may be likewise the sum of their Foci. So that all the Glasses are Distant from the next adjacent Glasses, the sum of their Foci. Only here it may be noted, that to cause Distinct Vision through this Telescope, 'tis not absolutely necessary that the second Eye-Glass k be exactly Distant from the Focus o of the first Eye-Glass b , the just length of its own Focus; For it may be more or less, but then the Visible Area, and Magnified Appearance of the Object shall be altered. As will be manifest after we have explained this Glass to those that consider it.

It is then evident that the first Eye-Glass b mixes all the Rays from different Points in the Focus at o ; from whence they flow forward, and fall upon the second Eye-Glass k , each parcel of Rays parallel amongst themselves: And by the Glass k are formed

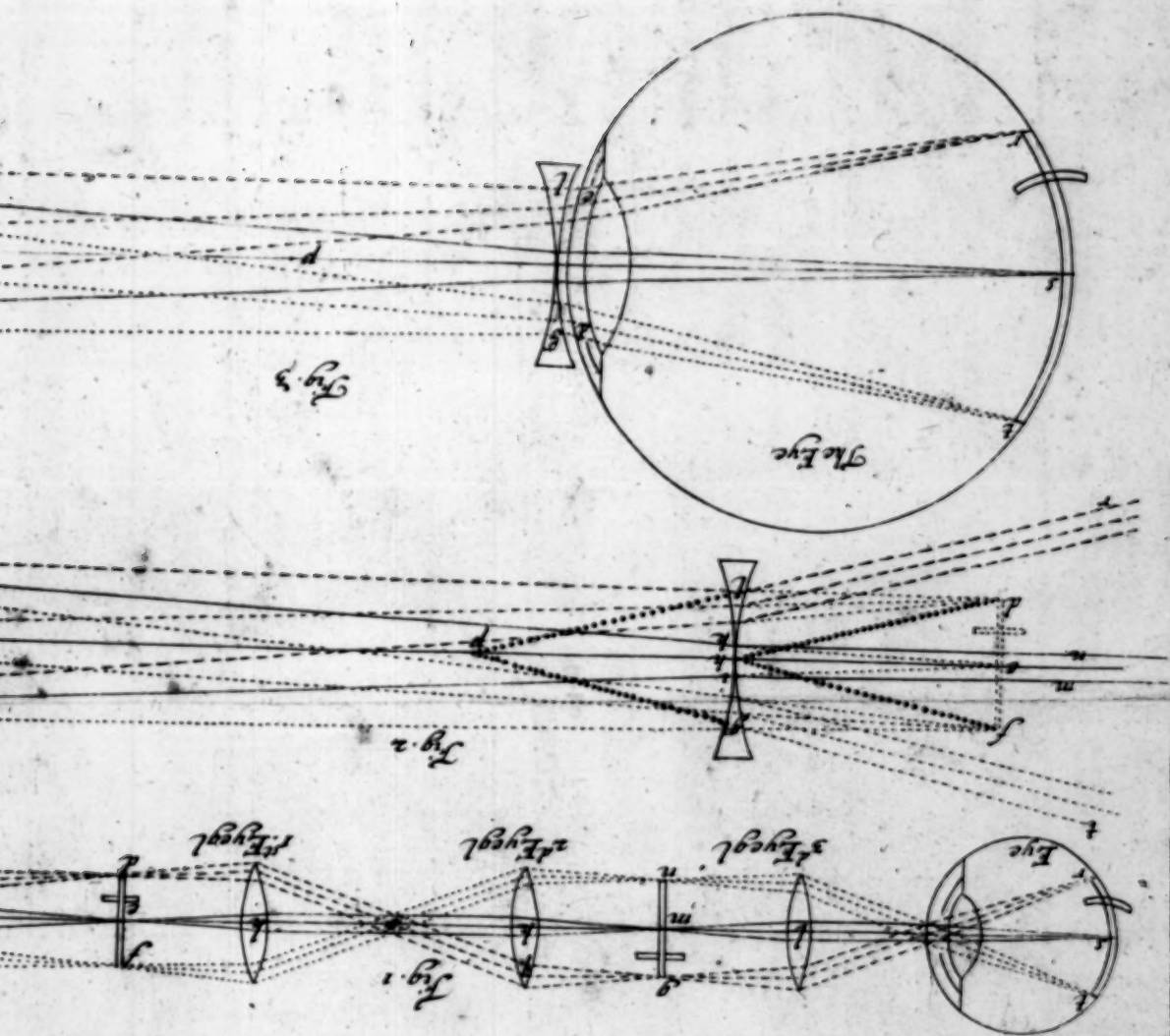
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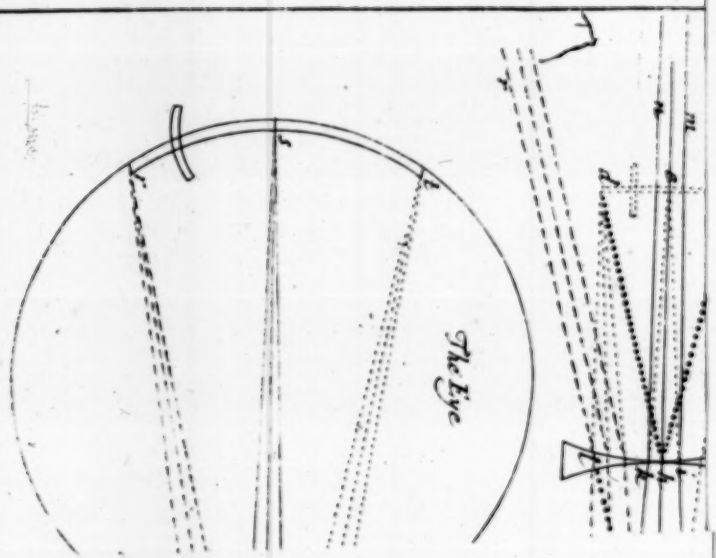
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Tab. 37. pag. 172



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formed into the second Distinct Base gmn . For we may imagine the middle Ray oq to proceed directly from o , the Focus of the Glas k ; wherefore oq shall be refracted by the Glas k , and be made to run in qg parallel to its Axis km . And then all the other Rays that are parallel to oq before they enter the Glas k , after they have passed the Glas k , do unite with qg in the Focus of the Glas k , and so the second Distinct Base gmn is formed.

Or otherwise. We may conceive the Glas k to be the Crystalline of the Eye, looking through the Telescope of Prop. L. by . And as the Crystalline in that Case does by means of the Glas b form in its Focus on the Retina the Image of the Distinct Base fed ; So may we imagine the Glas k to form in its Focus gmn , by means of the Glas b , the Image of the Distinct Base fed .

Then from the second Distinct Base gmn the Rays proceed as is expressed in the Scheme, and fall on the third Eye Glas l ; By whose means we may imagine the Distinct Base gmn projected distinctly on the Retina of the Eye rst , in the same manner as is shewn before in the Telescope of Prop. L.

And here we may observe that the Image on the Retina rst is *Inverted*, therefore the Object shall appear *Erect*. Prop. XXVIII. Sec. 4, 5.

And we may conceive this sort of Telescope as a double Telescope of Prop. L. For the Glasses b, y , make one Telescope; And the Glasses k, l , an other. And as the former by it self *Inverts* the Object; so the latter with the former *Reverts* the *Inverted* Image; and consequently makes the Object appear *Erect*. Yet it has been lately Publish'd in the *Journal des Scavans* 17. Sept. 1685. as a very difficult Problem in Dioptricks, why four Glasses in this kind of Telescope represent Objects *Erect*? I think I have solved this Problem to satisfaction, and my Answer is Publish'd Num. 187. of the *London Philosophical*

phical Transactions. As also in the *Bibliothèque Universelle & Historique de l'Année 1688. Tome 3. pag. 329.* But the Learned Author of this latter in his Translation has mistaken my Sense in one Particular. I shall therefore give it here again in the Second Part of this Work *Chap. 2.*

*Dioptrick
Problem
solved in
the Second
Part of
this Trea-
tise
Chap. 2.*

Concerning the Magnifying Power of this Telescope; our *Prop. L.* will direct us how to Calculate it. For by that Proposition 'tis manifest, that if the several Eye-Glasses *b, k, l,* be of equal *Foci*, and the Distance between *b* and *k* be the sum of both their *Foci*, that then the apparent Diametral Magnitude of an Object through the Glass, is to the Diametral Magnitude viewed by the naked Eye, as the Focus of the Object-Glass, to the Focus of any one of the Eye-Glasses. But if the *Foci* of the Eye-Glasses *b, k, l,* be different, or the Distance between *b* and *k* different from the sum of their *Foci*, then to obtain the magnified Appearance, we must have Recourse to Calculation. Wherein the Cases are so very various, that to insist on them all would be very tedious, and infinitely laborious. For the Focus of *b* may be greater, equal, or less than of *k* or of *l*; and so of *k* than of *l* or *b*; and so of *l* than of *k* or *b*; As likewise the Distance between *k* and *b* may be infinitely varied. I shall therefore pass this over; A little Consideration of the several Varieties will make any of them plain, and shew how they may be easily Calculated by those versed in the foregoing Doctrine. For 'tis but considering, how the Distinct Base *gmn* is Projected, whether equal to, greater or less than the Distinct Base *fed*; And how the Eye-Glass *l* conveys this Distinct Base *gmn* to the Eye.

In like manner, by *Prop. LIV.* may the Angle received, or Visible Area of an Object through this kind of double Telescope, be Determined, as in the single Telescope; Respect being had to the several Apertures, *Foci*, and Distances of the several Eye-Glasses *b, k, l.*

Scholium.

Scholium.

From the Explication of this kind of Telescope, and of that in *Prop. L.* may we easily apprehend the Theories of the various Combinations of Convex-Glasses in the Compositions of divers Telescopes of 3, 4, 5, 6, 7, 8, &c. Glasses.

Wherefore in explaining any kind of Telescope, we are first to obtain (by some Practical Rules to be deliver'd hereafter *Part II. Chap. 4. Sec. 3.*) the Focal length of each particular Glass by it self. Then we are to consider the Distances of each of these Glasses (as they lye in the Tube) from the Glasses before and behind it. Afterwards we are to consider, where the Distinct Base, or Distinct Bases are formed by these several Glasses, and how they are Projected as to Amplification or Diminution, which is easily found by the Doctrine before delivered. And then how the Eye-Glasses affect these Distinct Bases; As how they Confound them, Rectifie them, Invert or Magnifie them.

For the Result of all is this, that the Rays from the several Images in the several Distinct Bases, shall be Confounded on the Pupil of the Eye, in order to be rectified by the Crystalline, which (as has often been intimated) we may consider as a Convex-Glass, whose Focus is on the *Retina*. And from a due Consideration of the Premisses, it will appear why some Telescopes consisting of Convex-Glasses represent the Object *Erect*, others Inverted; in some one Glass is to be taken as one, in others two Glasses perform the Effect but of one.

And thus all the Combinations of Glasses expressed in the 8, 9, 10, 11, 12. *Iconismes of Zahn Telescop. Fund. 2. Syntag. 3. Cap. 6, 7, 8, 9.* are easily explained, with a thousand other Varieties.

PROP.

P R O P. LVII.

The Telescope Compos'd of a Concave Eye-Glass, and Convex Object-Glass of a larger Sphere is Explain'd.

The Posture of the Glasses in this Telescope is this ; The Distance of the Glasses is to be the *Difference* of their *Foci*, that is, the Concave Eye-Glass is to be placed so much nigher the Object-Glass than the Focal length of this Object-Glass, as is the virtual Focus of this Eye-Glass. And the Eye is to be placed as nigh the Eye-Glass as possible.

Note. I shall call the *virtual Focus* of the Concave, simply its *Focus*, it being well known that a Concave has no other *Focus*, but a *virtual Focus*.

I shall now shew, that through this Glass the Object appears *Distinct*, *Erect*, and *Magnify'd*. And shall shew the *Angle received* or *visible Area*, according to the several Postures of the Eye.

Only premising in this Proposition (as I have done in several former) that we suppose the Glasses of the least thickness imaginable ; and especially the *Concave-Glass* in its middle Point is supposed of no thickness at all, but the two Surfaces to touch.

To explain this Glass the better, I shall express the Figure
T 37. F 2 very large, *Tab. 37. f. 2.* Wherein we shall first consider the Telescope it self separate from the Eye. Wherefore, let some Distant Object (as suppose a *Cross*) send the Rays *a a a* from its upper Point, *b b b* from its middle Point, and *c c c* from its lower Point. These falling on the Object-Glass *x y z* are formed thereby into the Distinct Base *f e d*. Let now the Concave-Glass *g h l*, be placed between the Distinct Base *f e d*, and the Object-Glass *x y z*, so far distant from the Distinct Base, as is the Focus of this Concave ; that is, let *e b* be the virtual Fo-

cus

cus of the Concave Eye-Glass. Then the Rays (for Instance) from the middle Point, $x i$, $y b$, $z k$, falling on the Concave and Converging towards its Focus e , after passing the Glass become Parallel (*Corol. Prop. XIII.*) and run onwards in $i m$, $b e$, $k n$. The same may be conceived of the Rays from the Collateral Points; which Converge towards the Focus in f and d ; viz. that these also after passing the Concave Eye-Glass, do proceed onwards Parallel amongst themselves.

But concerning these Rays from the Collateral Points, we must note also; That, as it is shewn before in *Prop. L. Sec. 3, 4.* Concerning the Convex Eye Glass $g h l$ (*Tab. 35. f. 1.*) that T.35. F.1. it brings the Rays of the Collateral Points $f g$, $d l$, Parallel amongst themselves into its Focus at o . So the Concave Eye-Glass $g h l$ (*Tab. 37. f. 2.*) for the same Reasons expressed be- T.37. F.2. fore in *Prop. L. Sec. 3, 4. Mutatis Mutandis*, makes the several parcels of Rays from the Collateral Points, after passing it, to Diverge, as if they proceeded directly, from the Point p ; $p b$ and $b e$ being equal, and each equal to the Focal length of the Concave Eye-Glass. These things being fully considered; we proceed in the Explication of this Telescope. And therefore now let us apply the Eye thereto; And this also in a large Figure, *Tab. 37. f. 3.* T.37. F.3.

I say first, the Object appears through this Glass *Distinct*; For by what foregoes the Rays from each single Point, do fall on the Eye Parallel amongst themselves; And therefore each Point (by *Pr. XXVIII.*) is *distinctly* represented on the Fund of the Eye.

I say secondly, the Object appears *Erect*. For 'tis manifest by the Inspection only of the Scheme, that the Rays flowing from the lower Point of the Object, are Terminated at t the upper Part of the Retina. And the Rays from the upper Point of the Object are Terminated at r the lower Part of the Retina. So that the Image is painted *inverted* on the Retina. And therefore by *Prop. XXVIII.* the Object appears *Erect*.

A a

I say

I say lastly, the Object is *Magnified* by this Glass. For the Proof of this we are to remember, what foregoes in *Prop. XXXVII*: 'Tis there declared, that if an Object be Projected by a Convex-Glass xyz (*Tab. 37. f. 2.*) in the Distinct Base fed ; And the Eye be placed any where between the Glass and the Distinct Base, as suppose at b , draw fb, db , and the Object appears under the Angle $fb d$, which is much greater than $fy d$, the natural Optick-Angle. The same will hold, though we interpose the Concave-Glass gl ; for the Ray xg from the Objects lower Point, that runs Parallel to the Axis $bybe$, is Refracted into gt , as if it came directly from the Point p . And so the Ray zl from the Objects upper Point, that runs Parallel to the Axis $bybe$ is Refracted into lr , as if it came directly from the same Point p . If therefore we suppose the Rays xgf, zld , Parallel to the Axis; then gl shall be equal to fd ; And pb being by Supposition equal to be , pgt shall be Parallel to bf ; And plr shall be Parallel to bd . And consequently the Angle gpl shall be equal to the Angle $fb d$. Wherefore the Object through this Glass appears under the same Angle, as we may imagine the *Apices* of the Pencils f, d , would appear to the naked Eye at b (the Concave Eye-Glass being removed) And consequently the Object appears *Magnified*.

The *Visible Area*, or *Angle Received* by this Glass, is Determined by the *Aperture* or *Breadth* of the Eyes Pupil. For 'tis manifest, from *Tab. 37. f. 3.* That if the Pupil de of the Eye were not large enough to receive the Rays from the extreme Points of the Object, it would not perceive the *whole* Object through this Glass. Wherefore by the *Breadth* of the Pupil given, as also by the Focal Distances of the Object-Glass and Eye-Glass being given, we may easily obtain the *Visible Area*, or *Angle Received*. For let us suppose that we find from these *Data*, that the whole Object Projecting the Distinct Base fed (*Tab. 37. f. 2.*) would be Projected in the breadth of the Pupil,

Pupil, at the Distance *by* from the Object-Glass. Then find the Angle *fyd* (as is easie from these *Data*, and the preceding Doctrine) and we have the *Angle Received*.

Corollary 1.

In the XXXI. *Prop. Sec. 9.* We have considered the *Locus Apparens* of an Object, Projected in the Distinct Base by a Convex-Glass, to the Eye placed between the Glass and Distinct Base. And if the *Affirmative* of the Quere, which I there propose, hold true; The *Locus Apparens* of the middle Point of the Object seen through the Glass of *Tab. 37. f. 2.* is at *p.* T. 37. F. 2.

Corollary 2.

If we suppose a Convex Eye-Glass, whose Focal length is equal to the Focal length *ph* or *eh* of this Concave Eye-Glass, apply'd (as directed in *Prop. L.*) to this same Object-Glass *xyz* in *Tab. 37. f. 2.* It would magnifie the Object equally with this Concave: And from hence it follows, that the preceding Proposition LIII. concerning the Magnifying of a Telescope, may be apply'd to this sort of Telescope furnished with a Concave Eye-Glass. But then the Advantage of the Telescope in *Prop. L.* beyond that of this Proposition, is most signal in this particular, That it receives a very much greater Angle, or shews to the Eye a much greater *Area* of the Object. The *Area* in this being Determined by the breadth of the Convex Eye-Glass; But in that of a Concave Eye-Glass, the *Area* is proportioned to the breadth of the Pupil.

Corollary 3.

If the Eye recede from the Concave Eye-Glass of this Proposition, it perceives not so great a space of the Object. This

T. 37.
F. 2, 3.

is manifest only by Inspection of the two Schemes (*Fig. 2*, and *3. Tab. 37.*) For if the Breadth of the Pupil *de*, be but just sufficient to receive the Rays from the extreme Points of the Object, after they have passed the Eye-Glass, and are thereby so much Divaricated; if the Eye recede, the Pupil shall not be broad enough; and consequently shall see less of the Object. Just as in the Telescope of *Prop. L.*

Corollary 4.

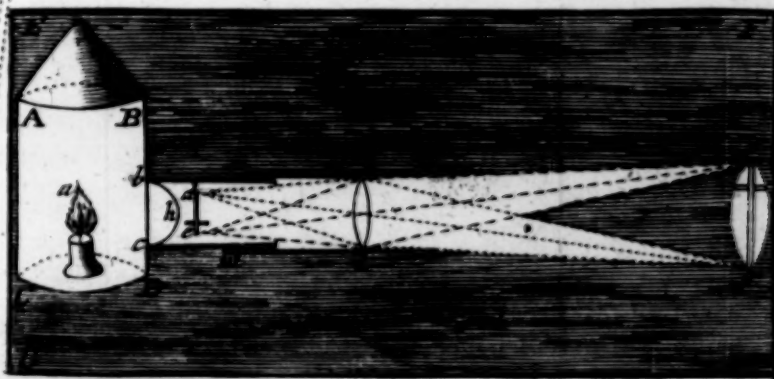
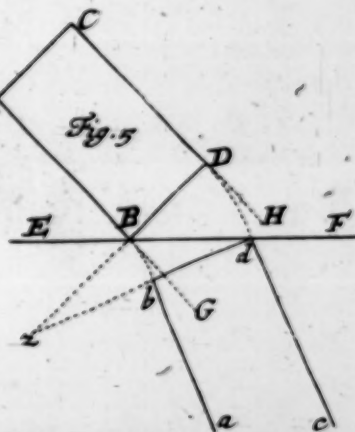
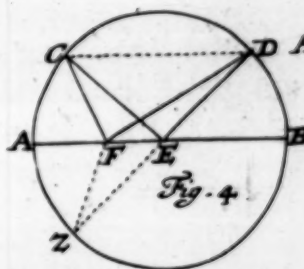
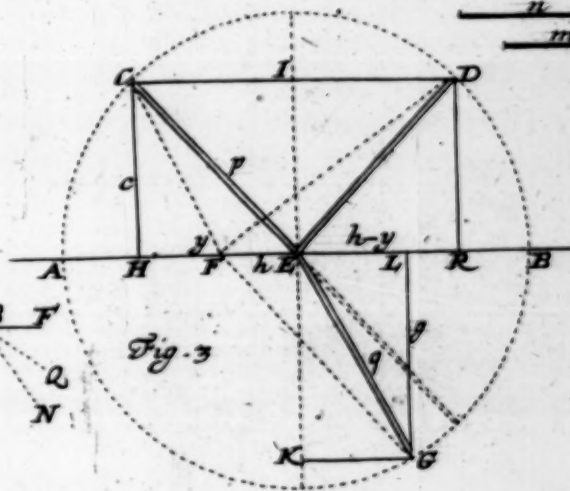
If the Eye move upwards or downwards, or to one side, or t'other of the Eye-Glass (supposing the Eye-Glass much broader than the Pupil) it perceives *consecutively* different parts of the Object. Thus suppose in *Fig. 2. Tab. 37.* The Pupil placed before the middle of the Eye-Glass at *b*, and not broad enough to receive the Rays from the upper and lower parts of the Object: If the Eye move upwards, it will meet with the Rays *gt* from the lower Point of the Object, which before escaped it; and so moving downwards it meets the Rays *lr* from the upper parts of the Object.

OF MICROSCOPES.

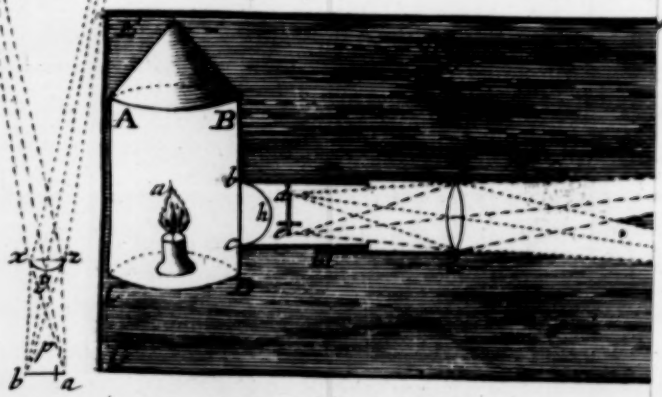
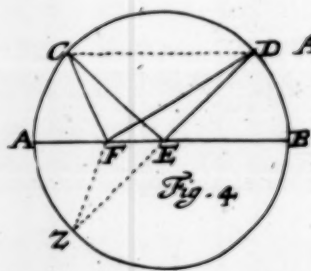
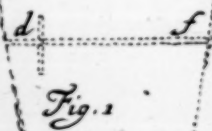
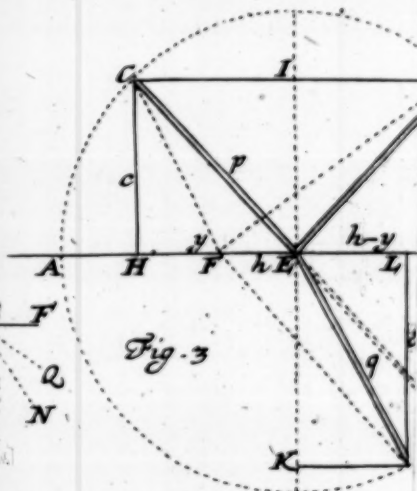
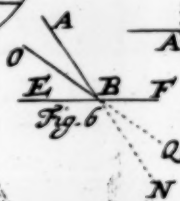
Hitherto of *single* Glasses, and of Glasses *combined* for viewing Distant Objects. We come now to treat of Glasses for viewing *minute* and *nigh* Objects, commonly called *Microscopes*. The Theory of these does so depend on what *foregoes*, that we shall have no occasion of insisting long upon them.

And first for *Microscopes* consisting of a *single* Convex-Glass. In these the Object is usually placed either in the Focus of the Glass, or a little nigher the Glass than the Focus; And the Eye is placed in or about the Focus on t'other side the Glass.

In



Tab. 38. pag. 182



In which Cases the Appearances were already solved. *Prop.* XXXI, XXXII, XXXIII, XXXIV, XXXV.

As to *double* Microscopes, or Microscopes consisting of more than one Convex-Glass, wherein the Object is Projected in a Distinct Base, before it be conveyed to the Eye; I explain them as follows, observing the Series of the Propositions.

P R O P. LVIII.

The double Microscope composed of a Convex Object Glass, and Convex Eye-Glass is Explained.

Tab. 38. Fig. 1. Let ab be a minute nigh Object exposed T. 39. F. 1. before the Object-Glass xyz , the Segment of a very small Sphere. Let the Focus of this Object-Glass be at p . Then the Object being something more distant from the Glass, than its Focal length yp , shall be Projected in the Distinct Base fd , somewhere on t'other side the Glass, according to the Doctrine before delivered *Prop. V.* And of what bigness the Image shall be Projected in the Distinct Base is determined by *Prop. XXVI.* Let the Eye-Glass lg be placed so far distant from the Distinct Base fd , as is the Focal length of this Eye-Glass; And the Eye ort placed where this Eye-Glass confounds all the Rays go, lo , which shall be about the Focus of the Eye-Glass.

All things being thus combined, The Effects of this Microscope are explained in all things, as in the Telescope of *Prop. L.* As to the Magnified, Inverted, and Distinct Appearance of the Object. And therefore 'tis needless to inlarge farther thereon: To those versed in what is already delivered, the very Inspection of the Scheme is sufficient.

Only as to the extraordinary Magnifying of these Microscopes, we may farther Remark; that whereas for viewing a Minute Object, a well-constituted Eye does usually Approach thereto

thereto about the distance of eight Inches: could we approach the Eye thereto, and view it distinctly at the distance of half an Inch, the Optick-Angle would be wonderfully magnify'd by *Prop. LI, LII.* that is to say, the apparent Magnitude of the Object would be increased at the rate of sixteen to one. Let us then suppose the Eye at *y*, viewing the Object *ab*, which would then appear under the Angle *ayb* equal to *dyf*; which is the same as were the Object increased to the bigness *df*, and viewed by the Eye at the same distance from it as *y* in the figure is now removed from *df*. Wherefore if by help of the Eye-Glass *gl*, the Eye can yet approach to the Distinct Base *df* (which we may now repute the real Object) suppose ten times nigher than *y* is to *df*; the apparent Magnitude of *df* shall again be increased ten times more than before, by the nigh approach of the Eye (suppose at *y*): that is, the Object by all these helps shall be magnified 160 times in length, or Diame-tral Magnitude.

As to the Calculation of all these Angles, and apparent Magnitudes, it cannot be difficult to those vers'd in what is before deliver'd.

There are various Combinations of Glasses in this kind of double Microscopes; for in some there are two Object-Glasses, that is to say, one Object-Glass of a very deep Convexity, and an other of a lesser Convexity, placed nigher the former than the Projection of the Distinct Base (according to *Schol. 1. Prop. XVI.*) which sometimes is called a Middle-Glass. In others there are two Eye-Glasses, &c. But the Theory of all these depends on, and is so manifest from what has been delivered, that 'tis needless to enlarge.

I shall conclude the First Part of this Treatise with a Piece of *Dioptricks*, which though *Ludicrous* affords an Appearance surprising and pleasant enough. The Explication thereof is much labour'd at by several, though it be very Obvious from what fore-goes. 'Tis this,

P R O P.

PROP. LIX.

*The Explication of the Magick Lantern, sometimes called
Lanterna Megalographica.*

The Contrivance is briefly this, *Tab. 38. f. 2.* $ABCD$ is T. 38. F. 2. a Tin Lantern, from whose side there proceeds a square or round Arm or Tube $b n k c m l$, consisting of two Parts, the outermost whereof $n k m l$ slides over the other, so as that the whole Tube may be lengthened or shortened thereby. In the end of the Arm $n k m l$ is fixt a Convex-Glass kl : about $d e$ there is a Contrivance for admitting and placing an Object $d e$ painted in dilute and transparent Colours on a plain thin Glass; which Object is there to be placed *Inverted*. This is usually some Ludicrous or frightful Representation, the more to divert the Spectators: $b h c$ is a deep Convex-Glass, so placed in the other end of the Prominent Tube, that it may strongly cast the light of the Flame a on the Picture $d e$ painted on the plain thin Glass. And here 'tis to be noted, that the Glass $b h c$ is only designed for the strong Illumination of the Picture $d e$, and has nothing to do in the Representation, and therefore in some of these Lanterns, instead of the Glass $b h c$, we shall find a *Concave-Speculum* so placed, that it may strongly cast the light of the Flame a on the Picture at $d e$.

Wherefore, Let us now consider the Picture $d e$ as a very lightsome Object of distinct Colours and Parts. And let us conceive $d e$ more remote from the Glass kl than its Focus. 'Tis then manifest, that the Distinct Image of the Object $d e$ shall be projected by the Glass kl on the opposite white Wall FH at fg ; And here it shall be represented *Erect*. For now the whole Chamber $EFGH$ is dark, the Lantern $ABCD$ inclosing all the Light; So that in Effect this Appearance of the *Magick Lan-*
tern

tern is no more than what is already declared concerning the Representation of outward Objects in a dark Room by a Convex Glas, after *Prop. IV.* (*vid. Tab. 14. f. 1.*) And here we may observe, that if the Tube be *Contracted*, and thereby the Glas *k l* brought *nigher* the Object *de*; the Representation *fg* shall be Projected so much the *larger*; and so much the *more Distant* from the Glas *k l*; according to the Rules before laid down. So that the smallest Picture at *de* may be Projected at *fg* in any greater Proportion required, within due limits. From whence the Name of *Lanterna Megalographica*. And consequently, protracting the Tube and drawing the Glas *k l* more distant from the Object *de*, will diminish the Representation *fg*, and Project it *nigher* the Glas *k l*.

As to the Mechanick Contrivance of this Lantern, the most convenient Proportion of the Glasses, &c. This is so ordinary amongst the common Glas-Grinders, that 'tis needless to insist farther thereon in this place. 'Tis sufficient to me that I have explained the Theory thereof.

Scholium.

On this depends the Theory of the *Optick Experiment* propos'd by Mr. *Hook* Num. 38. Pag. 741. *Philosoph. Transact.* For representing strange Visions and Appearances.

The End of the First Part.

DIOPTRICKS.

THE

SECOND PART

Containing

Various Dioptrick Miscellanies.

DIOPTRICKS.

THE

SECOND PART.

Containing

Various Dioptrick Miscellanies.

THE
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To My esteemed Friend

Henry Osborn

OF

Dardys-Town in the County of Meath

ESQUIRE.

THE Respect which I have ever had for you since our first Acquaintance, and which on all Occasions I have expressed in private, I have now an Opportunity of declaring to the Publick: And that too so very apposite, that it would be unpardonable in me to omit it at this time; by presenting the following Sheets to you, Dedicating them to your Name, and Devoting them and their Authour to your Use and Service.

You may well remember the frequent Discourses we have had on several Subjects treated of in the following Chapters, and on account whereof I first set on this Dioptrical Work: And particularly, I think, our Disquisitions

Dedication.

quisitions concerning the *Fustness* of *Telescopick Sights* adapted to *Astronomical Instruments*, and our *Considerations* of the *Micrometer*, were the first *Occasion* of my *Thoughts* turning this *Way*; And therefore the ensuing *Discourses* belong to you of *Right*. But if to this I add, the *Advantage* I have received by your *Acquaintance*, and the repeated *Satisfaction* I have had in your agreeable *Conversation*; I am bound by *indispensible Tyes* to make this *Acknowledgment*.

I cannot but admire your prudent *Choice* of a private, retired *Life*; notwithstanding your great *Advantages* both of *Nature* and *Fortune*, that render you capable of the most publick and weighty *Imploy*. By this *Course*, you have an *Opportunity* of enjoying your self, and improving your *Philosophical Thoughts* beyond the common pitch: You can look on unconcern'd, and securely observe the froathy *Sea* of *Business*, wherein *Men* fluctuate; and some are shipwreckt, sink, and perish.

And because you seem careless of propagating your *Name* the common ways; suffer me to erect this slight *Monument* to it: Though I am certain at the same time, that, if you pleased, you may raise a lasting *Mausolæum* to your *Memory*; but you seem above these *Desires*; yet you'll permit

Dedication.

mit your Friend to Honour it as far as he can ;
and if the Materials or Workmanship do not seem
to promise a long Duration to Posterity, this only
reflects on my Abilities , (which I shall never
vindicate) but cannot lessen the sincere Intention
of

Your most affectionate

Ap^{rl} 17.
1690.

Humble Servant,

WILL. MOLYNEUX.

DIOPTRICKS.

DIOPTRICKS.

PART II.

CHAP. I.

Of Refraction and Light.

- (1.) *This Discourse promised.* (2.) *Leibnitz's Universal Principle in Opticks, &c.* (3.) *On Occasion whereof; Of Final Causes.* (4.) *Farther Explication of Refraction.* (5.) *Light a Body, from several Arguments.* (6.) *From its being resisted in its passage through Diaphanous Mediums.* (7.) *Its Requiring time to move from place to place.* (8.) *Impossible to be increased, but by robbing some other Place of its Light.* (9.) *Therefore impossible to be augmented uniformly.*

(1.) **I**N the second Experiment of Part I. I had occasion ^{(1.) *This Discourse promised.*} to mention the *Natural Cause* of *Refraction*; or why the Rays of *Light* passing through different Mediums are refracted at their *Immersion* or *Emersion*. I then avoided any farther enquiry into the reason thereof, as being more of a *Physical* than *Mathematical* Consideration. But at the same time, I promised a farther Disquisition thereon, to be borrowed from a most Learned Author; whose reason we shall find briefly comprehended in this, That the Different Resistance that a Ray of *Light* finds in passing (for instance)

stance) through Air and Glass, is the cause, why 'tis bent from its direct Course. But how this Refraction comes to be from the Perpendicular, in proceeding from a Dense to a more Rare Medium; or towards the Perpendicular, when from a Rare to a more Dense Medium; we shall more fully apprehend by the Discourse it self, which I here subjoyn from the *Acta Erud. Lipsiæ, Ann. 1682. pag. 185.*

(2.) Leib-
nitz uni-
versal
Principle
in Op-
ticks.

(2.) One Universal Principle of Opticks, Catoptricks, and Dioptricks. By the Learned and Ingenious G. G. Leibnutzius.

The chief Hypothesis common to all these Sciences, and by which the Progress of all Rays of Light is geometrically determined, may be thus laid down. Light proceeds from the Radiating Point, to the Point to be enlightened, that way, which is of all the most easie; and this is first to be Determined in respect to plain Surfaces, and then is accommodated to Concave or Convex Surfaces, by considering the Planes, that are Tangents to these Surfaces. But here I take no notice of some Irregularities, which perhaps may conduce to the Generation of Colours, and to some other extraordinary Phenomena, which in practical Opticks are not at all considered.

Hence, In plain or simple Opticks, Tab. 38. Fig. 3. The direct Ray proceeds from the Radiating Point C, to the Point to be illustrated E, by the shortest direct way, the same Medium continuing all along, that is, in the Right Line CE.

In Catoptricks the Angle of Incidence CEA, and of Reflection DEB are equal. Let C be a Radiating Point; D the Point to be illustrated, and AB a Plain Speculum: 'Tis required to find in the Speculum the Point E, that reflects the Ray to D. I say, that it shall be such a Point, that the whole Progress, Way or Journey of the Ray CE + ED, may be the least or shortest, that is possible; or less than CF + FD, supposing that we take any other Point F in the Speculum. And this shall be obtained, if E be taken such, that the Angles CEA, DEB,

DEB, may be equal; as is manifest from Geometry. *Tab. T. 38. F. 4.*
 38. Fig. 4. Produce DE to Z, and joyn FZ.

Then $AZ = (DB =) AC$. And $FZ = FC$.

Therefore $CE + ED (=DZ)$ is less than CF or $FZ + FD$.

Protony and other Ancients insist on this Demonstration; and 'tis extant both elsewhere, and also in *Heliodorus Larissans*.

T. 38. F. 3.

In Dioptricks, * The Sines EH, EL, of the Angles of Incidence CEI, and Refracted Angle GEK are to each other reciprocally as the Resistances of the Mediums. Let IE be Air, and EK Water, Glais, or any other Diaphanous Medium more Dense than Air, C a Radiating Point in the Air, G the Point to be illustrated under the Glas: 'Tis inquired, by what Way or Path shall C radiate to G; or 'tis required to determine, in the Surface of the Glas AB, the Point E, which refracting the Ray that comes from C, sends it to G. Here this Point E must be taken such, that the Way, which the Ray takes, may be of all ways the *easiest*. But now in different Mediums, the Difficulties of the Way or Progress are in a Ratio compounded of the *Length* of the Way, and of the *Resistance* of the Mediums. Let the Right Line *m* represent the Resistance that Light finds in its passage through Air, and *n* the Resistance of its Passage through Glas: The *Difficulty* of the Way from C to E shall be as the Rectangle under CE and *m*; and from E to G as the Rectangle under EG and *n*. Therefore that the *Difficulty* of the whole Way CEG may be the *least* possible, the Sum of the Rectangles $CE * m + EG * n$ ought to be the least of all possible; or less than $CF * m + FG * n$, supposing any other Point F taken besides E. The Point E is now required. Wherefore, seeing the Points C and G, and also the Right Line AB are given by Position, therefore the Lines CH, GL, Perpendiculars to the Plain AB, and the Line HL, are given also. Let us call CH, *c*, and GL, *g*, and HL, *b*; but the sought

* Here I translate according to the 2.4. Definit. of Part I and not according to the Author.

Cc

Line

Line E H, let us call y ; then E L shall be $b - y$, and C E shall be $\sqrt{cc + yy}$, which we shall call p ; and E G shall be $\sqrt{gg + yy - 2by + bh}$, which we shall call q .

Wherefore $m \cdot \sqrt{cc + yy} : +n \cdot \sqrt{gg + yy - 2by + bh}$ (or $mp + +nq$) ought to be the least of all those Quantities that can possibly be so expressed; and 'tis required to determine y , that so it may be. By my Method *De Maximis & Minimis* (*Vid. Act. Lips. Ann. 1684. pag. 467. 472.*) which, beyond all that are hitherto known, does wonderfully shorten the Calculation, it is manifest at the first sight, almost without any Calculation, That $m q y$ shall be equal to $n p \cdot b - y$, or that $n p \cdot m q :: y \cdot b - y$, that is, the Rectangle of C E $\cdot n$ shall be the to Rectangle of E G $\cdot m ::$ As E H to E L. Therefore C E and E G being put equal, n the Resistance of Glass to Light shall be to m the Resistance of Air to Light :: As E H the Sine of the Angle of Incidence in Air C E I: To E L the Sine of the Refracted Angle in Glass G E K. Or these Sines shall be to each other reciprocally as the Resistances of the Mediums. Which was the Assertion to be proved.

Wherefore, if in one Example or Experiment E L be found $\frac{2}{3}$ of E H, the same Proportion shall hold in all other Experiments, wherever C and G be taken, that in Air, this in Glass. If C be in Air, and G in Water, Experiment shews, that E L shall be about $\frac{3}{4}$ of E H.

Des-Cart.
Notion of
Refraction
rectified.

Thus far this ingenious Author. The rest of his Discourse is chiefly employed in rectifying *Des-Cartes* Notion of Refraction, which tho founded on the same Principle here expressed, yet has this peculiar, that he makes Water, or Glass, or any other more Dense Medium, resist the Progress of Light less than Air. But in this particular he is abundantly rectified by *Leibnuzius*, who shews the Incongruity of that Supposition.

(3.) One thing more there is remarkable in the Learned *Leibnuzius* Discourse, which I cannot here pass over, and that is, A pious Reflection which he makes on this occasion, concerning *Final Causes*. For 'tis manifest, that the Ray proceeding from C, does not consult with it self, how it may with the greatest ease arrive at the Point E, or D, or G; neither is it carried by it self to those Points. But the *Great Creator* of all things, has so made *Light*, that this most beautiful, orderly, and admirable Event should result from its very Nature. Wherefore they are in a great Error, who reject *Final Causes* in Natural Philosophy, which, besides affording us occasion of admiring and adoring the *Divine Wisdom*, do often discover to us a curious Principle of finding out the Properties of those things whose inward Nature is not so clearly known by us, as that we can explain the immediate *efficient Causes* and *Instruments*, which the *Almighty Mover* employs in producing those *Effects*, and obtaining those *Ends*.

Remar
on Final
Causes.

Indeed I should think it an Attempt worth the Thought of some profound Philosopher, to give an Account of those admirable, orderly, and beautiful Appearances in Nature, whereof we can most plainly apprehend the *Designs* and *Final Causes*, but can hardly proceed to any farther Knowledge of them. (Thus for instance, suppose it were asked, *What is the cause of Refraction?* Were it not much satisfactory to answer, *That thereby the Ray may proceed the easiest way possible*.) This surely might be able to convince the most obstinate Opposers of Divinity: For certainly, if we can rely upon any *Deduction* or *Consequence* drawn out by the *Mind* of *Man*, we may assuredly rest satisfied in this; that so many *Phenomena*, stupendous and surprising for their designed *Contrivance*, could not proceed but from an *Omnipotent* and *Designing Being*. But if after all, they will arrive to such an height of Extravagance, as to say, We cannot rely on these Conclusions, as being all in the dark, and

Final Causes
highly
deserve
our notice.

knowing nothing; let them look to the hazard of their own Principles, who endanger their *eternal Happiness* on confidence of their own Arguments. But to resume our Subject.

Fermat
demon-
strates the
same.

The Famous Mons. Fermat has written a long Demonstration of this same Principle in *Dioptricks*. 'Tis publish'd amongst the French Letters, at the end of his *Opera Mathematica*. Tolose 1679. Pag. 158. To which I refer the Reader.

Farther
Explica-
tion of Re-
fraction
from
D. Barrow.
T. 38. F. 5.

(4.) But after all, perhaps it may not be amiss, to illustrate this Business of Refraction, by some more familiar and sensible Instance. Wherefore Tab. 38. Fig. 5. Let the Parallelogram $ABCD$ represent a Ray of Light of this breadth, and let it fall on the plain Surface of the Glass EF ; this in some measure does stop its Course; and the Point B entering the Glass, shall endeavour (but slower) to proceed onwards directly to G in the Line AB produced. But all this while the Point D , continuing yet in the Air, shall continue its former motion in the Right Line CDH ; but now 'tis impossible for both the Points to obtain what both endeavour; for each cannot perform his *direct Motion*, one *slower*, and t'other *quicker*. And therefore that they may both come nighest to what each endeavours, they shall both be turned about some certain Point Z in the Right Line DB produced. So that whilst the Point D in the *thinner Medium* proceeding *quicker* describes the greater Arch Dd ; the Point B , proceeding *more slowly* in the more *Dense Medium*, describes the lesser Arch Bb ; and when they have thus run through both these Arches, the Right Line BD shall obtain the Posture bd . And now that the Point D also is emerged into the more *Dense Medium* at d ; and it also is now as much retarded as b ; these circular Motions shall now cease; for D is not now carried quicker than B , and therefore describes not, as before, a greater Arch. Wherefore forsaking, as soon as they can, their former Progress, they shall both proceed in dc , ba , the Tangents to their

their Arches. And the whole Ray $ABCD$ thus bent, and brought into the Posture $abcd$ proceeds onwards directly in that Course. And here 'tis to be noted, That whatever Inclination AB has on EF , the Arches Dd , Bb , or their Semidiameters ZD , ZB , have always the same Proportion; to wit, such a Proportion, as the peculiar Difference in the Resistance or Density of one and t'other Medium does require (which is to be determined by Experiment): For *Tab. 38. F. 5.*

Fig. 6. Let us suppose the Point B thrust forwards towards Q or N , and that the Medium below EF is perfectly *Homogeneous*, that is, equally resisting in all Parts thereof; there is then no reason, why this Point should not be carried with an equal Celerity towards whatever Part, that is, it shall tend equally quick towards Q in the Right Line OBQ (supposing its direction lye that way) as towards N in the Right Line ABN . And therefore the Rays of Light AB , OB , however differently inclined, shall find an equal Resistance; and the Point B , whether it tend towards Q or N , shall be equally retarded. And also seeing the Point D (*Tab. 38. Fig. 5.*) continues in the *first Medium*, it shall be thrust forwards with the same Celerity, whatever is its Inclination. Whence 'tis manifest, that these Motions, or Paths passed over in the same time, to wit, the circular Arches Dd , Bb , shall always observe the same Proportion, that is, the Proportion of their Semidiameters ZD , ZB , or Zd , Zb ; which Proportion therefore principally and chiefly measures and determines the Refractions of Rays in the same two Mediums. And this is the same Proportion, as is between the Sines of the Angles opposite to Zd , ZB , in the Triangle ZdB , that is, of the Angles ZBd , (or ZBE) and ZdB . But ZBE is the Complement of the Angle ABE , and therefore (by *Def. 2. Part. I.*) ZBE is the Angle of Incidence or Inclination of AB to EF ; and the Angle ZdB is the Complement of the Angle Fdc ; and therefore ZdB is the Refracted

Refracted Angle (by *Def. 3. Part. I.*). From hence is manifest what we assert in the 7 *Experiment, Part I.* And here we shall note; that according to *Exper. 6. Part I.* *ZD* shall be to *ZB* :: As 300 To 193; or as 14 to 9.

*Light a
Body.*

(5.) And thus much concerning *Refraction*. The Consideration whereof does naturally suggest unto us, That *Light is a Body*. For however the Ancient *Aristotelians* defined it, *Actus perspicui quatenus perspicuum*, which is perfectly unintelligible; yet so much we may perceive hereby, that they designed to exclude it from all *Corporeal Notion*. But the various Properties of *Light*, that do necessarily belong to a *Body*, are so many and evident, that they leave no room for any farther doubt in this matter. I shall mention but a few.

*From its
being resi-
sted in its
passage
through
Diapha-
nous Me-
diums.*

(6.) And first, by this Affection of being *refracted*, 'tis manifest that *Light*, in its passage through this and other *Diaphanous Body* does find a *different Resistance*. Now 'tis unconceivable, how any thing, but *Body*, should suffer *Resistance*; but we may conceive the *Resistance*, that *Light* suffers in its passage through different *Diaphanous Bodies*, to proceed from the Medium *Hindering* of the *Diffusion* or *Distribution* of *Light* through more of the *Parts* of this Medium, and consequently it may be said to be *less illuminable*. For the Nature of *Light* endeavours to *diffuse* it self. And on the contrary, by how much *Light* does more equably or uniformly affect the *Parts* of the Medium, which it enlightens; or by how much it communicates its *Energy* to more of the *Particles* of the enlightened Space; that Medium may be said to be by so much the more *illuminable*, or *less to resist* the Progress of *Light*. Whence it is, that by how much the affected *Parts* of the Medium are more *solid* and *small*, and admit between them the *less Space* for any other *Heterogeneous Matter*, that suffers not by *Light*; by so much the Medium is said to be more *enlightened*. But leaving these *Philosophical Refinements*; 'tis manifest

nifest that Resistance must proceed from *Contact* of two Bodies. And *Contact*, either *Active* or *Passive*, belongs only to *Body*; according to that of the Philosophick Poet, *Tangere enim & tangi nisi Corpus nulla potest Res.* And our Saviour himself, the Fountain of all Wisdom and Philosophy, Divine and Natural, seems to confirm this Notion; when to prove himself a *True Body* after his Resurrection, He commands his Diffident Disciple *To Touch Him.*

(7.) The Second Property, that confirms *Light* to be a *Body*, and a *Body* moved or thrust forward, is, That it requires *time* to pass from one place to another, and does it not in an *instant*, but is only of all Motions the *quickest*. For the Experiment proving this, we are obliged to the Ingenious *Monf. Romer*, who has demonstrated beyond all Contradiction, from the Observations of the *Immersion* and *Emersions* of the *Satellites* of *Jupiter*; That *Light* requires the *Time* of one *Second* to move the space of 3000 *Leagues*, or 9000 *Miles*, which is near the *Earths* Diameter. He that requires a farther Account hereof, may consult the *Journal des Sçavans* 1676. Decemb. 7. *Philosoph. Transact.* Num. 136. Or *Mr. Newton's* Incomparable Piece, *Philosophiæ Natur. Princ. Mathem. Lib. I. Schol. Prop. 96.* Where 'tis asserted, That *Light* requires about ten Minutes time to come from the *Sun* to the *Earth*. And 'tis most evident, without this Allowance for the Time spent in *Lights* Motion, the Appearances of the *Satellites* Eclipses and Emersions are not to be explicated by any *Excentricity* or other Hypothesis. But by this Allowance, they answer to the greatest exactness. And this is a Part of *Astronomy* the most correct and accurately determined, as well as the most useful, of all others: For hereby *Geography* may be rectified, the *Longitude* determined, and *Navigation* made more easie and secure. For a Confirmation of all which, I appeal to the Labours of the Ingenious *Mr. Flamsteed* and *Mr. Halley*, to whom

whom the Learned World is for ever obliged by their Advancements of Astronomy.

Light not
to be in-
creased,
but by rob-
bing some
other place
of its
Light.

(8.) A third proof that Light is a Body, is, That it cannot by any Art or Contrivance whatsoever be *increased* or *diminished*; that is to say, we cannot magnifie (for instance) the Light of the Sun or a Candle, no more than we can magnifie a Cubick Inch of Gold, or make it *more* than a Cubick Inch. But in this particular I desire to be rightly understood, lest I seem herein to advance a *Paradox*. I say therefore, Whenever we see Light increased, 'tis by Robbing of some other part of the Medium of its Light; or, by bringing the Light, that naturally should have been diffused through some other part, to the more enlightened place. Thus for Example, In a Burning-Glass, by which the Light of the Sun is highly encreased in its *Focus*, or Burning-Spot: We are first to consider, that in this Focus the Image of the Sun is projected, as being the Distinct Base of the Glass. And secondly we may observe all round about this bright Spot of the Suns Image, there is cast the strong Shadow of the whole breadth of the Burning-Glass. For all the Rays from the Sun, that would have fallen on this broad shaded space, are now brought together and crowded close in this bright Spot, there raising a vigorous *Light* and violent *Heat*. This is abundantly confirmed by an easie Experiment: For cover all the Burning-Glass, except one small round space in its middle, just the bigness of the bright burning Spot in its Focus; and tho there be a shaded space round the bright Speck, as before, yet we shall not be sensible of any *Increase* either of Light or Heat; which plainly shews, that this *Increase* of Light (when the Glass is all bare) proceeds from the crowding together of those Rays that would have fallen on the rest of the Glass, and which (were not the Glass interposed) would have fallen on the shaded space round about the bright Speck.

There

There seems but one Objection against what is here laid down; and that is, that Light is *increased* by Reflection, without depriving any place of the Light it would otherwise receive; or, without bringing to the enlightened part any Light that would otherwise escape it, or never come at it. But if we consider the matter more attentively, we shall find it otherwise. For let us suppose an *Hole* of a Foot square in the side of a Chamber, and that a Candle were placed close to, and just before the middle of this Hole; there is but half this Candle that now enlightens this Room, the other half of the Rays proceeding directly out at the Hole: Let now a Looking-Glass be placed, so as just to fill up this Hole; the Rays which before would have gone out at the Hole, are now reflected into the Room; so that the Hemisphere without the Chamber, which was enlightened whilst the Hole continu'd open, is now robb'd of its Light; and all this Light is now reflected into the Room; whereby the *averse* side of the Flame is made to enlighten, as well as the side *directly* exposed to the Chamber. What is said of this Case, may be accommodated to all: For so a Looking-Glass lying Horizontal, and reflecting the Sun-Beams to the Ceiling of the Room, does plainly hinder the direct progress of the Rays to some other part, and consequently robs that part of its Light. This is evident, by supposing an Hole behind the Glass; as in the former Case.

(9.) From all which 'tis manifest, how vainly they attempt, who offer at *increasing Light Uniformly*, that is, *equally* throughout the whole Sphere of a Luminous Body, or Radiating Point. Such are the Pretences of those that would persuade the World of Contrivances for making the small Flame of a Lamp enlighten *strongly* a whole Chappel, Hall, or Court, by being hung up in the midst thereof. For these things are *impossible* to be effected in Nature, and they had as well pretend to create Light; for there is no other way of *increasing* it; un-

less by robbing another part of its Light; and then 'tis not *uniformly increased*. We have a very sensible Instance of this in the *New-invented Lanthorns*, now much used in *London*; which by the Convex-Glasses in their sides, do strongly throw those Rays along the Walks of the Passengers, which would otherwise (were the Glasses away, and the round Holes left open) be spent on parts of the Streets not frequented; whereby the untrodden parts of the Streets are robb'd of their Light, more strongly to supply and enlighten the Paths where Light is requisite.

I have insisted the longer on this particular, because there is nothing more commonly pretended, than this Invention of *increasing Light uniformly*, by those that do not consider how vain the Attempt is. I must confess, could the thing be effected, it would be a piece of *Oeconomick Philosophy*, the most pleasant and useful imaginable, and equivalent to the *perpetual Lamp* (if ever there was any such thing, as I very much doubt). The Student in his Closet, the Merchant in his Shop, the Housewife in her Offices, would find a great and pleasant Advantage therein. But above all, the *dark Northern Climates*, who are many Days, Weeks, and Months deprived of the Sun, would be infinitely obliged to the Inventor, who could make the Flame of a Lamp, no bigger than a Barly-Corn, supply the Presence of that *glorious Body*.

CHAP. II.

Dioptrick Problem.

Why Four Convex-Glasses in a Telescope shew Objects Erect.

I Was unwilling to burden the First Part with Digressions; and therefore in *Prop. LVI.* thereof, I promised this Discourie in this place.

In the *Journal der Scavans.* For Monday 17. Septemb. 1685. Pag. 466. *Amst. Edition* We find this Passage.

‘As Perspectives of *One Convex-Glass* make Objects appear upright, which those of *Two Convex-Glasses* invert; and again those of *Three* rectifie: So it should seem, that those of *Four* ought to invert. And yet Experience shews us that Objects appear upright through these Glasses. The singularity of this Phenomenon obliges all skill’d in *Dioptricks*, to enquire the reason thereof; but hitherto they have found none. *Monf. Regis*, who applies himself particularly to this Part of *Natural Philosophy*, believes that he has hit upon the Reason, and makes us hope that he will suddenly publish it.

Thus far the *Journal*: But does not tell us whose Remark this is: I am apt to believe, ’twas written by *Monf. Regis* himself, to the Publisher of the *Journal*.

To me this Phenomenon appears very easily explicable from the consideration of placing the Glasses in a Telescope. And I wonder that any one, who pretends to Skill in *Dioptricks*, should make a Difficulty of it. The posture of the Glasses in the Tube is thus; After the Object-Glass, the first Eye-Glass is placed so much distant (towards the Eye) from the Focus of the Object-Glass, as is this Focus of the Eye-Glass;

then the second or middle Eye-Glass is placed so much distant from the Focus of the first Eye-Glass, as is the Focus of this middle Eye-Glass. Lastly, the nearest or third Eye-Glass is placed so much distant from the Focus of the middle Eye-Glass, as is the Focus of this nearest Eye-Glass; and the Eye, looking through them all, is placed in the Focus of this nearest Eye-Glass.

I say therefore first, that *one single Convex-Glass*, cannot properly be said *by it self*, to shew Objects *erect* or *reverse*; but only *in respect of placing the Eye* that looks through it; and *in respect of the Objects Distance from it*. For if the Eye that looks through such a single Convex Glass, be placed higher thereto than the Glasses Focus or *Distinct Base*, distant Objects appear *erect*: If the Eye be placed just in the Focus or *Distinct Base*, distant Objects are neither *erect* or *reversed*, but all in *Confusion between both*. And if the Eye be placed *farther* from the Glass than the Focus or *Distinct Base*, distant Objects are *reversed*.

This being laid down, I assert, secondly, That the Object-Glass of a Telescope, consisting of a Convex Object-Glass and Convex Eye-Glass, *reverses* the Object, both to the Eye-Glass, and to the Eye that looks through it. For the Eye-Glass is placed *farther* from the Object-Glass, than is the Focus of the Object-Glass. And the Eye-Glass does nothing towards the *rectifying* or *reversing*. Thus we see the *reversing* of Objects in a Telescope of two Convex-Glasses, proceeds wholly from the *Object-Glass* and its *Position*; and the Eye-Glass has nothing to do in the Affair: For were the Eye it self in the place of the *Eye-Glass*, it would see the Object *inverted* through the single *Object-Glass*.

I come now to consider the second Eye-Glass placed after the first Eye-Glass. And here it is manifest, that placing this as it ought in a Telescope; if we place our Eye nearer to this middle Eye-Glass than its Focus the Eye sees the Object *inverted*

verted and confused : Place the Eye in the Focus, it sees the Object all in confusion, neither *erect* nor *reversed*. For here again, there is a distinct Representation of the Object to be received on a piece of Paper, as in the Focus of the Object-Glass ; and the Eye being placed at any time in this place, being in the *Distinct Base*, sees all in confusion, (all which is manifest from the First Part, and therefore but lightly touch'd here). But then let the Eye be placed farther from this middle Eye-Glass than its Focus (for so is the third or immediate Eye-Glass, it being always distant from the middle Eye-Glass, the Aggregate of both their *Foci*), it perceives the Object *erect* and *confused*.

Lastly, the third or immediate Eye-Glass does nothing towards the *erecting* or *reversing* the *Species*, which it receives *erect* from the middle Eye-Glass ; no more than, in a Telescope of two Convex-Glasses, the Eye-Glass does to the *Species* it receives from the Object-Glass ; as is shewn before. All which will be manifest from inspection only of *Tab. 37. Fig. 1.*

Wherefore we are to consider the Telescope consisting of a Convex Object-Glass and three Convex Eye-Glasses, as two Telescopes, each consisting of two Convex-Glasses. The first consists of the Object-Glass and first Eye-Glass, and this *inverts* the *Species* ; that is, the *Species* is *inverted* in the *Distinct Base* of the Object Glass, and is so brought to the Eye. The second Telescope consists of the two immediate Eye-Glasses, and this *erects* what the former *inverted* ; that is, the *Species* in the *Distinct Base* of the middle Eye-Glass is *erect*, and is so brought to the Eye, by the Eye-Glass. The Eye-Glasses themselves, in either Case, having nothing to do with the *erecting* or *inverting*, but merely in *representing* in the same posture the Image immediately before them.

The French Problem should not therefore have broken a Telescope of four Convex-Glasses into four Pieces, but into
two ;

two; and the Case would then have been plain; whereas, by breaking it into *four* Perspective-Glasses, that is attributed to *two* of them, which *neither* of them does, *viz.* *Inverting* and *Erecting*.

Therefore I say lastly, That *one* Convex-Glass (that is the Object-Glass) as posited in this Telescope, *inverts*: The *second* (that is the *first* Eye-Glass) does nothing towards *erecting* or *reversing*; but *represents* the Image as it is in the *Distinct Base* of the Object-Glass before it, that is, *inverted*. The *third* Glass *erects*, or rather *restores* what was before *inverted*. The *fourth* represents the Image, as it receives it from the *Distinct Base* of the *third*, that is, *erect*.

And this I think a sufficient and easie Answer to what the French Man makes a great Difficulty.

As a Corollary to what has been laid down in this Chapter, we may deduce this Practical Rule for combining or putting together a Telescope of *Prop. LVI*.

Take the two first Eye-Glasses, and combine them by Tryals, so as to make a Distinct Inverting Telescope of *Prop. L*.

Then take the Object-Glass and first Eye-Glass, and by Tryals combine them likewise.

Lastly, take both these Telescopes, and without altering the Distances of their Glasses in either of them singly, by Tryals combine both these Telescopes, till the Appearance be clear and distinct.

What is here done by Tryals, may be effected by actual Mensuration, or designing out the Distances of the Glasses from each other by knowing their Focal Lengths.

C H A P. III.

Of Glasses for defective Eyes.

- (1.) *Spectacles for old and pur-blind Men an Invention in Dioptricks of great use.* (2.) *Some Rules for choosing Spectacles both for old and pur-blind.* (3.) *Observations on Mr. Hook's Invention for helping Myopes by Convex Glasses.* (4.) *Telescopes and Microscopes adapted to defective Eyes.*

(1.) **W**ERE there no farther Use of *Dioptricks* than the ^{*Spectacles for old and pur-blind Men, an Invention of great use.*} Invention of *Spectacles* for the Help of defective Eyes; whether they be those of *old Men*, or those of *pur-blind Men*; I should think the Advantage that Mankind receives thereby, inferiour to no other Benefit whatsoever, not absolutely requisite to the support of Life. For as the *Sight* is the most noble and extensive of all our Senses; as we make the most frequent and constant use of our Eyes in all the actions and concerns of human Life; surely that Instrument that relieves the Eyes when decay'd, and supplies their Defects, rendring them useful, when otherwise almost useless, must needs, of all others, be esteemed of the greatest Advantage. In what a ^{*Condition of old Men deplorable without Spectacles.*} miserable condition do we count those, in whom it hath pleased the *great Contriver of the Eyes and Sight*, to shut those two little Windows of the Soul? And we may imagine, that they, in whom these Lights are but *partly* obscured, do in some measure partake of the Misery of the blind. How melancholy is the condition of him, who only enjoys the Sight of what is immediately about him? With what Disadvantage is he engaged in most of the Concerns of human Life? Reading is to him troublesome, War more than ordinary dangerous, Trade and

and Commerce toilsome and unpleasant. And so likewise, on the other hand; How forlorn would the latter part of most Mens Lives prove, unless *Spectacles* were at hand to help our Eyes, and a little form'd piece of Glass supply'd the Decays of Nature? The curious Mechanick, engaged in any minute Works, could no longer follow his Trade than till the 50th. or 60th. Year of his Age: The Scholar no longer converse with his Books, or with an absent Friend in a Letter. All after would be melancholy Idleness, or he must content himself to use another Man's Eyes for every Line. Thus forlorn was the state of most *old Men*, and many *young*, before this admirable Invention; which, on this very account, can never be prized too highly.

*Rules for
Choosing
Spectacles
both for
old and
pur-blind.*

(2.) And because in the First Part hereof, *Prop. XXVIII.* and *XLV.* I have but slightly touched on the proper Method for helping *defective Eyes*; I think it convenient to prosecute that matter more fully in this place. Always supposing what foregoes in the First Part as understood.

First therefore for helping the Eyes of *old Men*, as being more frequently and universally requisite than the Relief of *pur-blind Eyes*.

In the First Part (*Prop. XXVIII. Sec. 8.* and *Prop. XXXI.*) we learn, that a *Convex-Glass* is here to be used: For these seeing *distant Objects distinctly*, and *nigh Objects confusedly*, must use such Glasses for reading, &c. as make *nigh Objects appear as distant*; or, which bring the Rays from each single Point in a *nigh Object*, as if they came from a more *distant Point*. Or thus, seeing the *CrySTALLINES* of *old Men* are too flat for *nigh Objects*, that is, want *Convexity*; we are to help them by adding to them an artificial or adventitious *Convexity* of a Glass. But then our enquiry must be, What is a *Proper Convexity* to this or that *particular Eye*? And because reading, or working *curious small Works*, as being engaged upon *nigh Objects*, are
the

the chief Employments wherein *Spectacles* are requisite. I shall suppose our Enquiry chiefly designed for this purpose. And indeed, tho there can hardly be any Rules laid down, *strictly* to determin this matter; for *distinct* Sight may consist within a great Latitude: Yet if we observe the following Directions, we shall be apt to err less, and to fit our Eyes better with *Spectacles*, than if we observed no Rule at all, but chose at a venture.

First, When we first find our selves begin to require *Spectacles*, let us make choice of the *flattest* *Convexities*, that will possibly help our Eyes. These are usually called *Young Spectacles*. There are many ways of finding out and trying such; but none more ready, obvious, and easie, than trying with which one can read a small Print distinctly, with the Book *farthest* from the Eyes; or try which *Spectacles* burn at the greatest distance; for these *Spectacles* are the most proper for those Eyes to use, and shall prejudice the Sight less, and preserve it longest of any.

We may note likewise, that the distance of the Print, or Object from the Eye continuing the same, a *Convex* Glass may be said to be older or younger, according as it is removed *farther* from or *nigher* to the Eye or Object. This is manifest from the Doctrine in the First Part, concerning the *Locus Apparens* of an Object through such a Glass. To which therefore I refer.

Secondly. If your naked Eyes can read a moderate Print at the full extent of your Arms, or at the distance of about two Feet or two and an half; and you desire a Pair of *Spectacles* to read with, at the usual Distance of reading, *viz.* about a Foot or little more: Procure a Pair of Glasses of such a Convexity, that an Object being exposed before them at the Distance of about a Foot, they may have their *Imaginary Focus*, or the *apparent Place* of the Object, distant from them about two, or two Feet and an half. All which may be easily obtained and effected by the Doctrine in the First Part.

Barrow, :
Lect. Opt
14. p. 103.

In the next place, for relieving the Eyes of the *short-sighted*, *pur-blind*, or *Myopes*. We must consider, that these, laboring under the contrary Defect with *old Men* (for they see *nigh* Objects *distinctly*, but *distant* Objects *confusedly*), must be relieved with a Remedy of a contrary effect; and therefore they are helped by *Concave-Glasses*, which bring the Rays of *distant* Objects into the Eye, as if they were *nigh*. And because we may conceive the CrySTALLINES of these Eyes as too *protuberant* or *convex*, therefore we are to take off from this too great *Convexity*, by adding an adventitious *Concavity*. But then, there is nothing so universally complained of by those who are thus affected, as the Difficulty they find in fitting proper Glasses to their Eyes. For the removal whereof, the following Rules may be observed.

First, That for viewing *distant* Objects, according to what is noted in the First Part, *Prop. XLV.* If a *short-sighted* Person can read distinctly, or see Objects at the distance of a Foot from his naked Eye; a *Concave-Glass*, whose *Virtual-Focal-Length* is a Foot, makes such a Person see *distant* Objects distinctly. And so of any otherwise disposed Eye. So that knowing the Distance at which a *pur-blind* Person reads distinctly with *un-armed* Eyes, 'tis easie, by the Doctrine in the First Part, to assign him a proper Glass for his Eye to see *distant* Objects.

Secondly, For Glasses proper for *Myopes* to read by, or to see Objects at the distance of about a Foot and half. Let us suppose Eyes so affected, as not to be able to read, but at the distance of Four Inches; and that we desire Glasses for these Eyes to read by, at the ordinary distance of about a Foot and half: Let us form such *Concave-Glasses*, which, being exposed to an Object at the distance of a Foot and half, may have their *Virtual* respective Focus at the distance of four Inches. And so likewise for any other Distance. All which is easily performed by those versed in the First Part of this Work.

Thirdly,

Thirdly, We are to note, that *Myops* shall require different Glasses for viewing Objects at different Distances. But the visive Faculty not being contained within such strict and determined Limitations; that Glass which is useful at an Object an hundred Foot distant, shall serve likewise at an Object distant fifty Foot; but then 'tis not so helpful at one distant five Foot. But another may be had proper even for this Distance, and not so useful at an Object distant an hundred or fifty Foot. All which is manifest from the Doctrine in the First Part.

Fourthly, There are some Eyes so ill conformed, that no Glasses whatever will relieve them. Of this I have often heard a very ingenious Man and great Philosopher Sir William Petty often complain in his own particular. But then this proceeds not from a too little, or too great a Convexity in the Crystalline; but from some other Indisposition or ill Configuration, not to be relieved by Glasses.

Lastly, Persons *pur-blind* labour under this great Inconvenience, that the Glasses which relieve them in one particular, do hinder their strong Vision of distant Objects in another particular. For as Concave-Glasses do order the Rays from any one single Point, properly to be received by the Eye of a *Myops*: So at the same time, they diminish the Appearance of the whole Object. And from hence it is, that tho these sort of Eyes may be well enough relieved for Reading and Writing at a convenient distance, and for seeing pretty large Objects at the distance of 100, 200, or 500 Foot; yet for Objects much farther, unless they be very large indeed, they are not so easily supply'd.

(3.) And here I cannot but take notice of an ingenious intimation of Mr. *Hook's* (to whom the World is certainly much obliged for his curious Contrivances in *Mechanicks*, and his other Philosophick Indeavours) published in Num. 3. of the *Philosophical Collections*. Lond. 1681. which he calls *Myopibus Juvamen*.

*M. Hook's
Contrivance for
helping
Myopes
considered.*

'Tis briefly this, That some sort of *short-sighted* Persons, who cannot be relieved by *Concave* Glasses, may perhaps find some help in *Convex*-Glasses; their Eyes being removed at a convenient distance farther from these Glasses than their *Distinct Bases*. As for reading by these Glasses; the Book must be *inverted*, and then the Image in the *Distinct Base* shall be *erect*, and the Eye shall perceive it *erect*. As for Writing, the Difficulty is greater than mentioned by the *Learned Author*: For the *Myops* must not only learn to write *inverted*, but also *retrograde*, viz. from the Right to the Left Hand; and, what is yet more inconvenient, from the bottom towards the top of the Page; which is hardly practicable on account of blotting the wet Writing. As for viewing *distant Objects* with these Glasses; I acknowledg with the ingenious Author, that much of the Disagreeableness of the *inverted Prospect* is taken off by use and custom, as I my self have experienced by my frequent use of inverting Telescopes. But yet I cannot go so far with the Author, as to assent to his Deduction from hence; which is, that 'tis only use and custom that makes us judge Objects *erect* that are perceived by an *inverted* Image on the Fund of the Eye. For then a Man *standing on his Head*, should judge the Trees and other Objects he sees *inverted*; which no one in his Senses will do, but rather judge what is right, viz. that he himself is *inverted*, whilst the circumjacent Objects continue *erect*. This will be more evident to us, by considering the case of an adult Person, who has been blind from his Birth, and now suddenly restored to his Sight: He is not prejudiced by custom, and yet (doubtless) would judge as is usual. But lastly, an other great Difficulty that will attend the use of these Spectacles for *Myopes*, is, that they must be carried at such a *distance* from the Eyes, that it will be very troublesom to manage them commodiously. And if to this again we add the distance requisite for the Object, I question whether some Mens Arms will be long enough

to manage a Pen for Writing, or turning the Leaves of a Book.

I have been the longer on this Proposal, because the *Worthy Author* does candidly invite all, to communicate to the Publick, what real Benefit by it, or Objections against it, they shall find.

(4.) I shall conclude this Chapter with the way of Adapting *Telescopes* and *Microscopes* to defective Eyes. Which is briefly thus; Adapting
Telescopes
and Mi-
croscopes
to defe-
ctive Eyes.

A Telescope composed of a *Convex* Object-Glass and *Concave* Eye-Glass, being apply'd to an *old* Eye, the Eye-Glass may be a little farther removed from the Object-Glass than ordinarily: On the contrary, in such a Telescope for the Eye of a *Myops*, the Eye-Glass may be removed a little nigher to the Object-Glass; the reason hereof is manifest from what has been delivered in the First Part. But then, how much farther, or nigher they are to be removed, is only to be determined by Experiment, and by every ones fitting the Glass to his own Eye.

And so likewise in Telescopes composed of a *Convex* Object-Glass, and *Convex* Eye-Glass: For the Eye of an *old* Man, the Eye-Glass may be removed a little farther from the Object-Glass, or from the *Distinct Base*. And for the Eye of a *Myops* a little nigher to the Object-Glass, than for Eyes naturally and orderly affected.

In like manner for *Microscopes*; First the simple or single *Convex* for an *old* Eye, is to be removed a little farther from the Object; and for a *Myops*, a little nigher the Object, than the usual posture.

And so in *Double Microscopes*. For an *old* Eye, the Eye-Glass is to be a little farther from the Object-Glass or *Distinct Base*; and for a *Myops*, a little nigher the Object-Glass or *Distinct Base*. And because in these, the *Distinct Base* is brought nigher to, or farther from the Eye-Glass (without altering the Distance

Distance of the Eye-Glass and Object-Glass), only by removing the whole Microscope *nigher to or farther from* the very Object: This latter Motion effects the same as the former; and therefore may be used for *old or short* Sight instead of the former.

C H A P. IV.

Of Mechanick-Dioptricks,

- (1.) Chief Authors that have treated of Grinding or Forming Optick-Glasses. (2.) For trying whether a Glass be not plain. (3.) For finding the Focal Lengths of Glasses. (4.) Concerning the Centre of a Glass. (5.) For trying the Regularity and Goodness of an Object-Glass. (6.) Managing great Glasses, and the Author's Treating thereof. (7.) Proportioning Glasses in Telescopes. (8.) Mr. Hooks's Contrivance for making an Object-Glass of a small Sphere serve a long Tube.

*Authors
treating of
Glass-
grinding.*

(1.) **I** Design not in this Chapter to deliver at large the several ways of Grinding and Forming Optick-Glasses, the manner of making the *Forms, Tools, or Dishes* wherein they are shaped; or the various *Machines* contrived by several ingenious Heads for this purpose: For this may justly require a particular Treatise by it self. And because nothing of this kind has ever yet appeared in English, according as I find the present Work accepted, I may perhaps hereafter attempt something in this way, for the satisfaction of some ingenious *English* Spirits, who may be inclinable to offer at these Exercises. In the mean time, they that are Masters of the Languages wherein they are written, may apply themselves to *Pere Cherubin's Dioptrique Oculaire*, and *Zahn's Oculus Artificialis*; wherein they will find an abundance on this Subject. But after all that can be writ concerning it, Practice and Experience will find out many

*Cherubin.
Zahn.*

many Conveniencies and Inconveniencies, - which can hardly be committed to Words, or described; and therefore this is at last the best Instructour. But here I shall mention the chief Modern Authors that have contrived *Engines* for Grinding Glasses, that those who please may consult them. Mr. *Hook* in *Hook* his *Micrographia* describes an Engine for this purpose. *Hevelius* in his *Selenographia*, Chap. I, II. describes another for Modelling the Forms or Dishes for Grinding Spherick Glasses. And in the First Part of his *Machina Cælestis*, Chap. XXIII. describes one for Grinding Conick-Glasses. *Ant. Mar. Schyrleus de Rheita* De Rheita in his *Oculus Enoch & Elie*, Lib. IV. has a Machine for Conick-Glasses. *Maignan* at the end of his *Perspectiva Horaria* describes Maignan. Machines both for Spherick and Conick-Glasses. *Des-Cartes* Des Cartes in his *Dioptricks* has another for Conick Glasses. *Mons. Borelly* Bore li. has given the World the Secret of his manner of Grinding great Glasses in Cypher, *Journal des Sçavans*, Ann. 1676. July 6. but has not yet obliged us with the Discovery: Tho he be a Person of the greatest Candor and Freedom, and the most communicative; as I am obliged to express with much Gratitude for his Civilities shew'd me in *Paris* 1685. at which time he gave me an Object-Glass formed by this way for a Telescope 24 Foot long. The celebrated *Mons. Fatio de Duillier* (of De Duillier whom *Dr. Burnet* gives deservedly so excellent a Character in his Letters of Travails) in the *Journal des Sçavans*, Ann. 1684. Novemb. 20. describes an Invention of his own, for exactly forming the Dishes for Grinding Spherick Glasses; which indeed is very ingenious, and perfectly new: But I cannot tell how easily it may be practised, without some farther Improvement; for his Contrivances of the Block *c*, and of the Screws *d*, *g*, (I refer to his Figure) seem not to have all the Motions requisite, to keep the Point *e* exactly true to the Axis of the little Telescope *b k*.

Wren.

The Incomparable Sir Christopher Wren, our English Archimedes, Apollonius, Diophantus, proposes his Contrivance for Forming Hyperbolick Glasses, Num. 48. and 53. of the Philosophic. Transact.

Conick-
Glasses
not better
than
Spherick
From
Mr. New-
ton.

But all farther Endeavours for Forming Conick-Glasses, which have hitherto been wholly frustrated and unsuccessful, may now be put to a full stop; when we hear in this matter the Opinion of as great a Philosopher and Mathematician, as this or any Age could ever boast of, The Celebrated Mr. Newton of Cambridge, who in his profound Treatise, *Philosophiæ Naturalis Principia Mathematica*, has fathom'd the greatest Depths of Nature, and laid a Foundation for Posterity to raise an infinite Superstructure. Thus he, in the First Book, Schol. ad Prop. XCVIII. *Ad usum autem Opticos, &c.* In English as follows, But for all Optick Uses, Spherick Figures are the most commodious. If the Object-Glasses of Telescopes were compos'd of two Spherick-Glasses containing Water between them, perhaps the Irregularity of the Refractions that are made on the Surfaces of the Glasses towards their edges, may be accurately enough corrected by the Refractions of the Water. And such Object-Glasses are preferable to Elleptick or Hyperbolick-Glasses; not only because they are easier and more accurately to be formed; but also because they refract more accurately those Pencils of Rays that are (collateral or) out of the Glasses Axis. But the different Refrangibility of different Rays, will for ever hinder us from perfecting Opticks by Glasses either of Spherick, or any other Figures whatsoever. And unless we can correct the Errors that arise from hence, all our Labour is lost in other Corrections.

And indeed if we consider it right, we shall find it impossible, by whatever Figures to render the Appearance of the Collateral Parts of an Object so distinct as the direct; for the very natural Eye does it not; and therefore we are forced to apply it successively directly before the Parts of any Object we design to view: And we may well despair to perform by Art,
more

more than what the Almighty Framers of the Eye has given us a Pattern for.

What this *Great Man* means by the *Different Refrangibility* of ^{Different Refrangibility of Rays.} *Different Rays*, We may find, Num. 80. p. 3075. N. 83. p. 4059. N. 84. p. 4087. N. 85. p. 5004. N. 88. p. 5084. N. 110. p. 217. N. 121. p. 499. N. 123. p. 556. N. 128. p. 692: of the *Philosoph. Transactions*; wherein he lays down a perfectly new and most ingenious Theory of *Light*.

In the same Tracts, Num. 81, 82, 83. we may find an ^{Cata-Dioptrick Telescope.} Account of a new *Cata-Dioptrical Telescope*, invented by this same excellent Person.

(2.) All that I shall offer at more in this Chapter, is, to lay down some practical Rules for finding the *Foci* and *Centres* of Glasses; with some other *Accidental Remarks*, that may ^{Trying whether a Glass be plain.} be requisite to the clearer understanding and performance of some Precepts delivered in this Treatise.

The Object-Glasses of Telescopes are generally of so little Curvature on their Surfaces, that by looking on them or by feeling them, it cannot be discovered, whether they are plain or of a Spherick Figure; or which is formed on a Sphere of a greater, which of a lesser Radius. To find this (as I have said before, in *Schol. Prop. XXXI. Part I.*) we are to shake the Glass nimbly at our Arms length before the Eye; and if the Objects seen through it seem to dance or move, the Glass is not plain. And that Glass which makes the Objects seem the most to move is formed on the less Sphere, whether Convex or Concave.

(3.) When we have thus found our Glasses to be Spherick. ^{For finding the Foci of Glasses.} Then supposing them Convex, there are several Methods for finding their *Foci*. I shall lay down some of the plainest and most certain.

First, for Glasses of pretty deep Convexities (that is, of small Spheres), apply them to the end of a Scale of Inches and Decimal Parts, and expose them before the Sun; and upon the

F f

Scale,

Scale, we shall find the bright Intersection of the Rays exactly measured out. Or expose them in the Hole of a dark Chamber; and where a white Paper receives the distinct Representation of *distant* Objects, there is the Focus of this Glass. This is an universal and certain way for all Convexes. For a Glass of a pretty long *Focus*, observe some distant Objects through it, and recede from the Glass, till the Eye perceive all in *confusion*, or till the Objects begin just to appear *inverted*; here the Eye is in the Focus. If it be a Plano-Convex Glass, make it reflect the Sun against a Wall; we shall on the Wall perceive two sorts of Light, one more *bright* within an other more *obscure*; withdraw the Glass from the Wall, till the *bright* Image is at its smallest; the Glass is then distant from the Wall about the fourth part of its *Focal* Length. If it be a double Convex, expose each side to the Sun in like manner, and observe *both* the Distances of the Glass from the Wall: The first Distance is about half the Radius of the Convexity turned from the Sun, and the second Distance is about half the Radius of t'other Convexity likewise: Thus we have the Radii of the two Convexities; whence the Focus is determined by *Prop. III. Part I.* The reason hereof depends on the Doctrine of *Catoptricks*.

But the most exact way of determining the just Focal Length of the Object-Glass of a Telescope, is what I shall lay down in *Chap. V. Sec. 4.* of this Part, which, because I must necessarily deliver in that place, I will not here anticipate.

*Foci of
Concaves.*

The Foci of Concaves are obtained by *Reflection*; for as a *Concave Miroir* or *Speculum* burns at the distance of about half the Radius of the Concavity; so a *Concave-Glass* being supposed a *Reflecting Speculum*, shall unite the Rays of the Sun, at the distance of about half the Radius of the Cavity.

*Concern-
ing the
Centres
of Glasses.*

(4.) Before we proceed to the *Centration* of Glasses, we are to recollect the 18th. *Definition* of the First Part, wherein we define the *Axis* of a Glass. I say therefore, when the *Axis* of

a Glass passes directly through the Centre of the Glasses Breadth, or roundness of its Aperture, that Glass is said to be *truly centred*. And this will always be so when the Glass is *equally thick* round the edges of its *Aperture*. But this will be more intelligible by a Scheme *Tab. 40. Fig. 1.* Let *agdea* be a Plano-Convex. T. 40. F. 1. Glass, much thicker towards the Edge *de*, than towards *a*; let *ag* be equal to *gd*, *g* is the *Centre* of this Glass, for 'tis the middle Point of its *Aperture* or *Breadth*. But then this Glass is not *truly centered*: For, to *l* the *Centre* of the *Convexity* *agd* draw *gl*, I say *gl* is not the *Axis* of this Glass; but if there may an other Line *lk* be drawn, whose Portion within the Glass *ik* is greater than *bg* (as in this Case it may easily be demonstrated. For $lk = lg$ and lh more than li . For $\angle lib = \text{Rect.}$ therefore bg less than ki) and which being perpendicular to the plain Surface *ae*, as well as to the *Convex Surface* *akd*; *lk* must consequently be the *Axis* of this Glass. Wherefore *k* is the *true Centre* of this Glass. And because, to compleat this Glass, there is wanting the Portion *dfe*; therefore to make the true *Centre* of the Glass *k*, coincident with the middle Point of its *Breadth* or *Aperture*; we are to make *kb* equal to *kd*; and then we are to cut off, or to cover the Portion of the Glass *bac* equal to *dfe*; and so we obtain the *compleat* and *truly Centered* Glass *cbkdec*. The same may be understood of *double Convexes*, without farther Explication.

But because in *Object-Glasses* of even moderate Lengths, 'tis impossible by the *Eye*, or any *Admeasurement* of their *Thickneſs*, to know whether they be *truly Centered*: Therefore we must have recourse to some other *Methods*, that may shew this. And these are various.

First, Holding the Glasses at a due distance from the *Eye*, let us observe the two reflected Images of a *Candle*; and where these two Images *unite* or *coalesce*, there is the *true Centre* of the Glass; if this be in the *middle* of the Glasses *Breadth*, the Glass

is *truly Centred*; if not, we are to rectifie it, shall be declared hereafter.

A second way is, By presenting the Glas before the Sun, and making it reflect the Light on a Plain nighly parallel to its Surface, at a proper distance; and we shall perceive two sorts of Light reflected; one *smaller*, but much more *strong* and *vigorous*, within another more faint and large. Then by a due posture of the Glas (found by Tryals) both these Lights are to be projected as round as possible; and at a proper distance from the Wall on which they are reflected; the *round brightest Spot* is to be brought into the smallest compass that it can (Tryal will make all this plain). When the Glas is in this posture, if the *bright Spot* be projected just in the middle of the fainter Light, the Glas is *well centred*. If it be projected to the Left Hand of this middle, the Glas is thickest towards the Left Hand Edge. And so to whatever side of the faint Light, this bright Spot is projected, on *that side* is the Glas *thickest*; and on *that side* lies the *true Centre*. The reason hereof I shall explain in a Plano-Convex-Glas, and is the same in a double Convex; *mutatis mutandis*. For the bright Spot is the Image of the Sun projected by the curve Surface of the Glas consider'd as a *reflecting Speculum* (whereof more hereafter), and the faint Light is the Reflection of the Sun from the other plain Surface.

Tab. 40. Fig. 2. *a de* is a Plano-Convex-Glas, thicker towards the side *de* than towards *a*. Let the Curvity *ad* (whose Centre is *c*) be exposed directly to the Sun: By the known Laws of *Catoptricks*, the Parallel Rays (from the middle Point of the Sun, for instance) falling on the Curvity (which parallel Rays in the Figure are expressed by continued Lines), are united in the Focus at *f*, distant from the Curvity about half its Radius, there causing a brisk Light and Heat. In the mean time these parallel Rays do fall on the plain Surface *a e* *obliquely*, and

and are reflected thereby alongst the prickd Lines; and these are they which cause the *faint* Light. Now 'tis manifest, that if a Plain at *f*, parallel to the plain Surface *ae*, received these two Reflections, the *bright* Light at *f* would not be found in the middle of the *faint* Light: For here we see the *faint* Light is thrown much to one side. And the *bright* Light *f* is projected upwards from the middle of the *faint* Light; for the Glas is thickest upwards towards its Edge *de*.

The third Way of Examining the Centres of Glasses is yet more compleat than the former; for it does not only discover the Fault (if there be any, as in long Object-Glasses 'tis very rare but there is; especially if they be wrought in the Form by the unguided Hand, and not by Engine), but withal, it rectifies the Fault. 'Tis thus; Cover the Surface of the Glas with a thin piece of Paper, in which there is cut a round Hole of about an Inch diameter, and round about this Hole there are to be struck two or three Concentrick Circles; move this Paper upon the Glas, till you see, on the plain that receives the reflected Light, that the *bright* Spot is exactly in the middle of the other fainter Light round it. This also one may measure by a Pair of Compasses, having, to that end, slightly fixed the Paper to the Glas, that we may more nicely determine, whether this *bright* Spot be exactly in the middle. This therefore being carefully adjusted by gently sliding the Paper on the Glas (if it be requisite), we are, without the least altering this *true* Position of the Paper, to fix it more firmly to the Glas. And laying it thus on a Table, let us mark on the Glas (by the Point of a Diamond) three Points in one of the Circumferences Concentrick to the round Hole in the Paper. And sticking a small piece of Cement on the Glas about the middle of the round Hole; by means of the three marked Points, let us find the exact Centre of this round Hole. Then uncovering the whole Glas (except only the Cement

ment in which the Centre is marked), with a Diamond-pointed Compass, let us strike as large a Circle on the Glass, as its Breadth will bear. Then round the Glass according to this Circle, and 'tis as exactly centred as the Sense can judge.

But note, That whereas I have said that the *brighter* Spot is smaller than the *fainter*; and that the *brighter* Light is to be reflected into the *middle* of the *fainter*. This is to be understood, supposing the Breadth of the Glass will allow it. For the Glass may be so narrow, that the Projection of the bright Image of the Sun may be *broader* than the Breadth of the Glass. But this happens so seldom in Practice, that I pass it over.

Pere Cherubin, who is often very nice in matters of little moment, and loose enough in those of greater weight and absolute necessity, describes an implicated Contrivance for true Centring of Glasses. *Vision Parfait. Tom. II. p. 109.*

For trying
the Regu-
larity and
Goodness
of an Ob-
ject-Glass.

(5.) The same Frier lays down a Way of Examining the Regularity and Goodness of an Object-Glass, *pag. 25.* which it may not be amiss here to insert. After we have centred the Object-Glass as well as we can, by the foregoing Method: To try the Regularity of its Form to the greatest exactness, says he, We must do thus. On a Paper strike two Concentrick Circles; one whose Diameter is the same with the Breadth of the Object-Glass; t'other of half that Diameter. This inward Circumference divide into six equal Parts, by the known way of applying the Radius six times in the Circumference, and making six fine small Holes therein with a Needle. Let us cover one side of the Glass with this Paper; and then exposing it to the Sun, we are to receive the Rays that pass through these six Holes on a Plain at a just distance from the Glass. And by withdrawing or approaching this Plain from or to the Glass, we shall find, whether the Rays that pass through these six Holes unite exactly together at any distance from

from the Glas; if they do, we may be assured of the Regularity of this Glas, that is, of its *just Form*. And at the same time we obtain exactly the Glasses *Focal Length*.

But after all, there is no better way for trying the Excellency of an Object-Glas, than by placing it in a Tube; and trying it with small Eye-Glasses at several distant Objects. For that Object-Glas that represents the Objects the *brightest* and most *distinct*, and bears the *greatest Aperture*, and most *Convex* or *Concave* Eye-Glas, without colouring or Haziness, is surely the best. The most convenient Object to try them at, is the *Title Page* of a large Book; wherein there are generally Letters printed of divers Magnitudes, and therefore affords variety of small Objects; whereby the comparative Excellency of Object-Glasses may be nicely estimated. This the Celebrated *Monf. Cassini*, the *French King's Astronomer*, shew'd me when I visited him at the *Observatory* in *Paris*, *Ann. 1685*. who tryed all his Glasses by the large *Title-Page* of a Book, fixt inverted on the *Jaume* of a *Steeple Window* more than $\frac{1}{4}$ of a *Mile* distant from the *Observatoire*.

(6) The next Piece of *Mechanick-Dioptricks*, which I shall mention, is, The *Managing Great Glasses*. And herein I shall not swell this Volume with describing those sumptuous Contrivances and costly Machines invented for this purpose: It shall suffice me to refer the Reader to the *Original Authors*, where he may find them described at large.

Hevelius in his *Machina Cælestis*, *Part. I. Cap. 19, 20, 21*, *Hevelius*, 22. describes the Engines he used for his Telescopes. And amongst others, a Contrivance for managing his Tube of 60 Foot, and another of 150 Foot long.

The deservedly Celebrated *Monf. Hugen*, one of the chief *Hugens* Mathematick Luminaries of the present Age, has publish'd a small Tract, *Astrosopia Compendiaria*, designed only for Describing his way of Managing great Glasses with very little trouble

ble, and without a Tube. This I am sure is no barren Speculation of the Ingenious Author's, but successfully practised by him; as I can gratefully testifie, having had the favour of being shewn the whole Contrivance by the Excellent Author himself in his Garden at the *Hague*, *Ann.* 1685. at which

*Planetary
Clock.*

time I had the happiness also of seeing his *Planetary Clock*, or *Moving Ephemeris*, a Machine that cannot be sufficiently admired.

Cusset.

Monsr. Cusset, an ingenious *French Man* of *Lions*, has publish'd his Contrivance for managing great Glasses, in the *Journal des Scavans*, *Ann.* 1685. *May* 18.

Monsr. Cassini, when I was with him at the *Observatoire* in *Paris* (amongst other Curiosities, which according to his usual Candour and Civility he communicated to me) shew'd me two very pretty Contrivances for managing great Glasses; which, because not yet publick, I shall describe as well as I can by Memory at this distance of time. The first was a plain Piece of Clock-work, moved by a Spring and regulated by a Pendulum Vibrating half Seconds. This carried an Arm that stood something prominent from the Body of the Work; which Arm at its extremity carried the Object-Glass fixt in a Ring. This Arm and the Object-Glass, by means of graduated Arches in the Clock-work could be turned *ad Libitum* directly to any Star. Thus suppose *Saturn* were to be observed; by a common *Ephemeris*, knowing his Longitude and Latitude, together with the Time of Day or Night, the Arches of the Clock being put to such and such Divisions, and the Machine it self placed with such or such a part horizontal or perpendicular; the Object-Glass was of course directly expos'd to the Star. Then the Pendulum being put in motion, the Machine kept the Glass constantly expos'd directly to the Star in its diurnal Motion. This whole Machine then, being placed upon an Height, carried the Object-Glass in its due position; and the *Observer below* managed the

the Eye-Glass by his free Hand, assisted with a *Rest*. Tho I cannot retain so exact a Remembrance of this Engine, as to venture at a Scheme of the particular Parts; yet this Description perhaps will be sufficient to give the ingenious Astronomer an Idea thereof, so as to apprehend the Contrivance in general. Something much of the same kind may we find described in Mr. *Hook's Animadversions on Hevelius Mach. Cœlestis*. p. 66, 67, 68, &c.

The other Contrivance he mentioned to me, if I forget not, he or *Monf. Borelly* told me, was due to *Monf. Azout*. It is thus; From the Tube of a small Telescope there stands out an Arm perpendicular to the side of this smaller Tube: This Arm carries the great Object-Glass; then an Observer upon an Height manages this small Telescope, following therewith the Motion of a Star; by which means the great Object-Glass (being parallel to the Object-Glass of this lesser Telescope) is kept constantly in prosecution of this same Star: Then the other Observer *below* manages the Eye-Glass as before.

These two Contrivances are indeed pretty Thoughts; but I cannot promise that they can be so easily practised; unless the Machines that carry the great Object-Glass be made to rise and fall at pleasure, as the Star rises or sets. For otherwise, the Observer that manages the Eye-Glass, shall soon lose the sight of his Object; unless he have the opportunity of rising and falling, by some such Contrivance as the fore-mentioned *Monf. Cusset* proposes in the fore-cited place. But this is vastly chargeable.

'Tis now above seven Years since *Monf. Boffat* of *Thoulouse* has promised the World his Contrivance for managing great Glasses; of which he has given a small Specimen in the *Journal des Scavans* 1682. Dec. 28. but we hear nothing farther of it; perhaps because the Contrivance requires reflecting *Speculums*, which much weaken the light of the Object, and are therefore found useless,

Proportioning of
Glasses in
Telescopes
and Microscopes.

(7.) I may reckon another Piece of *Mechanick Dioptricks*, The proportioning of the Glasses in Telescopes and Microscopes. We have said before, that, of two or more Object-Glasses of the same Focal Lengths; that is the best, which will bear an Eye-Glass of the greatest Convexity (this is usually called the Deepest Charge): But yet there are some Proportions to be observed, which Experience has found out, as the most convenient and best adapted for most Mens Eyes, that are well disposed. For so a good Object-Glass of 12 or 13 Feet, will bear a Charge of 3 Inches, better than a Charge much deeper or shallower.

But moreover, In adapting an Eye-Glass to an Object-Glass, respect is likewise to be had to the *Object* we contemplate; for Objects of a sedate Light, as *Saturn*, *Jupiter*, &c. will allow deeper Charges, than those of a more brisk and strong Light, as *Venus*, &c.

Wherefore this whole Affair being only the Subject of Experiment, to that I shall refer, and only hint by the bye; That for Telescopes of three Convex Eye-Glasses, *Cberubin* advises (*Dioptrique Oculaire*, III. Par. Sec. 2. Cap. VII. pag. 188.) that, of the three Eye-Glasses, that next the Object-Glass should be of the deepest Charge; the middle Eye-Glass ought to be something shallower; and the immediate Eye-Glass the shallowest of all.

For Proportioning Glasses in double Microscopes, we may consult the same Author. But 'tis tedious to transcribe.

The LV. Prop. of the First Part, relates to the *Apertures* of Object-Glasses. After which, we have nothing to add in this place; which otherwise might have been challenged by a Discourse thereon, as being of a *Mechanick* Consideration.

M. Hooks
Contrivance to
make a
Glas of a
small
Sphere
serve a
long Tube.

(8.) The last Piece of *Mechanick Dioptricks* I shall mention, is an ingenious Thought of Mr. *Hooks*, for making a short Object-Glas perform the part of one formed on a much larger Sphere.

Prepare

" Prepare (says he) two Glasses, the one exactly flat on
 " both sides, the other flat on the one side, and Convex on
 " the other, of what Sphere you please. Let the flat Glas be
 " a little broader than t'other. Then let there be made a
 " Cell or Ring of Brass very exactly turned, into which these
 " two Glasses may be so fastened with Cement, that the plain
 " Surfaces of them may lye exactly parallel, and that the Con-
 " vex-side of the Plano Convex-Glas may lye inward, but
 " so as not to touch the flat of the other Glas. These being
 " cemented into the Ring very closely about the edges: By a
 " small Hole in the side of the Brass Ring or Cell, fill the
 " interposed space between these two with *Water, Oyl of Tur-*
 " *pentine, Spirit of Wine, Saline Liquors, &c.* then stop the Hole
 " with a Screw: And according to the differing Refraction of
 " the interposed Liquors, so shall the Focus of this compound
 " Glas be longer or shorter. *Vid. Philosoph. Transf. Num. 12.*
pag. 202.

This, I must confess is an ingenious Hint: But I doubt
 the desired Effect will not be so successfully attained thereby,
 so as to constitute an Object-Glas for a Telescope. For cer-
 tainly, were it effectual; 'tis so easie and withal so useful,
 that before this time it would have obtained, and been pra-
 ctised *universally*. And this makes me question, whether it
 would be of any better effect, than a *Meniscus-Glas*, or a Com-
 bined Glas of *Prop. XVII. Part. I.*

CHAP. V.

Of Telescopick-Instruments.

- (1.) *Controversie between Hevelius and Hook concerning Telescopick Sights.* (2.) *Hevelius's Objections against them.* *His Mistake concerning them.* (3.) *Their Fabrick or Contrivance.* (4.) *Adjusting them to plain Rulers or Tubes.* (5.) *To Quadrants, Sextants, &c.* (6.) *Dioptrick Reason of their Performance.* (7.) *Adapting the Micrometer to a Telescope.*

Contro-
versie be-
tween He-
velius and
Hook con-
cerning
Telesco-
pick-
Sights.

(1.) **T**HE Fame of *Johannes Hevelius*, Consul of *Dantzick*, is deservedly celebrated by all that delight in Astronomy. His Performances herein are highly extoll'd by all; and the sumptuous Volumes of his Labours and Studies, which he has published, have procured him an immortal Name of Honour amongst the Literate. But notwithstanding all his commendable Endeavours, he has not yet arrived at the height of perfection; but something is yet wanting and deficient even in his most costly Machines. Whoever peruses the large and elegant Volume of his First *Machina Cœlestis*, will admire at the vast Treasure he has expended on Astronomical Instruments of all sorts; when he sees even the very Description of them so very sumptuous. And yet, at the same time, whoever peruses a small Book of *Animadversions on this Machina Cœlestis*, by the ingenious Mr. *Hook*, will find one grand Defect does attend the noble *Hevelius* Instruments, which renders them (I will not say, useless, faulty, or no better than *Ticho's*, yet) not so compleat and perfect, as otherwise they had been by the Addition of *Telescopick Sights*. For *Hevelius* wholly used *Plain Sights*, which certainly are not so accurate as *Telescopick*.

copick. And tho I must confess ingeniously, that this renowned Astronomer, by his extraordinary Diligence, great Care, and perpetual long-continued Practice, but chiefly by his peculiar sharpness of Sight, had arrived to a great exactness of Observation by plain Sights (as I find by comparing the Observations made by the most curious Astronomers of our Age, *Flamsteed*, *Halley*, *Cassini*, &c. by Telescopic Sights, with those Observations made by *Hevelius*;) yet this we are to attribute more to the peculiar acuteness of his Eye, and to his extraordinary Diligence and Care in Observation, than to the exactness of plain Sights. For to me it seems manifest, from what the Learned Mr. *Hook* lays down in the forementioned Book, that the naked Eye cannot ordinarily perceive an Angle (or Object that subtends an Angle) less than a Minute, or half a Minute at the smallest. For tho we perceive Stars of that magnitude that their Diameters are not half a Minute; yet this is by a sort of adventitious or glaring Light, that is caused by the Refraction of their Rays in the Air; which makes them appear to us much bigger than really they are; as is manifest, when we come to look at them with a Telescope that takes off this glaring Light.

(2.) The want therefore of Telescopic Sights is what Mr. *Hook* chiefly insists upon, as defective in *Hevelius* costly Astronomical Apparatus: But yet, in his whole Book of *Animadversions*, he takes no notice of the chief Objections, which *Hevelius* uses against them. And I am perswaded the Candour of that Noble Astronomer (whose Memory must now be sacred) was so great, that upon the removal of these Difficulties, he would have given up the Cause; for it seems the Controversie was long agitated between them.

These Objections we shall find in the First Part of the *Machina Cœlestis*, Cap. XIV. pag. 296. *Accedit, si quando Observator non æquè directè & præcisè semper, ut sæpius, crede, contingeret,*

Hevelius
Object: 2. 13
against
them.

tingeret, per Centra lentium collineat, facile diversitas aliqua aspectus observationibus possit induci, quæ suo tempore Cæli scrutatores jugiter seduceret. Caterum, cum Acus vel Fila adeo prope lentem Ocularem ad observatoris oculum vix in remotione aliquot digitorum subsistunt; dubito an Dioptra hæc oculo tam propinqua, multo accuratius Stellas quasvis minimas, quam Pinnacidia nostra, ad sex novemve pedes ab invicem remota, possit detegere. Nam etiamsi objectum distinctius videas; in eo tamen, quod Dioptra tua oculo propius adheret, plus à vero deflectere poteris, quam nos circa nostra Pinnacidia, quæ tanto spatio ab invicem removentur. Ut taceam, quod intersectio filorum minimas Stellas tibi tegat, &c.

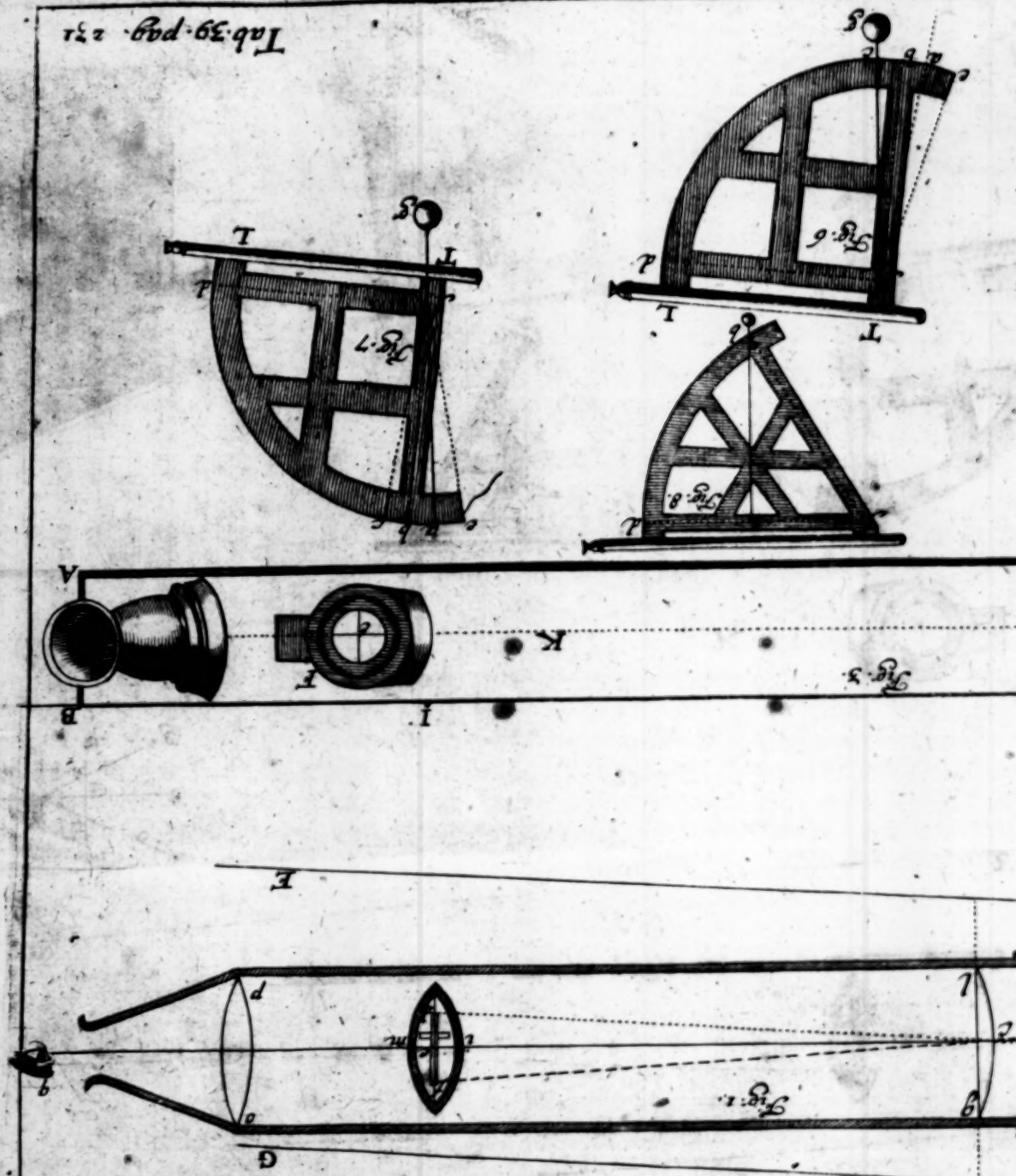
For English Readers thus,

“Add to this, That if at any time the Observator chances
 “not to look directly and precisely through the midst of the
 “Glasses (as believe me it may often happen) some Varieties
 “may easily intermingle with the Observations, which in time
 “may egregiously deceive the Astronomer. Moreover, seeing
 “the Needle or cross Threads, do stand so close to the Eye-
 “Glass, and near the Eye of the Observator; I question whether these Sights, so near the Eye, can discover the smallest
 “Stars much more accurately, than our plain Sights, which
 “are distant from each other Six or Nine Feet. For, tho by
 “these Telescopick Sights, one may see the Object more distinctly; yet because they are so nigh to the Eye, one may
 “err, more than tis possible by our plain Sights, that are so
 “far asunder; so that I shall take no farther notice of another
 “Inconvenience, which is, that the Intersection of the Threads
 “shall cover the smallest Stars from your Sight.

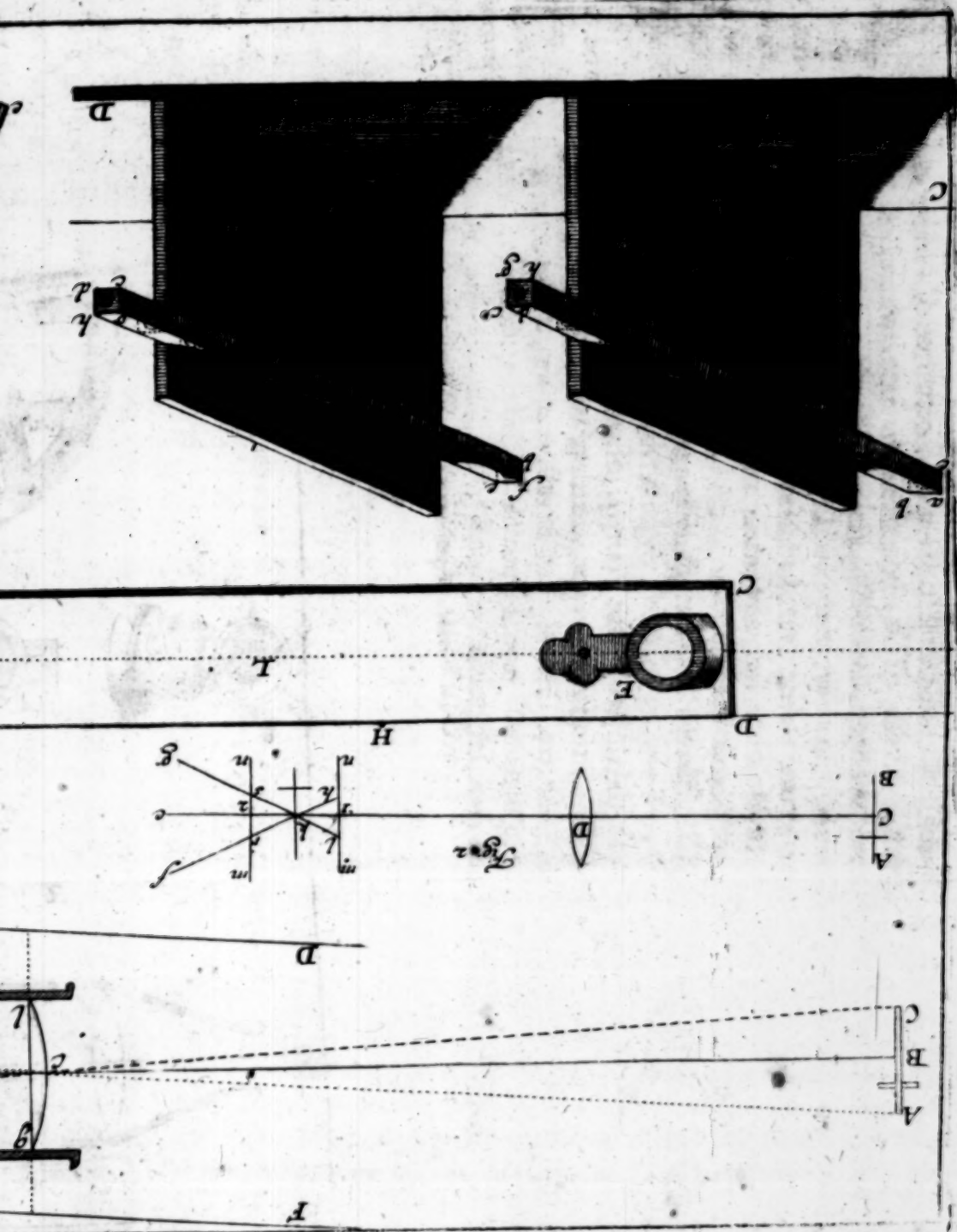
His Mistake concerning them.

Thus far the Learned *Hevelius*. Which shews plainly, that he had no right apprehension of the Nature of these Sights. And therefore the best way of reconciling him to them, had been, fairly to have laid down the *Dioptrical Reasons* of their Performance and Exactness. Upon a right understanding whereof,

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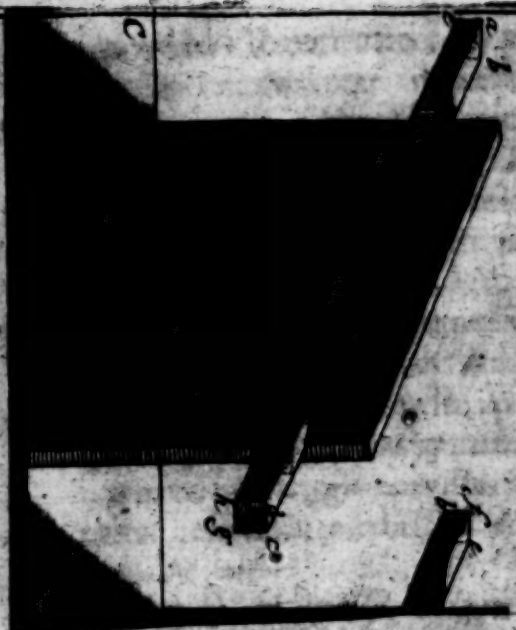
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A
C
B

D

Fig. 2



whereof, all those Objections would be answered, and would naturally vanish. This had been the right Method of proceeding amongst *Candid Philosophers*: Whilst vilifying his Instruments, and slighting his Performances with them as no better than those in the Age before him, did but exasperate the Noble old Man, and made him adhere more obstinately to his former Practice.

That *Hevelius* did not rightly apprehend the Nature of *Telescopick Sights*, is manifest by this Objection which he makes against them, from the *shortness* of the *Line of Collimation*, which he imagins no longer, than between the Eye or Eye-Glass, and cross Hairs; but is really as long as between the Object-Glass and cross Hairs: As shall be evident from what I shall now lay down.

Wherein I shall briefly explain their usual Fabrick or Contrivance; their adjusting to a plain Ruler, Cylindrick, or square Tube; their adjusting to Quadrants, Sextants, and other Instruments; and the Dioptrick Reason of their Performance and Exactness.

(3.) And first, the Fabrick or contrivance of these Telescopick Sights is briefly thus, *Tab. 39. Fig. 1.* Choosing an Object-glass *gcl* and convex-Eye-glass *op* proper for the length of the Ruler or Tube *gopl* which we are to use. Let us take care that the Object-glass be pretty well centred (by Chap. 4. Sect. 4. of this part) but in this particular the greatest exactness is not requisite (whatever *Pere. Cherubin d'Orleans* may say to the contrary, in the second Tome of his *Vision Parfait*. Paris 1681. fol. in his description of Levels. pag. 23, 81, 106, 108, 109. &c. but chiefly pag. 107. wherein the Friar is most grossly mistaken) 'tis sufficient, if the Glass be pretty nigh the matter, as usually most Glasses are, immediately out of the Workmans hands. This Object-glass and Eye-glass are each to be fixed strongly on a brass Ring in the Ruler at their proper distance.

And

Their Fabrick or Contrivance.
T. 39. F. 1.

True Centration of Glasses not absolutely requisite.

And exactly in the Focus of the Object-glass $f m d i$ in an other brass Ring are strain'd the finest cross-Hairs $f d, i m$; This Ring is contrived to be moveable to the Right and Left hand, towards f , or towards d , and also (if the nature of the Instrument, to which we affix these Sights, require it) *upwards* and *downwards*; and to be steadily fixed in any posture by screws or otherwise. The Mechanick contrivance where of is obvious enough, and needs not here be described, every one pleasing himself in his own way.

By the Doctrine in the first part 'tis manifest, that this Tube being thus disposed and presented before a Distant Object $A B C$, the Picture or Image of the Object is projected in the Focus of the Object-glass $f e d$; and this Image in the distinct Base is at the plain of the Cross-hairs. Wherefore all the Rays that compose this Image, which escape, or do not fall on, the Cross-hairs, shall arrive at the Eye freely, and distinctly. But the Points in the Image, which are projected, and fall just on the Cross-hairs, are hid by the Cross-hairs from the Eye: and the Hairs themselves appear as if they were *really* stretch'd upon the *very Object*. For they are extended in the distinct Base, which, in this kind of Telescope, is the *Locus apparens* of the Object, by *prop. 50. schol.* At the same time the Hairs themselves appear very distinctly to the Eye q in the outward Focus of the Eye-glass by *prop. 32, 33. schol.*

We may then conceive, that there is *some one* Point in the Object, as suppose B , which sending a Cone of Rays on the Object-glass, the Principal Ray of this Cone, or the Axis thereof, after passing the Object-glass, runs parallel in $c e$ to the side of the Ruler or Tube $l p$. Wherefore, if the intersection of the Cross-hairs, e , without removing or stirring the Object-glass in the Tube, be brought to meet with this Line, or to cover this Point B in the Object, and be there strongly fixed: Whatever Point, in any Object, shall hereafter be found covered by this crossing of the
Hairs;

Hairs; we may be assured that this line of Collimation, viz. the line from the Eye to this point in the Object, or the line from this Intersection e to the point in the Object, runs parallel to the side of the Ruler or Tube, and therefore the side of the Ruler is directly pointed towards that mark in the Object. Respect being had to the Breadth of the Ruler.

In like manner, supposing the sides of the Tube were in the lines DE, FG ; and the Object-glass fixed upon it in the posture expressed in the Figure. We may conceive some point A in the Object, the Axis of whose Cone of Rays $Ac d$ after passing the Object-glass, runs parallel to the sides DE, FG . If then the Intersection of the Cross-hairs e , without stirring the Object-glass in the Tube, be brought downwards to d , so that it may meet with the line cd , and cover the point A in the Object, and the Ring of the Cross-hairs be there fixed. Whatever point in an Object shall hereafter be found covered by this crossing of the Hairs, the line from the Intersection of the Hairs to this point in the Object runs parallel to the sides of the Tube or Ruler DE, FG . And therefore we may be sure, the sides of the Ruler are directly pointed towards the mark A in the distant Object. Respect being had to the Rulers Breadth.

For from whatever point on the inner surface of the Object-glass (how thick soever it be, or how ill soever centred) the Ray cd emerges; no other Ray can emerge from that same point, and fall on the point d , but it must necessarily be a principal Ray or Axis of some Cone, and must run parallel to the side of the Ruler DE . For whatever Optick Angle the length AB in any Object subtends; that length shall be projected by this Object-glass in de subtending the same Angle dce .

(4) Wherefore we now come to shew how to find out the Ray cd , that runs parallel to the side of the Ruler DE ; or how to rectifie the Cross-hairs on the Ruler.

*Adjusting
them to
plain Ru-
lers or
Tubes.*

H h

And

Exactly
determin-
ing the
Focus of
the Object-
Glass.

And first for, adjusting the Cross-hairs at their *exact distance* from the Object-glass, that is, in its *exact Focus*. This is easily performed thus, Let us look at some Object distant 3 or 4 miles; and moving or shaking our Eye before the Eye-glass upwards and downwards, or to one and t'other hand, let us observe whether the Cross-hairs seem to *move*, or *dance*, upon the said Object: for if it do, then the Cross-hairs are *not* at their *exact distance* from the Object-glass; but they must be moved farther from or nigher to the Object-glass; till the Eye, looking at such a distant Object, and moving before the Eye-glass, perceives the Cross-hairs, as it were, *fixed* and *immoveable* on the Object.

(Note. *This is the way for exactly determining the Focal length of an Object-glass, to which I have referred in Chap. 4. Sec. 3.*)

If in *raising* the Eye, the Object seems to *fall down* on the Cross-hairs, or if in *depressing* the Eye, the Object seems to *rise* on the Cross-hairs, then are the Cross-hairs *too nigh* the Object-glass: but if in *raising* the Eye, the Object seems to *rise* on the Cross-hairs; or in *depressing* the Eye, the Object seems to *sink* or *fall* on the Cross-hairs; then are the Cross-hairs *too far* from the Object-glass. All which will be evident from *Tab. 39. f. 2.* wherein let *A B* be a distant Object, whose middle point *C* is projected by the Object-glass, *D* at *k*. Let *m n 1* be the Cross-hairs *too nigh* the Object-glass and *m n 2* the same *too far* from the Object-glass *e, f, g*, the Eye placed at three different stations. In the case of the first Cross-hairs, if the Eye *rise* from *e* to *f*, it perceives the point *k* *depressed* from *1* to *h*, or if the Eye *fall* from *e* to *g*, it perceives the point *k* *raised* from *1* to *l*: and here the Cross-hairs are *too nigh* the Object-glass. But in case of the second Cross-hairs; if the Eye *rise* from *e* to *f*, the point *k* seems to *rise* on the Cross-hairs from *2* to *r*; or if the Eye *fall* from *e* to *g*, it perceives the point *k* *fallen* from *2* to *s*: and in this case the Cross-hairs are *too far distant* from the Object-glass. But if the Cross-hairs are exactly in the *Focus* at *k*, let the Eye *rise*
or

or fall, the Cross-hairs seem fixed and steady on the Object. And this is the first thing requisite for adjusting these Sights.

This I have borrowed from my own *Sciathericum Telescopicum*, Published at Dublin 1686. quarto. And I hope, one may be allow'd to transcribe from himself without being called a Plagiary.

When this Affair is well adjusted; we may proceed to the second Rectification, which consists in making the *line of sight*, *mire*, or *collimation*, exactly parallel to the Sides of the Tube or Ruler, to which the Telescopic Sights are to be adapted. And for the easier obtaining of this, we are first to be ascertain'd, that even the *two sides* of the Ruler or Tube are exactly parallel. And hereof we may be informed after this manner. *Tab. 39.* T. 39. R. 3.

Fig. 3. On an even board draw the right line D H I B, let A B C D be a Ruler, to which the Telescopic Sights are to be fitted, E the Ring carrying the Object-glass, F the Ring carrying the Cross-hairs, G the Snout carrying the Eye-glass. To the line B D apply the side B D of the Ruler, and looking through the Glasses observe the point in an Object distant a mile or two whereon the Cross-hairs fall. Then remove the Ruler, and apply its other side A C to the line B D, and observe whether the Cross-hairs fall on the same point of the Object, as before. If they do so, then are the sides of the Ruler A C, B D, parallel; if not, then the sides are not parallel. The reason that so remote an Object must be chosen, is, that the Breadth of the Ruler may subtend an imperceptible Angle in a circle whose Radius is the distance of the Object from the Object-glass.

Or otherwise, In the plain Board strike two round brass-wire Pinns. Suppose H, I, which having their roundness from their being drawn, must needs have their sides parallel. To these Pins apply one and t'other side of the Ruler, and observe as before.

Having thus found, or made the *two sides* of the Ruler (or four sides of the Tube, if need be) parallel; the next thing is to make the line of Collimation L K parallel to these sides; or to

H h 2

bring

bring the Intersection of the Cross-hairs *e* to meet with the Axis *LK* of some of the *Radions Pencils*, which Axis, after passing the Object-glass, runs parallel to these Sides.

Tab. 39.
Fig. 4. 5.

To effect this (*Tab. 39. f. 4, 5.*) we are to raise the plain Board *AB* edgewise on the Board of a window or table *CD*, so that we may rest or sustain the Ruler or parallelipiped Tube *ab c d e f g h*, on the round Pins *i, k*: then looking through the hole in the end *g b d c* designed for the Eye, let us observe the point in a far distant Object, which falls exactly on the Intersection of the Cross-hairs. Afterwards, let us invert the Ruler or Tube (as we have it in *fig. 5.*) making the side *a b c d*, which in *fig. 4.* was *uppermost*, now *undermost* in this *fig. 5.* by which means, the side of the Tube *a d h e*, which in *fig. 4.* was *farthest* from the Board, in *fig. 5.* is *next* the Board. Then looking through the Tube, let us observe, whether the Cross-hairs fall now on the same point in the Object, as in the first posture: If they *doe* agree exactly, then is the *line of collimation* parallel to the sides *a b, b g*; if they do *not*, but fall to the *right* hand apparently of the said point in the Object, then are the Cross-hairs to be removed (by whatever Contrivance they are made moveable) to the *Left* Hand (the contrary requiring the contrary), so that the Tube continuing in this latter posture, the cross-Hairs may cover a Point in the Object, middle between the Point covered in the posture of *Fig. 4.* and in the posture of *Fig. 5.* at the first sight. And thus by frequent Repetitions and Tryals, we at last bring all to rights. After the same manner that we have rectified the Line of *Collimation* to run *parallel* to any two parallel Sides of the Tube, we may rectifie it to a *Parallelism* with the other two parallel Sides of the Tube (supposing the Ring that carries the cross-Hairs to have all the Motions requisite to such Rectification). And so we fix all strongly, chiefly the Object-Glass and cross-Hairs; and the Operation is compleat.

By

By this Method the *Line of Sight* of any Cylandrick or square Tube may be made to run parallel to its Sides, for many Operations and Observations Mathematical and Natural: Amongst others, for finding the *Declination* of the *Magnet*, according to the Methods lately proposed by Monsr. *Hautefeville*, and M. *Sturmius* in the *Journal des Scavans*, 23. Aug. 1683. And in the *Acta Eruditorum*, *Lipsie*, Ann. 1684. Decemb. And for want of this Method; what Monsr. *Sturmius* says in the foresaid *Act. Lips.* pag. 579. is very defective. For thus he, *Sola Tubi locatio, ut Axis Visionis per medias Lentes excurrrens Meridiana Linea exacte respondeat, difficultatis quippiam habere videbatur; verum & huic infirmitati præsens, uti credo, inventum est Remedium, &c.* And the Remedy he tells us is, That the Tube be made a *Parallelipiped* of Wood or Brass; for then, says he, Applying the Side of your Tube to the *Meridian Line*, the *Axis of Vision* will be parallel to the said *Meridian Line*. But with the Leave of so Great a Man, I deny this; unless first it be rectified; so that this *Axis* runs parallel to the Side of the Tube. And let us take what care soever possible for truly centring the *Object-Glass*, and placing it, and the *cross-Hairs* exactly in the Tube; we must after all rectifie these *Sights* by some such Method as I have laid down, or else we may be egregiously deceived. And on this account, all the *Levels* and *In-* Sturmius
Mistake. *struments*, to which *Pere Cherubin D'Orleans* has adapted *Tele-* Cherubins
Gro's Er-
ror. *scopick-Sights*, and which he has so neatly and sumptuously described by curious Schemes, and a large Volume, *La Vision Parfait. Tome II. A Paris 1681. Fol.* are deficient and useles. For he places the whole *Rectification* of this *Line of Collimation* in the true *Centration* of the *Glasses*, pag. 107. And rejects the Right *Rectification* by moving of the *cross-Hairs* as erroneous, pag. 107. But in this the *Friar* betrays his Ignorance; for tho the true *Centration* of the *Object-Glass* be of good *Con-* 2 *venience* and *Advantage*; yet it does not perfect the Instrument without

without farther Rectification; as being impossible to be obtained to sufficient accuracy; and therefore we must have recourse to some such Method as foregoes.

I have hitherto mentioned only two of the finest cross-Hairs to be extended as a Mensurator in the Focus of the Object-Glass: But for some Uses, perhaps the *finest Silver*, or *Gold Wire*, is better; as not being disordered by *Heat* and *Cold*: Or else, the *Point* of the smallest and most curious *Needle*, on whose Extremity, the smallest *Telescopick-Star* may be visible.

See the
end of this
Chapter.

Constancy
and Security
of
these Tele-
scopick-
Sights.

When these *Telescopick-Sights* are rightly adjusted in the *Tube*, and strongly fixt in their due Posture by *Screws*, and all covered over from outward Injuries and Accidents, they are of all *Sights* the most constant and lasting, and the least subject to be disordered: So that, when one finds the *Great Hevelius* objecting against them, their *Aptness to be out of order*, one would think the most commodious *Fabrick* of them was never explained to him; tho I am sure, his Instructor *Mr. Hook* was as able, as any in the *World*, to inform him rightly in this *Matter*.

To Qua-
drants,
Sextants,
&c.

Tab. 39.
Fig. 6, 7.

(5.) I come now to the Rectification of these *Sights* on *Quadrants* and *Sextants*, for taking *Angles*. This is done either *before* or *after* the *Divisions* into *Degrees*, &c. are made on the *Limb* of the *Quadrant*. If it be done *before*, then we suppose the *Telescope T L* (*Tab. 39. Fig. 6, 7.*) fixt to the *Quadrant*, which we suppose continued a little farther than the *Fourth* part of a *Circle*. Choosing then an *Object* pretty near the *Horizon*; let us look through the *Telescope*, in the usual *Posture* of *Observation*, as *Fig. 6.* and observe the *Point* in the *Object* marked by the cross-Hairs; and at the same time we are to note most nicely the *Point c*, which the *Plumb-Line f c g*, hung from the *Centre f* of the *Quadrant*, cuts on the *Limb*. Then we are to invert the *Quadrant* into the *Posture* of *Fig. 7.* (which is easily done by the usual *Contrivances*

vances for managing great Quadrants, by tooth'd Semicircles and endless Screws) keeping still the Telescope T L nighly upon the same height from the Ground, as before) unless the Object we look at, be so far distant, that the Breadth of the Quadrant subtends but an insensible Angle. But yet for certainty, 'tis better to keep the Telescope, as 'tis said, upon the same height from the Floor); then direct the Telescope T L, that the cross-Hairs may cover exactly the same Point in the Object, as before in the Posture of Fig. 6. And hanging now the Plumb-Line afg on the Limb of the Quadrant; let us remove it *to* and *fro*, till we find out the exact Point a , from which the Plumb-Line being hung, shall most nicely hang over the Centre of the Quadrant f . Then carefully marking the Point a , let us divide the Arch ca into two equal Parts in b ; and drawing bf , the Point b is the Point from which we are to begin the Divisions of the Quadrant: And the Line of Collimation through the Telescopic-Sight, stands exactly at Right Angles to the Line bf . So that the Quadrant bfd being compleated and divided, the said Line of Sight through the Telescope runs exquisitely parallel to the Line fd .

In the next place, supposing the Quadrant bfd truly compleated and divided; and that we designed to fix thereto the Telescopic-Sight T L; so that the Line of Sight may run exactly at Right Angles to the Line bf , or parallel to the Line df . We are to do as in the foregoing Praxis. And if in dividing the Arch ac , we find its half exactly coincident with the Point b , we have our desire. But if it differ from the Point b , and fall *between* b and d , then the Line of Collimation through the Telescope stands at an *obtuse* Angle with the Line bf ; and the Instrument errs in *excess*: If this half Arch fall *without* b and d , then the Line of Collimation makes an *acute* Angle with the Line bf ; and the Instrument errs in *defect*.

defect. And by often Tryals, we are to remove the cross-Hairs within the Tube, so much, as is requisite to correct this Error. And when we have thus rectified them to their due place, there they are to be strongly fixt. Or else, in Observations taken by this Instrument, we are to *make allowance* for this Error; by *subtracting from* (if it be in *excess*) or by *adding to* (if it be in *defect*) each Observation so much, as we find the Error to be.

The reason of this Rectification is most plain; for 'tis manifest, that *cf d* (Fig. 6.) *wants* of a full Quadrant, as much as *af d* (Fig. 7.) *exceeds* a Quadrant. So the difference of the two Arches in the two Postures being *ac*; half this difference *bc* added in Fig. 6. or *ab* subtracted in Fig. 7. makes *bd* a compleat Quadrant.

If we find our Instrument *err* in taking Angles, and we desire to know the Error *more nicely*, than perhaps the Divisions of the Instrument it self will shew it: We are to do thus; Let us observe diligently the Object pointed at, in the Posture the Instrument discovers its Error; and the Object pointed at when the Instrument lies *truly*. Then, with a large Telescope and Micrometer (as is used in taking the Planets Diameters, as shall be declared hereafter, Sec. 7.): Let us take the Angle subtended at the Object-Glass of the Quadrants Telescope by the length between these two Objects, and we obtain the Error of our Instrument *most nicely*. Thus for Example; Supposing the Quadrant *bfd* already accurately divided, and that the Plumb-Line, Fig. 6. plays over the Point *c*: And upon the Inversion of the Instrument, Fig. 7. we find that before we can get it to play exactly over the Centre *f*, we must hang it over the Point *e*; so that the Arch *eb* exceeds *bc* by the Arch *ea*; 'tis plain that the Angle *efa* is the Error of the Instrument: For had the Plumb-Line hung over *a*, and over the Centre *f* in this latter Posture, the Instrument

strument had been *exact*; because *a* is as much on one side *b*, as *c* is on t'other side *b*. Wherefore *ef a* being the Angle, by which our Instrument *errs* in observation: Let us turn the Instrument into the usual Posture of Observation, as in *Fig. 6.* and hanging the Plumb-Line on the Centre *f*; let us bring it to play nicely on the Point *e*, and observe what distant Object is covered by the cross-Hairs: Then let us bring it to play exactly on the Point *a*, and observe likewise what distant Object is pointed at by the Telescope Hairs. Lastly, by a large Telescope and Micrometer, let us measure the Angle between these *two Objects*, and we shall have the Angle of Error much more nicely, than 'tis possible the Angle *ef a* should be given by the Divisions on the Limb of the Quadrant *e a*. And thus much for adjusting a Quadrant.

A Sextant is rectifi'd in like manner; If we consider (Tab. ^{T 39 FS.} Rectifica-
39. *Fig. 8.*) that if from the Centre *f* to the beginning of the Divisions *d* there be drawn the Radius *fd*; and it be divided equally in *c*; and from *c* there be suspended the Plumb-Line *cb*:—When the Plumb-Line hangs over the 60th Degree at *b*; then the Line *fd* lies horizontal: And consequently, if the Line of Collimation through the Tube be parallel to *fd*, this Line also lies horizontal. To try which, Whilst the Sextant stands in this Posture, observe the Object marked by the cross-Hairs; then invert the Sextant; and over the Point *b* hang the Plumb-Line; and when from the Point *b* the Plumb-Line hangs over the middle Point *c*, then again is the Line *fd* horizontal in this Posture. Mark then, whether the cross-Hairs cover the same Object as before: If they do, then the Line of Collimation is parallel to *fd*: If they do not; but the Point in the Object marked in this latter Posture be *higher* than the Point marked in the first Posture, the Instrument *errs* in *excess*; if it be *lower*, the Instrument *errs* in *defect*. And either we are to remove the cross-Hairs, till we

bring all to rights, and there fix them : Or by the Methods before laid down in the *Rectification* of the *Quadrant*, we are to find the Quantity of this erroneous Angle, and to allow for it in Observation.

*Rectifica-
tion of a
moveable
Sight.*

In Instruments furnished with two pair of Telescopic-Sights, one on a *fixt* Arm, and t'other on a *moveable* Arm (by the Ancients termed an *Alidade*) ; 'tis easie rectifying the *Sights* on the *moveable* Arm thus : After the *Sights* on the *fixt* Arm are rectifi'd by what foregoes ; bring the Index of the *moveable* Arm to the beginning of the Divisions on the Limb of the Instrument, be it *Quadrant* or *Sextant*, &c. 'tis then manifest, that the *Line of Collimation* through the *moveable* Telescope (if it be right) should lye parallel to the *Line of Collimation* through the *fixt* Telescope. Observe therefore, whether the cross-Hairs in *both* Telescopes do at the *same time* cut the *same* Star, or fall on the *same* Point in an Object distant three or four Miles. If they do, then the *moveable* Telescope agreeing with the *fixt*, and the *fixt* being supposed rectifi'd to the Divisions on the Instrument, the *moveable* is right likewise. But if the Hairs in the *moveable* Telescope do not agree in marking the *same* Point with the cross-Hairs in the *fixt* Telescope ; then the Hairs in this *moveable* Telescope are to be removed (by whatever Contrivance there is for that purpose) and brought to rights, and there fixt.

There are other Methods propounded for rectifying Telescopic-Sights on other sorts of Instruments, by means of Observations towards the *Zenith*, as our former Methods have been employed towards the *Horizon*. But 'tis sufficient here to lay down only what foregoes, as being of the greatest and most frequent use : Referring for the others to M. Picard's Treatise of the *Measure of a Degree of a great Circle of the Earth* ; publish'd at the end of *Memoirs for a Natural History of Animals*, &c. By the *Academy Royal at Paris* ; lately translated into English, and printed at *London*, 1688. Fol. Before

Before I quit this Point, it may not be amiss to intimate ^{Further Use of a} one Use, to which a plain Ruler furnished with Telescopick-Sights (such as is expressed *Tab. 39. Fig. 3.*) may be apply'd ^{Telescopick-Ruler} and that is, not only for trying the exquisite *straitness* of either of its *own* Edges, and *parallelism* of its *own* two Sides; but also, for the ready Tryal of the same in any other Ruler: For 'tis but affixing (by a little Cement, or otherwise) this *Telescopick-Ruler* over the Ruler to be try'd, and resting the Edge of this latter against the Pins H, I, and gently sliding the Edge alongst these Pins, and always touching them; looking all the while through the Telescope, observe whether the cross-Hairs do steadily adhere to the same Point in an Object: For if the Edge of the Ruler have the least irregular Crookedness, the cross-Hairs will move from the Point first observed. And this shall detect the least Curvity in the Edge of a Ruler (especially if the Ruler be long, and the Distance of the Pins be considerable) that shall escape the most exquisite Eye of a Workman. The way of trying the Parallelism of the two Sides of this latter Ruler, is the same with what foregoes for the Telescopick-Ruler it self: For when the Telescopick-Ruler is adjoyned over the other, they may both be taken but as one Ruler with Telescope-Sights affix'd.

(6.) I come now to the last thing propos'd concerning ^{Dioptrick-Reason of their Performances} Telescopick-Sights; and that is, To shew the *Dioptrick-Reason* of their Performance and Exactness. But herein there will be little requisite to be added to what foregoes, both in the First Part concerning Telescopes in general, and to what is laid down in this Chapter concerning Telescopick-Sights. 'Tis manifest by Experiments, that the ordinary Power of Man's Eye extends no farther than perceiving what subtends an Angle of about a Minute, or something less. But when an Eye is arm'd with a Telescope, it may discern an Angle

less than a Second. The Telescope that magnifies distinctly the Appearance of *Body*, magnifies also distinctly the Appearance of *Extension*, *Space*, and *Motion* through this Space; so if the Minute-Hand of a Watch, which can but just be perceived to move, be looked upon with a Magnifying-Glass, we shall see it give a considerable Leap at every Stroak of the Balance. And thus likewise the slow diurnal Motion of the Sun or Stars, which is hardly perceivable by the bare Eye, unless assisted by an Instrument of a vast Radius, is most easily perceived through an ordinary Telescope of 18 Inches long: Insomuch that we may determine to the greatest Niceity and Exactness, when a Star passes just over the cross-Hairs, even to the single Beat of a Second-Pendulum. And let an Object in the Heavens rise never so little, the Image in the Distinct-Base falls correspondently at the cross-Hairs; and the Eye, by means of the Eye-Glass, perceives this Motion, be it never so small. Thus suppose (*Tab. 39. Fig. 11*) that a Star rise from B to A, the Image falls at the cross-Hairs in the Distinct Base from *e* to *d*; then by means of the Eye-Glass *p*, the Space *ed* is mightily magnified, and consequently the Angle B *e* A, equal to *e c d* by which the Star is risen, is made most sensible to the Eye *q*. By whatforegoes in the First Part concerning the magnifying of this sort of Telescope.

Hevelius
Mistake
farther
manifest.

By this we may perceive, how the Noble *Hevelius* was mistaken in his Estimate of these Sights; when he imagin'd the Line of Collimation therein was no longer than between the cross-Hairs and Eye-Glass: Whereas this Distance is not at all to be consider'd in their Performance; the Line of Collimation being full as long as the Distance between the Object-Glass and cross-Hairs. I am perswaded, had he been rectifi'd in this particular, he would never have adhered so obstinately to the Use of plain Sights upon his most costly Instruments. Tho I must confess, 'tis difficult to wean a Man from the Use

Use of what he has been accustomed to for so many Years; and upon the Exactness of which, the Accuracy of all his former Labors did depend.

As to all other Objections which he makes against them, as that they are easily disordered, that the Glasses are easily vitiated by the Breath of the Observer, &c. They are not of any the least moment. For 'tis manifest, they may be contrived so, as to be *more secure*, and *less* subject to Injuries, than any other plain Sights whatsoever: And in this particular, Telescope-Sights are so far from being obnoxious, that certainly they are preferable to the best contrived plain Sights; for what can be more simple and easier preserved, than the forementioned small (but strong) Brass Rings defended by a Tin or Brass Tube covering all? When once these are adjusted and fixt, nothing can possibly injure them. 'Tis true, the Breath of the Observer, if puft into the Telescope, will fully the Eye-Glass; but how easily is this avoided? Who is it goes purposely to make a *speaking-Trumpet* of a Telescope? The other most considerable Objection against their Use is, That in dark Nights, at the smaller Stars, the cross-Hairs in the Telescope require a little *enlightening*, or else they are *invisible*, and cannot be seen when the Star just applies to them. This is so easily remedi'd, by admitting to them, through a small opening purposely left in the side of the Tube, the least glimmering Light of a Lanthorn; or by placing a Lanthorn a little aside before the Object-Glass; that 'tis not worth mentioning as a Difficulty, much less is it to be made an Argument for their utter rejection. As to what he says of the Hairs being so gross as to cover the smaller Stars, this only relates to the Material we employ; and the finest Silk-Worms Clue will be found small enough almost to bisect the smallest Stars: If not, let us use the *finest Needle*, on whose slender Point we may distinctly receive the most minute Star.

And

Further
Uses of
Tele-
scopick
Sights.

And thus much concerning *Telescopick-Sights*; from whose application to Mathematick-Instruments, Astronomy, and Geography may expect their utmost Advancements. And even Natural Philosophy it self may hereby receive the greatest Help, when we consider how Telescopes may be apply'd to many Experiments therein; amongst others, to make the most nice *Hygroscope*; and has already been used for accurately determining the capricious *Variations* of the *Magnet*. *Telescopick-Sights* have been already successfully apply'd to most exquisite Levels; wherein *Monf. Picard* in his Curious Treatise *Du Nivellement* has prevented any farther Explication: And I doubt not, but every Day will find new Uses for these Sights. Amongst others, I'll presume to mention my own *Telescopick-Dial* already publish'd, *Anno 1686*: A Contrivance, which, without Vanity I may say, has not displeased at *Home*, and has been well received *Abroad*.

Adapting
the Mi-
crometer
to a Tele-
scope.

(7.) The next *Telescopick Instrument* which I shall explain, is the *Micrometer*. Concerning the *Invention* of this Ingenious Instrument, I have only this to say, That for the Honour thereof, there are several Competitors: *Monf. Petit*, Surveyor of the Fortifications in *France*, was the first that publish'd to the World the rough Draught hereof, 12. Mar. 1667. *Vid. Journal des Sçavans*, 16. May 1667. After him *Monf. Azout*, another Ingenious *Frenchman*, publish'd a Tract concerning the exact Mensuration of the Planets Diameters, wherein he seems to challenge the Invention of this Instrument to himself and *Monf. Picard*, *Journ. des Sçavans*, 28. Jun. 1667. and *Philosoph. Transact. Num. 21. pag. 373*. But last of all a Candid *Englishman* of our own, *Mr. Rich. Townley*, does vindicate the first Contrivance hereof to its true and original Author, *Mr. Gascoigne* an *English Gentleman*, who was kill'd in King *Charles I.* Service, *Vid. Philosoph. Transact. Num. 25. pag. 457*. wherein *Mr. Townley* (who is of undoubted Credit) asserts, that

Inventor
of the Mi-
crometer.

that Mr. *Gascoigne* made and used this Instrument before the Civil Wars in *England*: And that Mr. *Townley* had then in his Custody two or three of these Instruments first devised by Mr. *Gascoigne*; to which Mr. *Townley* himself had added some considerable Improvements. All which, with the exact Fa-
brick, and fitting of the Body of the Instrument to a Tele-
scope, we shall find accurately described in *Num. 29. p. 541. Philosoph. Transact.* to which I shall therefore refer the Reader; and shall hint only such things concerning it in this place; as may be *there wanting* for the clearer Instruction of the un-
exercised Beginner.

First therefore for a brief Description thereof (as much as is requisite to maintain the order of our Discourse); 'tis in short this. In the Focus of the Object-Glass of a Telescope, there are placed two fine parallel Hairs, or smooth Edges of Brass Plates; these are made by Screws to open or close at pleasure, as wide as the Telescope admits. The Turns of these Screws are reckon'd out by proper *Indices*; so that in opening the Edges of the Micrometer, the Indices do shew, how many *Revolutions* of the Screws, and Parts of a *Revolution* are compleated in that Opening. Suppose therefore the Screws to be of so fine a Thread, as to contain 30. Threads in an Inch length; then every Revolution of the Screw opens or closes the Edges of the Micrometer a thirtieth part of an Inch. By one Revolution of the Screw, the *Index* receives one Revolution: Then, the Circumference of the Plate, over which the *Index* moves (as the Hand of a Watch over the Hour-Plate) being divided into 100 Parts; when the *Index* moves one of these Parts, the Screw moves the Edges a three thousandth part of an Inch; (or the one thirty six thousandth part of a Foot; by which we find how easie 'tis to divide a Foot into thirty six or forty thousand Parts) And this Motion, tho every Minute, is made, by the Eye-Glass of the Telescope, perceivable.

The

The way of taking small Angles by this Instrument is thus; Suppose it were the Diameter of the Moon; Open the Micrometer till the two Edges do just clasp or touch the Moons Edges; then observe by the Indices how many Revolutions and parts of a Revolution were compleated to this opening; and by a proper Table (the way of composing which I shall shew presently), convert these Revolutions and Parts into Minutes and Seconds. In like manner, for observing small Angles on the Earth, the Diameters of the other Planets, the Distances of *Jupiter's* *Satellites* from his Body, or the Moons Spots, &c.

But now for making the Table, First we are to fix the Micrometer exactly in the *Focus* of the Object-Glass (by the Rules before given, *Chap. 5. Sect. 4.*) if it be at very distant Objects we design to use it: Or otherwise in the *respective Focus*, if it be designed for nigh Objects. We may then compose the Table two manner of ways: The first is more easie, tho not so very certain and accurate, yet exact enough for most Uses. Measure by Inches and Decimal Parts the Distance between the Object-Glass and Micrometer, taking into the Account two Thirds of the Object-Glass's Thickness: Let us suppose the Distance 10 Foot, or 120 Inches, or 120000 Parts; and we desire to know what Angle is shewn by the Micrometer, being open 2 Inches, or 2000 Parts.

T. 25. F. 4. The Computation is plain (*Tab. 35. Fig. 4.*) $ey = 120000$, $fd = 2000$, then $ed = ef = 1000$. And As ey : To Rad.: So ed : To Tang. $\angle eyd = \angle eyf = 0^\circ 28' 38''$, and therefore $\angle fyd$ is equal to $0^\circ 57' 16''$. Then finding by accurate Admeasurement, how many Revolutions of the Screws or *Index*, are requisite to open the Edges 2 Inches; the same compleats the Angle $0^\circ 57' 16''$. Suppose therefore 60 Revolutions open the Micrometer 2 Inches; then 60 Revolutions shew, that the Object, that just appears through the Edges.

Edges at that opening, subtends an Angle of $0^{\circ} 57' 16''$. Then 30 (*viz.* half 60) Revolutions give $28^{\circ} 38''$, *viz.* half $57' 16''$. And one Revolution gives an Angle of $57' 16''$, and the hundredth part of a Revolution gives $34''$. And thus the Table is compos'd to any Number of Parts and Revolutions requisite. But this Way, depending on the exact Admeasurement of the Distance of the Micrometer's Edges (which can hardly be obtained to sufficient Accuracy, unless we know most nicely what Number of Threads in the Screw there were in an Inch length; for then we know what Number of Revolutions compleat an Inch), 'tis not so accurate as what follows, which is,

The second way for composing the Table, is this: Having fix'd the Micrometer at its due Distance from the Object-Glass; on the side of a Wall or House far distant mark out two conspicuous Objects, that may both at a time be received into the Telescope: Measure nicely the distance of these Objects from each other; and also the distance of either of them from the Object-Glass (which we suppose directly before the Point in the Wall middle between the two Objects). And by Trigonometry calculate the Angle, which the distance between these two Objects subtends before the Object-Glass. Then looking through the Telescope, open the Micrometer, till the two Edges thereof exactly meet with or embrace these two Objects; and observe, how many Revolutions and parts of a Revolution are performed in this opening; for so many compleat the Angle before calculated. And having the Revolutions and Parts that compleat any one Angle, we may easily find all the rest, as aforelaid. For in these small Angles, the Angles and Revolutions are proportional; that is, if a certain Number of Revolutions give a certain Angle; half this Number gives half this Angle; and the hundredth part of this Number gives the hundredth part of the Angle, &c.

In the first Method that I propos'd for adapting the Micrometer, and composing the Table, I have allowed for the Object-Glasses Thickness in measuring its distance from the Micrometer. But this Nicety is hardly requisite; unless it be in short Tubes. For at the Radius of 10 Foot, 1 Inch is the Tangent of $28^{\circ} 38''$; and at the Radius of 10 foot $\frac{1}{10}$ of an Inch, 1 Inch is the Tangent of $28^{\circ} 37''$; so there is but *one Second* difference; tho we should err one tenth of an Inch in admeasuring the distance between the Object-Glass and Micrometer.

Other Te-
lescopic-
Instru-
ments.

I might now mention the Application of a *Lattice of fine Hairs* in the Focus of the Object-Glass of a Telescope, as an help to draw distant Objects in *Perspective*: And of applying there a pretty contriv'd Parallelogram for the same purpose. But the first is obvious enough by the least intimation thereof; and the latter is so amply described by *Pere Cherubin d'Orleans* in his *Dioptrique Oculaire*; that 'tis needless to add any thing farther in this place.

Instead
of cross-
Hairs.

I conclude this Chapter with a brief hint of what I have found very commodious for many purposes; that is; instead of the forementioned cross-Hairs, I have often used a curious piece of clear, thin, flat Glass, whereon there are drawn two very fine cross-Lines by the curious Point of a Diamond, smaller than the most fine Wyre or Hair; not easily disturbed by a sleight Touch (unless we break the Glass), nor alterable by Heat and Cold. Thus also may we make a Lattice.

CHAP. VI.

Of the Invention, Discoveries made by, and other Applications, of Optick-Glasses.

- (1.) Optick-Glasses unknown to the Ancients. (2.) Pretended Passage in Plautus. (3.) An other Passage in Pliny. (4.) Probably invented about 1300. (5.) Friar Bacon's Pretence. (6.) Inventers of the Telescope. (7.) Optick-Glasses long known before the Telescope. Remark thereon. (8.) Celestial Discoveries by the Telescope. (9.) In the fixt Stars. (10.) In Saturn. Examination of Gallies Hypothesis. (11.) In Jupiter. Motion of his Satellits diligently prosecuted by Cassini and Flamsteed. Satellits all disappearing. (12.) Reflection on the Motions of Saturn's and Jupiter's Satellits. (13.) In Mars. (14.) In the Sun. (15.) In Venus and Mercury. Hence the Falsity of the Ptolemaick Hypothesis. (16.) In the Moon. (17.) Planets whether inhabited. (18.) Telescopes Use on Earth. (19.) Uses of the Celestial Discoveries of the Telescope. (20.) Microscopick Discoveries and Writers. (21.) Viewing nigh Objects with a Telescope. Use thereof in Miniature-Painting. (22.) Measuring Distances at one Station by the Telescope.

(1.) **T**HAT the Ancients had no knowledg of Optick-Glasses, ^{Optick-Glasses unknown to the Ancients:} is most evident from their universal silence in this Matter: Their most learned and inquisitive Philosophers making no mention, or the least hint thereof, in their Writings. And doubtless a Contrivance of that universal Use, beneficial to all old Men, both in Reading and Writing, could never have been so concealed, as that not the least Footsteps

thereof should remain to Posterity. The only Reliefs they had for their decayed Sights were certain *Collyria* or *Eye-Salves*; and when these fail'd them, they were left almost in the dark for *minute* and *close* Objects.

We hear indeed mighty Stories of *Archimedes* burning the Ships of *Marcellus*, at a great distance from the Walls of *Syracuse*. But whether the Matter of Fact be *true* or *false* (as I am very inclinable to believe it *false*), yet there is no mention of his performing this admirable Effect by *Optick-Glasses*. Perhaps, if there were any such thing done at all; it was performed by *Concave Speculums*: And no one denies the Ancients the knowledg of *Catoptricks*. For *Archimedes* himself writ a Book (as 'tis said) *De Speculis Ustoriiis Parabolicis*; but it has never yet seen the Light.

And yet there are in the World a sort of Men, so devoted to the past Ages, that they will not allow any Improvements of Arts in the modern Generation, unknown to the Ages some Centuries before us. Of this Class was he, (whoever he was) that, rather than the Ancients should be ignorant of *Optick-Glasses*, would forge a Passage in *Plautus* (which really is not at all to be found in him), for Confirmation of his Opinion.

Pretend-
ed Pas-
sage in
Plautus.

(2.) *Pancirollus* (who surely was too candid a Person to be the first Author of this Fiction) in the Second Book *De Rebus Inventis*, Tit. 15. quotes this Passage from *Plautus*, *Cedò Vitrum, necesse est Conspicilio uti*: Which, says he, cannot possibly be meant of any other thing but of the Glasses which we call *Spectacles*. And his Commentator *Salmuth* takes some pains to cite *Christianus Becmannus* (I suppose in his *Oratio de Barbarie & Superstitione superiorum Temporum*) for clearing this Passage of *Plautus*: But yet he is so hard pressed with it, that by no Art, but by main strength he breaks through it, and says, That notwithstanding that Passage, yet certainly *Optick-Glasses* are a modern Invention.

Whereas

Whereas, had he been aware, that that Quotation from *Plautus* is a mere *Fiction*; and that no such Passage can be found in all his Writings; he might easily have avoided its Force, without all that stir. For so we shall find it answered in the *Lettere Memorabili del Abbate Michele Giustiani Parte Terza, Let. 16.* Passage out of Plautus forged.

(3.) Another place cited for the Antiquity of *Optick-Glasses*, is that of *Pliny, Lib. 7. Cap. 53. Hist. Nat.* wherein we find the word *Specillum*. To this Passage we have this Answer in the forementioned Letters of *Giustiani*; that *Specillum* cannot possibly be here meant of a *Spectacle-Glass*, seeing we find the Expression, *Inungit Specillum*; which, says he, cannot be understood of *Spectacles*, which we rather *wipe and cleanse*, than *anoint and grease*. But this Construction of the Learned Authors is much *forced and unnatural*: For the plain sense of that Passage in *Pliny* is this. *Pliny* in that Chapter is giving Instances of the *sudden Deaths* of many Men; and telling how they were seized, whilst they were doing *so or so*, and wholly thoughtless of that fatal moment. Amongst many other Examples, he has this; *Super omnes C. Julius Medicus dum inungit, Specillum per Oculum trahens.* Specillum a Chyrurgeon's Probe. The meaning whereof is no more, than that the *Physician C. Julius* was on a sudden seized by Death, whilst he was applying an *Unguent* to his Patients Eye, and drawing his *Probe* (called *Specillum*) through it. Whereas, to joyn *inungit* and *Specillum*, spoils the Grammatical Sense of the whole, and renders it unintelligible.

'Tis evident therefore, that from neither of these Passages can we draw any Argument for the Antiquity of *Optick-Glasses*. Optick-Glasses probably invented about 1300.

(4.) Wherefore seeing we must necessarily allow this Invention due to the Modern Age of the World; our next Enquiry shall be, Where first to fix it. But herein we shall find but faint Traces to direct us.

Monf. Menage,

Menage's
Opinion.

Monsieur Menage a learned and ingenious French-man, in his *Origini della Lingua Italiana*, Geneva, 1685, commenting on the Word *Occhiali del Galilai*, discourses there of the Time of the invention of *Spectacles*: And after relating the known Story of *Frier Jordan*, (of which more anon) he has this notable Passage; That *Monsieur du Cange* had told the Author (*Monsieur Menage*) of a Greek Poem, the Manuscript whereof is now in the French King's Library, wherein the Poet, who lived An. 1150, Jest'ing on the Physicians of those Times, lays of them to this Purport in French, *Qu'ils tatent le Poux, & qu'ils Regardent les Excremens du Malade avec une Verre*. That they observe the Excrements of their Patients with a Glass. But *Monf. Menage* is of Opinion, that this was a Transparent Glass, whelm'd over the Vessel, more for the Relief of their Nose against the Stench, than of their Eyes.

But however we may doubt of *Spectacles* being so ancient as 1150. We may ^{probably Guen} be ~~certain~~ that about the Thirteenth Century, they were commonly known and used. For (beside what we shall say hereafter of our Country-man *Frier Bacon*) the most learned *Monf. Spon* in his *Recherches Curieuses D'Antiquité*, Differt. 16. inserts a Letter of Signior *Redi* to *Paulus Falconerius*, concerning the Time when *Spectacles* were invented; and this he fixes between 1280 and 1311. from the Testimony of a Manuscript Chronicle in Latin, in the Library of the *Friers Preachers* of St. *Katherine* at *Pisa*, Fol. 16. Wherein 'tis said, that *Frater Alexander de Spina*, *Vir modestus & bonus, quæcunque vidit aut audivit facta, scrivit & facere. Ocularia ab aliquo primo facta, & communicare nolente, ipse fecit & communicavit corde hilari & volente*. And this *Alexander de Spina* was a Native of *Pisa*, and dyed there, An. 1313.

Spina's
Pretense.

Another
Authority

Signior Redi has in his Library a Manuscript written An. 1299. *Di Governo della Famiglia de Scandro di Pipozzo*. In which there is this Passage; *Mi truovo cosi Gravofo di Anni che non arei Valenza*

lenza Di Leggere e Scrivere senza Vetri appellati Okiali, Trovati novellamente per Commodità delli Poveri Veki, quando affiebolano del Vedere. Thus in English, I find my self so pressed by Age, that I can neither read or write without those Glasses they call Spectacles, lately invented, to the great Advantage of poor Old Men, when their Sight grows weak.

The Italian Dictionary, *de la Crusca*, on the Word *Occhiale*, Frier Jordan's Authority. makes this remark, That *Frier Jordan de Rivalto*, who dyed at *Pisa*, *An. 1311.* in a Book of Sermons which he writ *An. 1305.* tells his Auditory in one of them, that it is not Twenty Years since the Art of making Spectacles was found out, and is indeed one of the best and most necessary Inventions in the World.

About the same time *viz. 1305.* *Bernard Gordon* a famous Gordon. Physician of *Montpelier*, in his *Lilium Medicinae*, thus commends a certain Eye-Salve: *Et est tanta Virtutis, quod decrepitem faceret legere Literas minutas absque Ocularibus.* And *An. 1363.* *Guido de Chauviac*, in his Book entituled *Grand Chirurgery*, after Chauviac. proposing several Collyria, saith; If these or the like will not do, you must make use of Spectacles.

From all which we may be pretty certain, That Spectacles were well known in the 13th. Century, and not much before. But who the Happy Man was, that first hit upon this lucky Thought, may yet be questioned. 'Tis true indeed, if we credit the forementioned Chronicle of the Convent at *Pisa*, *Frier Spina* makes as fair a Challenge to the Invention, as the first Author, who refused to communicate it. But I am apt to believe, That, whoever this close Man was that would not impart to *Spina*, He was a Frier; and that these Monkish Men, and *Jordan* amongst the rest, had this Invention whispered amongst themselves, before it was publick; and that they all had the First Hint thereof from our Country-Man *Frier Roger Bacon*.

Bacon's
Presence.

(5) That this learned Frier Bacon who dyed An. 1292. and lyes buried at Oxford) did perfectly well understand all sorts of Optick-Glasses, shall be plainly made out, from the natural and easie sense of his own Words, in his Book of *Perspective*: Whereby we shall find, that he not only understood the Effects of single *Convex* and *Concave-Glasses*; but knew likewise the way of combining them, so as to compose some such Instrument as our *Telescope*. This perhaps will be looked upon as a *Great Paradox*, and as great Partiality in an *English* Author to his Country-Man; especially considering, how universally the contrary has prevail'd; the Votes of most learned Men having conferr'd the Honor of this Invention on other Pretenders. But if, from the unconstrain'd Words of his Books, we plainly make out this Assertion, I hope the Attempt may not be counted unreasonable or partial.

And First in his Book of *Perspective* Part III. Dis. 2. C. 3. he has these Words; *Si vero Corpora non sunt plana* (having treated of them before) *per qua Visus videt, sed spherica; tunc est magna Diversitas, nam vel Concavitas Corporis est versus oculum, vel Convexitas, &c.* By which 'tis manifest, he knew what a *Concave* and *Convex-Glass* was. Moreover, in the same Place Dis. ult. he proceeds thus; *De Visione fractâ majora sunt, nam de facili patet, maxima posse apparere minima, & è contra; & longè distantia videbuntur propinquissimè, & è converso: Sic etiam faceremus Solem & Lunam & Stellas descendere secundum Apparentiam hic inferius, &c.* Thus in English, *Greater Wonders than all these are performed by refracted Vision; For thereby, 'tis easily made appear, that the Greatest Object may be represented as very little, and contrarily; And so likewise, the most distant Objects as just at hand, and contrarily. Hereby also may we bring the Sun and Moon and Stars down here below in Appearance, &c.* This, I think, is so express in the Point, that it leaves no room to doubt, but that he had some admirable Secret in Optick Glasses. Add

to this what he has in his Epistle *ad Parisiensem*, of the Secrets of Art and Nature, Cap. 5. *Possunt etiam sic figurari Perspicua, ut longissimè posita appareant propinquissima, & è contrario ; Ita quod ex incredibili Distantia legeremus literas minutissimas, & numeraremus Res quantumcunque parvas, & stellas faceremus apparere quò vellemus.* Glasses or Diaphanous Bodies, says he, may be so formed, that the most remote Objects may appear as just at hand, and contrarily ; So that we may read the smallest Letters at an incredible Distance, and may number things though never so small, and may make the Stars appear as near as we please.

And that these Things may not seem incredible of this Great Man ; who, in that dark, ignorant Age could be master of these admirable Inventions ; I shall refer the Reader, for a more compleat Account of him, to *Ant. a Wood Hist. & Antiquit. Universit. Oxoniensis*, Lib. 1. Pag. 136. and to Dr. Plott's *Nat. Hist. of Oxfordshire*, Cap. 9. Sect. 2, 3, &c. and Sect. 39, 40, 41. Where we may find, how he was persecuted by the ignorant malicious Friars of his Order, as practising Magick and Necromancy : for which they cast him into Prison, and there detain'd him for a long time, some say to his Death, in the 78th. Year of his Age. There we shall find, how he was the first Promoter of the Emendation of the Calendar. compleated afterwards in the Time of Pope Gregory II, But above all, his Pretense to the first Invention of Gunpowder seems as well founded, as possible, on this Passage in his *Epistola ad Parisiensem*, Cap. 6. (a Hundred years before Barthold. Swartz. lived) *In omnem Distantiam quam volumus, possumus artificialiter componere Ignem comburentem, ex sale Petreæ & Aliis ;* (These *Alia*, in another Manuscript Copy, are, Sulfur & Carbonum Pulvis) And soon after he adds, *Præter hæc (i.e. Combustionem) sunt alia stupenda Naturæ, nam soni velut Tonitrus & Coruscationes possunt fieri in Aere, imò majore Horrore*
Bacon in-
vented
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quam illa quæ sunt per Naturam : Nam modica materia adapta, sc. ad Quantitatem unius Pollicis, sonum facit horribilem, & Coruscationem ostendit violentem, & hoc fit multis modis, quibus Civitas aut Exercitus destruat. : Igne exsiliante cum Fragore inestimabili. Mira hæc sunt, si quis sciret uti, in debita Quantitate & Materia. By which last Passage we may guess, he had not the way of applying it to a Gun; though 'tis manifest, he was sensible that some such Use might be made of it. But the particular Manner did not offer it self to him at first.

Note.

I Confess, I have not by me at this time the Originals, from whence these Passages are quoted; the present Distractions of our miserable Country having separated me and my Books; and the Place, where I am, affords not the Copies : Therefore, if in these Quotations I am any wise mistaken, I must not be blamed, acknowledging that I have them at *second hand* from the forenamed Authors.

But to return to our *Optick-Glasses*. 'Tis evident that Bacon was acquainted with them; and probably knew how to adapt them in a *Telescope*. But the long and close Imprisonment he suffer'd before his Death (for 'tis said no one was permitted to speak to him; and that all his Writings, Books and Instruments were seized and burnt; except only those few Fragments of his which we have saved accidentally) was the Reason, that we have no farther Advancements of his in this kind transmitted to Posterity. But 'tis very probable that the use of single Glasses in *Spectacles*, as being an Invention of immediate Advantage to Human Life, and in it self very easie and simple, might therefore be presently catched at by the World, and put into Practice: Whilst his other more curious Combinations of Glasses might be lost and forgot. And this I am the more inclinable to believe; First, because *Frier Bacon's* Time agrees so well with *Frier Jordan's* forecited Testimony *An. 1305*. That it was not then twenty years

years since the Invention of *Spe&ctacles*: And secondly, because we find this sort of *Monkish Men* first take notice of the Invention, before all other Men; which shews, they had it delivered amongst themselves only, for a while before others.

(6.) And thus much concerning *Frier Bacon's* Pretense. But that I may not seem altogether partial, I shall here add the Opinions of others, concerning *Other Inventors* of the *Telescope*. For I find no other Pretenders to the Invention of single *Convex* and *Concave* Glasses, but the forenamed. *Borellus* has written a small Tract purposely on this Subject, *De vero Telescopii Inventore*: Wherein, Cap. 12. he seems to give the Invention to *Zacharias Joannides* of *Middleburg* in *Zeland*, An. 1590. (6.) More Modern Inventors of the Telescope. Another Candidate for this Discovery, he names *Johannes Lipperboy*, or *La Prey*, An. 1609. Zacharias Johnson. A *Dutchman* also, whom *Surturus* calls *Lippersein*. Jo. Lipperboy. *Adrianus Metius* Mathematick Professor at *Franequer* says, his Brother *Jacobus Metius* of *Alkmaer* was certainly the first Inventor of the *Telescope*. Ja. Metius. And if we believe the *Italians*, we shall have the Honour of inventing this Instrument conferr'd on the incomparable *Galileo*. But he himself in his *Nuncius sidereus* confesses, that the first Intimation he received of this Instrument, was, that a *Dutchman* had then lately made one; which set him (*Galileo*) upon the thought how to effect it; which, he says, he successfully discover'd by the consideration of *Refraction*, and found that a *Concave* and a *Convex* Glas rightly adapted would perform what he only heard in general of the *Dutch* Invention.

But certainly the first Publick notice of this Contrivance came from some of the forementioned *Dutchmen* (for *Frier Bacon's* Hint mentions not the particular Combination of the Glasses) and therefore the Instrument is deservedly called *Tubus Batavus*. Though we must confess at the same time, that *Galileo*, An. 1610. (see his *Nuncius sidereus*) did first apply this curious Instrument to Celestial Observations; and had then

Baptista
Porta.

made such wonderful Discoveries in the Heavens thereby, that all his Philosophick Successors have ever since attempted to climb higher, by lengthening their *Ladders*, and advancing this Instrument by many Degrees. However I must not here conceal the Pretense of *Baptista Porta*, who, in his *Magia Naturalis*, Lib. 17. Cap. 10. Printed An. 1589. has these Words, *Si utramque (Lentem sc. Concavam & Convexam) rectè componere noveris, & longinqua & proxima, majora & clara videbis.* But *Porta's* Character is so well known, that we may easily imagine, he had got this Hint from *Holland*.

Fontana.

Franciscus Fontana a Neapolitan, in his *Observationes cælestium terrestriumque rerum*, contends that he himself An. 1608. first invented the *Telescope*, composed of a *Convex Object-Glass* and *Convex Eye-Glass*: For the *Tubus Batavius*, and *Galileo's* Tube was furnish'd with a *Concave Eye-Glass*; and *Fontana* confesses, it was before his; and that An. 1618. he first invented the double *Microscope*. *Rheita* in his *Oculus Enoch & Elia*, Lib. 4. towards the end, pretends to be the first Discoverer of the erecting *Telescope* of three *Convex Eye-Glasses*, as also of the *Telescope* for looking with both Eyes, called *Telescopium binoculum*: Of which latter, *Cherubin* has writ his whole Volume, *La Vision parfait, &c.*

Rheita.

(7.) Op-
tick Glas-
ses long
known be-
fore the
Telescope.

(7.) Thus we see, how long the Use of single Optick Glasses was common in the World (even about 300 years) before Men rightly understood their due Application, in the Composition of this admirable Instrument. They had them in their hands, they look'd through them, now a *Convex*, then a *Concave*, and admired their Effects, and the Help they gave to disorder'd Eyes: but still were ignorant of the vast Advantage the most acute Eye might receive by them, even to the Increase of its Power, some Thousands of Degrees beyond its natural Abilities. This was reserved for some lucky Chance in a future Age, to be discovered by him that should first

first be so fortunate, as to adapt these Glasses at their *due Distance*: for to some such happy Hitt, I imagine the Invention is due; and not to any profound Thought on the nature and properties of Glasses, that first suggested the Contrivance to the *Dutch Mechanick*, that was its Author.

And this does naturally suggest a Thought to us, of some encouragements in natural Enquiries, by the method of *experimental Philosophy*; that perhaps we are every day ingaged amongst some *particular Things*, which we commonly see, handle, use, and are conversant with; and which have in them some latent, hidden Properties, which, upon a right Application, (to be discovered perhaps by some lucky Hitt) may be of the most useful and surprising Effects. And that therefore, we should not despair of making the greatest Discoveries about even the *meanest Things*. Who could expect to see such Wonders from an easie Composition of three such plain, simple Bodies, *Niter, Sulfur, and Charcole*, as we daily see from *Gunpowder*? And the Property of the *Magnet's* drawing Iron was commonly known many generations, before it was so happily applied to guiding a Ship: Who could have thought, by looking upon that dark unpromising stone, that future Ages should use it to such a stupendous and advantageous a Purpose, far exceeding the Virtues of the most illustrious Gems? Hence may we learn, not to despise the Products of Nature, even of the meanest Appearance. And let us not say, that any Discovery is *useless*, since we know not what Time and Posterity may produce from the simplest Truth. And this naturally leads me to the discoveries made by *Optick Glasses*.

(8.) *Galileo* (as is noted before) is deservedly reputed the first that raised up this *Gigantick Instrument*, that ventures to climb Heaven and from thence brings down the Stars. He first, was surprized and struck with wonder, to see *four little Moons* dancing round *Jupiter*, that from their first Creation

Remark
thereon.
Uses of
Discoveries
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ately
known.

(8.) Celestial Discoveries of the Telescope.

Whether
all for the
use of
Man.

to the lucky Moment when he first discovered them, had never struck the eye of any mortal Inhabitant of this Globe. Were these then made for the *Use of poor Man*, from whose Knowledge they were concealed for 5000 years together? Vain Man! that thus presumes to confine the Designs of the *Almighty Creator* to miserable Dust and Ashes; when his *infinite Power* can make Millions of *intelligent Beings*, and all intelligent after different ways, to serve and praise him: And these perhaps are the Inhabitants of these *distant Worlds*, and of those again infinitely extended beyond these. 'Tis true indeed, now these little Planets are discovered, we have happily applied them to an advantageous purpose (as shall be shew'd hereafter) But this we are to esteem as a particular Benefit of Providence to these latter Generations; and respects not all the general Race of Mankind, that lived and were busie for 5000 years together; and knew nothing of them. But in this stupendous Enquiry I stop, as not being able to reach it with the *longest Telescope*.

To keep therefore to our Subject: I shall take the Heavens in order, as they lie; considering first the *uppermost*, and so descend *down* to our Earth, and shall briefly declare the *Discoveries* made in each, and (as far as I can attain it) by *whom* and *when*; with farther References to those Authors, where each particular may be found more fully treated of.

(9.) In the
first Stars.

(9.) And First, for the *Fixt Stars*: That whitish Band or Zone, the *Galaxia* or milky Way, that so irregularly compasses a great scope in the Heavens, and of which the Ancients could give no tolerable Account, is found by the Telescope to be no other, than an heap of very minute Stars thickly set together; which, by their great Distance, Smalness and Closeness, appear to the naked Eye, as one united whitish Cloud. In like manner, the *Nebulosa Orionis*, *Præsepe Cancræ*, &c. are found to be a Congeries of small Stars closely set together, but easily distinguishable by the Telescope. The *Pleiades* or
seven

seven Stars (tho scarce more than six appear) are found by an ordinary Glass to be nigh forty. And in the single Constellation of *Orion*, the Telescope discovers more Stars than the naked Eye can number in all the Heavens. On this Account, the Seed of *Abraham*, that was to be made *numerous* as the Stars in the Firmament, may yet (for ought we know) admit of Propagations through many future Generations, before it comes up to its Limits. And the number, which *Archimedes* demonstrated greater than that of the Grains of Sand composing this Globe of Earth, may perhaps fall short of the Stars in the Heavens: For hardly any Corner of the Firmament so dark; But the Telescope, turn'd towards it, descries Multitudes of *glittering Spangles* therein.

(10.) From the *fixt Stars* let us contract our Prospect, and in a vast, long, and almost immense Course homewards, we first meet with *Saturn*. By his slow Motion he takes State upon him, as carrying about him something more weighty than ordinary. But the short sight perceives nothing thereof, and sees only a plain round Globe, as the rest of the *Chorus* dancing round the Sun. All his Equipage and Attendants are hid from our View, 'till surveyed more closely by the Telescope: And then behold a mighty Ring parallel to the Equator, bright as the Planets own Face, encompassing round his Body; very thin, and separated in all Appearance on all sides from his Globe: sometimes appearing broader, sometimes narrower, and sometimes almost vanishing; then again returning by a regular Period, and resuming by Degrees its former Shape; which again by degrees it looses according to his own Periodical Motion. But this is not all his Equipage, for besides this Throne of Light, this Majestick Planet is constantly attended by a Guard of *five Satellits*, that follow his Motion and dance round him continually in a Circle.

Galileo.

Galileo was the first that observed any thing extraordinary in Saturns Appearance An. 1610. Octob. as he tells us in some of his *Italian Letters*: But his Glasses were too short to give the true Shape of this Planet. All that he descry'd was something appendent on each side of him, which he took to be two Globes much less than Saturns own Body; and therefore he first publish'd (and at the same time conceal'd) this Discovery, by transposing the Letters of this Sentence, *altissimum Planetam tergeminum observavi*. But when the Telescope was better advanced, (as what Invention is it, that receives not Advancements in Time?) the true and genuine Appearance of Saturn began to shew it self, and its regular Changes were taken notice of. But though several Authors writ Treatises of this surprising Appearance, and particularly the celebrated Hevelius (*de nativa Saturni facie*) *Hodierna*, &c. yet all their Observations were imperfect and deficient; and chiefly for want of excellent Glasses: Till the incomparable *Christ. Hugenius* has put the last hand to this Affair; and in his ingenious Treatise, *Systema Saturnium*, Hag. Comit. 1659. 4°. has publish'd to the World a compleat History of all Observations of this Planets Appearances with a most ingenious Theory for their Explication. In the beginning of the year 1655. his excellent Person first discovered the biggest of Saturns *Satellites* with a Telescope of 12 feet, charged with an Eye-Glass of 3 Inches; afterwards, An. 1656. he doubled that Length, retaining the same Eye-Glass. The *Satellit* he discover'd, is the *Fourth* from Saturn; and in the forenamed Treatise, he gives us the *Epochæ* and *Tables* of its Motion; But our most ingenious Countryman, Mr. *Halley*, deservedly celebrated for his Astronomical Labours, discovered in the year 1682. that *Hugenius's* Numbers were considerably run out; and therefore he set himself to correct the Period of this *Satellit*, which he has done accordingly, Num. 145. Pag. 82. *Philosoph. Transact.* And in Num. 187. Pag. 299.

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we shall find *Monf. Cassini's* Tables of the Motions of all *Saturn's Satellites*, together with their distances from *Saturn* correspondent to their Periodical Times: Of which more hereafter.

The other four *Satellites* were all discovered by *Monf. Cassini* in the Order following. The third and fifth were first seen by him, *An. 1671, 72 and 73.* by a 17 Foot Glass of *Campani*, and 36 Foot Glass of *Divini*, and by such another of *Borelli*. An Account whereof may be seen at large in Num. 92. of the *Philosoph. Transact.* But the innermost or first, and the second were not seen by him till the year 1684. at which time, having procured Glasses of an extraordinary length, as 80, 100, 150 and 200 Feet; the vast distance and smallness of these Planets could no longer conceal them from his sight. *Vid. Philosoph. Transact. Num. 181.*

Cassini's Discoveries about Saturn.

The last thing I shall take notice of, relating to this Planet, is, That *Monf. Gallet*, *Provost* of *S. Symphorian* at *Avignon*, in the year 1684. has advanced an Hypothesis for solving its Appearances; which, as it relates to the Telescope, may properly be here considered. I shall therefore briefly propose some of the chief difficulties, that seem to attend this Theory: And that I may not be prolix, I shall suppose the Reader acquainted with what *Monf. Gallet* lays down in the *Journal des Sçavans, An. 1684. May 15. & June 12.* and in Latin in the *Acta Lipsie, An. 1684. Septemb. Pag. 421.*

Examination of Gallet's Hypothesis.

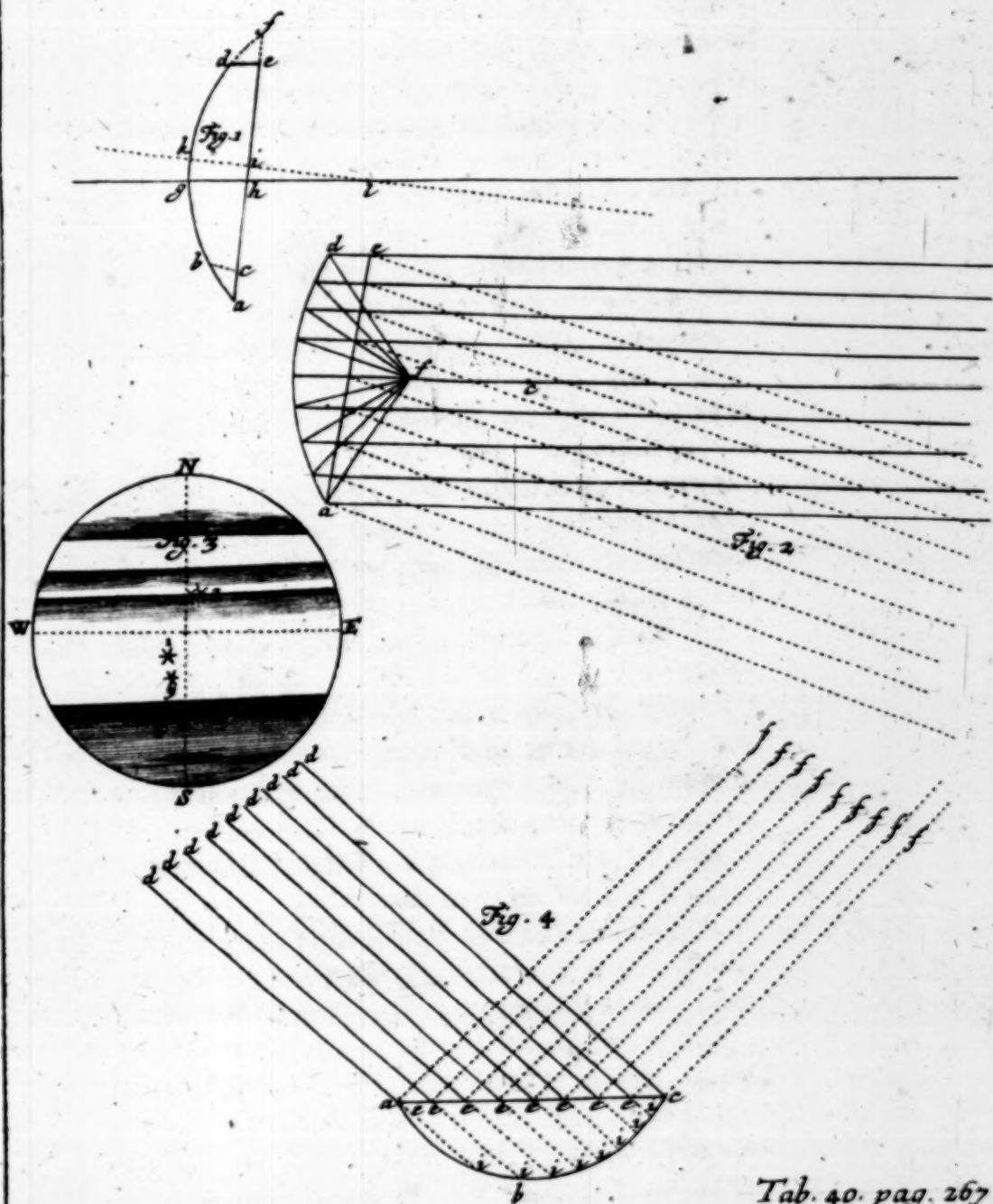
First therefore, he supposes *Saturn* and the other Planets, except the Moon, *polite Globes*, reflecting the Image of the Sun as a *Convex Speculum*. Which seems not at all to be founded on more than mere Conjecture: For we have no Reason to think them different in this particular from the Moon; which is found of a rugged uneven Surface.

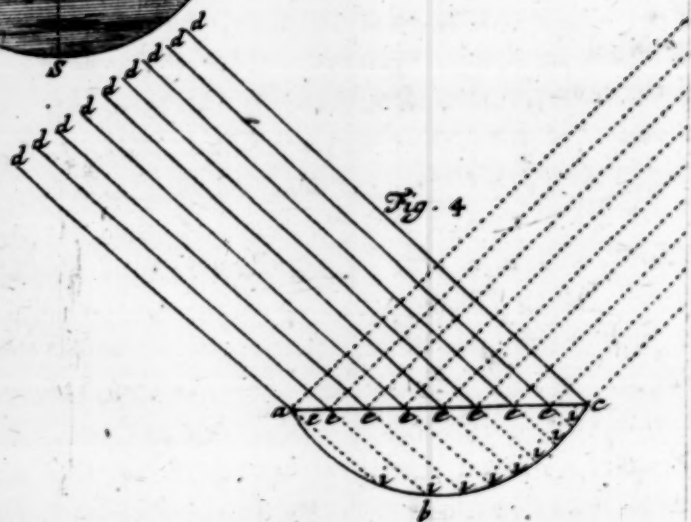
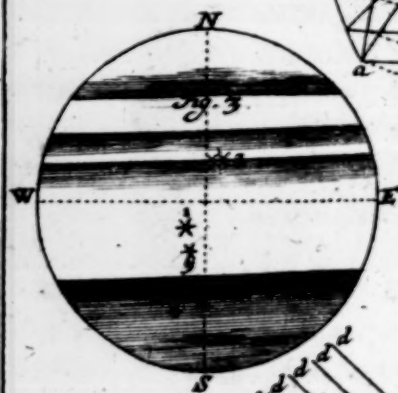
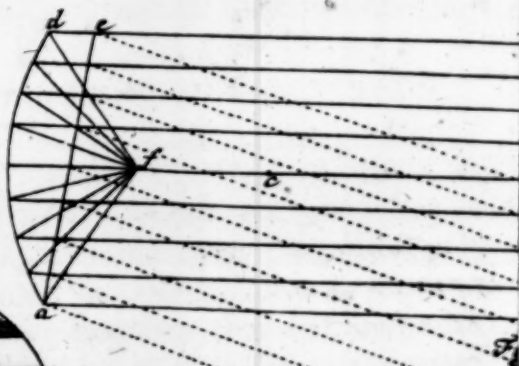
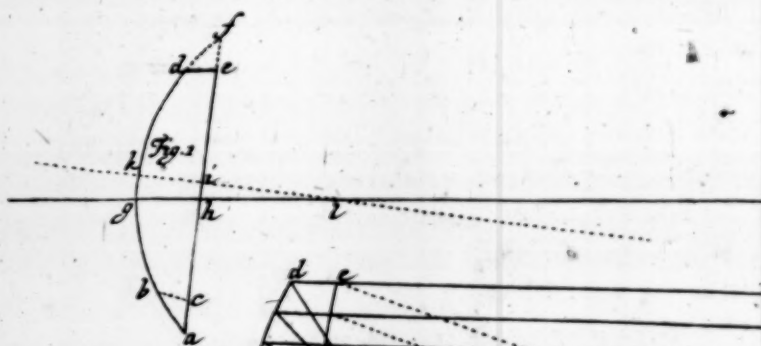
But secondly, granting that they (and especially *Saturn*) may be *polite Spheres*, (for we will not confine the infinite Variety of the Creation;) and granting that *Saturn* reflects two

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sorts of Light ; one whereby his whole Body becomes visible ; and the other the bright Image of the Sun from his Convex Surface ; as we see a Convex Speculum is it self visible, by the Rays it reflects disorderly from its whole Surface ; (which so far partakes of a little Roughness) and at the same time reflects a bright and orderly Image of the Sun, from one certain part of this Surface to the Eye rightly posited : Yet this bright Image of the Sun, which is reflected from *Saturn*, (how far soever *Saturn* be removed) can never be projected by an Object-Glass, in its distinct Base, *greater* than the Projection of the whole Body of *Saturn* in the same distinct Base. And yet (if I mistake not) this is the Foundation of *Monf. Gallet's Theory*. This is so evident to any one the least versed in *Dioptricks* and *Catoptricks*, that 'tis needless to insist upon it any longer. We may make a convincing Experiment hereof : Expose a reflecting Convex-Speculum before the Sun, and by a Convex-Glass project the Image of this Speculum on a Paper in a dark Room : we shall there see the Representation of the Speculum it self, and of the bright Image of the Sun on the Speculum. And indeed by the least Consideration of the matter, it will be evident to us, that 'tis impossible it should be otherwise : For the little Image of the Sun, reflected from the Convex Speculum, possesses but a very small part of the Speculum's Surface ; and therefore cannot possibly be projected, by the Object-Glass in its distinct Base, *greater* than the Projection of the Convex Speculum itself. The same may we conceive of *Saturn*, by supposing him a Convex, polite, reflecting Speculum ; for though he should then, besides the Figure of his own Body, reflect the bright Image of the Sun, from a small part of his Spherick Surface on the Object-Glass ; yet the Object-Glass could never project, in its distinct Base, the Representation of this Image of the Sun *greater* than the Representation of the whole Body of *Saturn*.





Saturn. And, notwithstanding the Evidence hereof, *Monf. Gallet* affirms, the Appearance of *Saturns Ring* or *Anſe* proceed to from hence; that the Object-Glaſs projects, in its diſtinct Baſe, the representation of the bright Image of the Sun, reflected from *Saturn's Convex polite Surface*, greater than (or clearly without) the Representation of his *Body* itſelf.

Thirdly, *Monf. Gallet* affirms, that the Reason why *Jupiter*, *Mars*, &c. are not projected by an Object-Glaſs, in its diſtinct Baſe, with a *Ring* or *Anſe*, is becauſe theſe Planets are nearer to us than *Saturn*: And therefore in the Projection of *Jupiter*, the bright Image of the Sun reflected from its polite Surface; is represented by the Object-Glaſs in its diſtinct Baſe, equal to the Representation of *Jupiters* whole Body; in the Projection of *Mars* it is leſs, &c. But the Diſtance of theſe Planets (even of the nigheſt) is ſo very great, (and as it were infinite) in reſpect of the ſmall Breadth of an Object-Glaſs; that in compariſon to this ſmall breadth of an Object-Glaſs, we can make no difference between the Diſtance of *Saturn* and even *Mercury*.

Laſtly, the Experiments, on which *Monf. Gallet* founds his whole *Hypotheſis*, ſeems not at all to confirm, or in the leaſt wiſe to reſpect what he builds thereon. The Phænomenon ariſing from reflecting the Sun-Beams by an Object-Glaſs on a Wall poſited obliquely, proceeds from the Object-Glaſs being conſider'd as a *Concave Reflecting Speculum*, having alſo another Surface either Plain or Convex beſides the Concave, (as I have noted before, Chap. 4. Sec. 4.) and yet he ſuperſtructs hereon a Theory, for explaining the Appearances of *Convex Polite Surfaces*, ſuch as he makes the Planets. This will be manifeſt by *Tab. 40. Fig. 4.* wherein, for eaſe ſake, we take a T. 40. F. 4. Plano-Convex Glaſs *abc*, and expoſing its plain ſide *aec* obliquely to the Suns Rays *de, de, de*, ſome of them ſhall be reflected by the plain Surface *aec* into *ef, ef, ef*; but others of them

M m 2 entering

entring the Glafs run on in ei, ei, ei , (we do not here confider the Refraction they fuffer) and fo falling on the Concave Surface $aibic$ (for fo I'll call it) are reflected by it, according to the Laws of *Spherick Catoptricks*: But the Reflections of thefe immerged Rays I have not expreffed, for avoiding Confufion in the Scheme. I acknowledge, the *Physical* Cause of this latter Reflection, from the Surface of the Glafs $aibic$, is perhaps not to be accounted for by Human Understanding; but the matter of Fact is certain. One fhould think, when the Rays are arrived at the extreme Points i, i, i , of the hindmoft Surface of the Glafs, they fhould, without any of them being reflected at all, emerge from the Glafs: But 'tis manifefl *some* of them are reflected, and that too, juft in the fame manner, as if the Surface $aibic$ were a Polite, Opake, Concave Surface; and not covered by the Surface aec . They that defire to enquire farther into the natural Cause hereof, may confult *Grimaldi Phyfico-Mathefis de Lumine & Coloribus*, Bonon. 1665: 4°.

'Tis then by the Reflection from this Concave Surface $aibic$, (give me leave fo to call it) that the fimilitude of *Saturn's Anſæ* are represented in *Gallet's* Experiment; and by the Reflection from the Plain Surface aec , that the Similitude of his Body is represented in the ſaid Experiment. And how this can be accommodated to the Reflection, which *Saturn* himſelf makes from his own Body, and to his Appearance through a Teleftope, I confeſs I cannot apprehend. Moreover *Gallet's* Hypotheſis gives no Account of the two dark Spaces on each ſide the Globe of *Saturn*, between his Body and the *Anſæ*. For the Experiment, on which he founds his Fancy, ſhews no ſuch Diſtinction; all being inlightened therein: As will be viſible to thoſe ſhall try it.

'Twere too tedious in this Place to confider particularly *Monſ. Gallet's* Scheme, and his *Particular System* of *Saturn*; which

which might easily be shewn defective. But thus much I thought requisite to say in this Place; because the Theory he proposes is of a *Dioptrical* Consideration: Because it has never yet been taken notice of by any other: And because he advances it in Opposition to Mons. *Hugens* System, which carries with it so much Probability.

(11.) *Jupiter* next presents himself less incumbred than *Saturn*, yet not wanting a Courtly Train; For tho his Guards^{(11) In Jupiter.} are but *four* in number, yet their size and brightness shew their *Strength*, and their quick Motion round him shews their Diligence.

Galileo was certainly the first Inhabitant of this Globe,^{Galileo.} that ever saw these *Satellites*, Jan. 7, 1610. And from that Moment to this, no more could ever be discovered about him: Though Vanity and desire of being the Author of some Novelty, made Frier *Ant. de Rheita* proceed so far, as to write^{Rheita.} a Tract of 5 more *Satellites* (9 in all) about *Jupiter*. But *Hevelius*, in the fourth Chap. of his *Selenography*, has demonstrated the Frier to be mistaken, and has shewn the small fixt Stars (only discoverable by the Telescope) that deceived him.

These *Satellites* are easily seen by a 3 Foot Glass, and I have just perceived them with one of 15 Inches. But to make exact Observations of their Motions, 'tis requisite, we use Tubes of 10 and 12 Feet and upwards. From the time of their Discovery, many curious Astronomers have attended their Motions round *Jupiter* with a diligent Eye; and have found, that sometimes falling into the Shadow of *Jupiters* Body, they disappear; and thence emerging, they again become visible: sometimes they are hid behind the very Globe of their great Lord; and sometimes being just in his Face, his Splendor overcomes theirs, and they become invisible, as a glimmering Lamp between the Eye and Sun.

Joh.

Borellus.

Cassini.
Flam-
steed.

Joh. Alfons Borellus has publish'd a Tract of the *Theoricks* of these *Medicea Sidera* (so named by *Galileo* in complement to his great Patron the Duke of Tuscany) *Theoricæ Mediceorum siderum, Florent. 1666. 4.* But none have laboured more to reduce the Observations of these little Planets to something of Use and Advantage to the World, than the two celebrated Astronomers of the present Age, *Cassini* and *Flamsteed*: The former has taken great pains in Publishing Hypotheses, Tables, and Precepts for calculating their Appearances and Eclipses, (in his *Ephemerides Mediceorum siderum, 1668. Bonon.*) in order to settling the *Longitudes* of Places to a great Certainty. And the latter finding the Numbers of *Cassini's* Tables not so just to the present Time, from most accurate Observations of his own, taken by the Telescope and Micrometer, has fixt new Numbers, that agree about this time most exactly to the *Phænomena*; but with the Liberty to himself of altering these Numbers, as by future Observations he shall find Occasion; For he is not so positive as to say, that what he settles (for 15 or 20 years) shall be perpetual. What he has hitherto published of this kind may be found dispersed in the *Philosoph. Transact.* wherein, he has given the World the Catalogues of these *Satellites Eclipses* for several years consecutively. In Num. 154. Pag. 404. he shews their Uses, and how (by their Help) the difference of *Longitude* betwixt any two Places on Earth, where they should be observed, might be determined: And teaches a Method of finding out, within what space on our Globe any of them are observable. These Directions he repeats in Latin, Num. 165. Pag. 760. for the use of Forreiners. In Num. 177 & 178. with the Catalogue of Eclipses for the year 1686. he describes a small Instrument; and shews how by the help of it, of the said Catalogue, and of the Tables of *Jupiters Geocentrick Places* and *Parallaxes*, the Appearances of the *Satellites* at any time in that year might be discovered, and delineated by Scale and

and Compass on Paper. But the Curiosities, which this excellent Astronomer has yet unpublished, relating to this useful Part of Astronomy, are very great and ingenious; which I hope, in time, he will impart to the World; as with much Freedom and Generosity, he now communicates them to his private Friends: In the Number of which I am very proud to reckon my self.

Before I quit this Article of the *Satellites*, I cannot omit taking notice of an Observation, which, by mere Accident, I made some years ago, of a *Total Disappearance* of all the *Satellites*. I had often attended their Motions with a good Telescope of 12 Foot; and often had them all four conspicuous at a time, very often *three*, frequently *two*, but never less than *one*, and this but very rarely; till *An. 1681. Novemb. 2. Hor. 10. p. m. Dublinii St. Vet.* there was not one of them visible; Jupiter there stood by himself, in all Appearance, without his Guards; and a bold *Lucian* might have pull'd him from his Throne without Resistance. Some years after, I obtain'd from my learned Friend Mr. *Flamsteed* his Tables of the Motions of these *Satellites*; And the Postures of these *Jovial Moons*, at that time, are found by them to be as is expressed in *Tab. 40. Fig. 3.* The first, third and fourth were *just in his Face*; and were therefore drown'd by his Light, and the second was *behind* his Body. The rarity of this Appearance (at least to me) makes me note it so particularly: perhaps those that are more frequently employ'd in watching their Motions may meet sometimes with the like Conjunctions; but I believe 'tis very seldom. *Hevelius*, in his constant Attendance on them, for more than a year and half, hit not upon such an Appearance; as may be seen by the History of his Observations at the end of his *Selenographia*; Nor *Cassini*, as is manifest from his forementioned Ephemerides.

Besides these *four little Moons* about *Jupiter*, the Telescope discovers other Remarkables even in his Body. As first: his

Face

*Satellites
all disappearing.*

T. 40. F. 3.

*Jupiter's
Beles.*

Borellus.

Cassini.
Flam-
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Face

*Satellites
all disappearing.*

T. 40. F. 3.

*Jupiter's
Belts.*

Face is not all of a Colour ; But there are in it *brighter* and *darker* Parts ; and these are drawn athwart him, like broad *Zones* or *Belts* almost parallel to the *Ecliptick*, as is expressed

T.40. F.3. in Fig. 3. Tab. 40.

Jupiter's
Spot and
Rotation.

Hence the
Rotation
of our
Earth.

The last thing that has been observed in this Planet is a *Spot*, first seen by our Ingenious Mr. *Hook* May 9. h. 9. p. m. 1665. By this *Spot* 'tis manifest, that *Jupiter* turns round his own *Axis* in the space of less than 10 hours, or about 9^h. 56'. A very strong Argument to prove, that our *Earth* may do so likewise ; since *Jupiter*, who is so considerably bigger than the *Earth*, has a Motion much more quick than ours in 24^h. *Kepler*, upon the Restauration of the *Pythagorean* or *Copernican Hypothesis*, did conjecture, from the Motion of the Primary Planets about the *Sun* as their Centre, That the *Sun* moved about its own *Axis* ; but could not evince it, till future Observations by the *Telescope* discovered *Spots* in the *Sun*, and by them demonstrated, that the *Sun* revolves on its own *Axis*, in 25 $\frac{1}{4}$ days. *Jupiter* has four *secondary* Planets moving round him, and he himself in their Centre turns on his *Axis* : our *Earth* has a *secondary* Planet, the *Moon*, that moves round her once a Month : is it not therefore highly probable, that the *Earth* also revolves diurnally on its *Axis* ? For a farther account of this *Spot* in *Jupiter*, I refer to the *Philosoph. Transact.* N.1. P.3. N.4. P.75. N.8. P.143. N.12. P.209. N.15. P.246. N.82. P.4039.

(12.) Re-
flection on
the Moti-
ons of Sa-
turn's and
Jupiter's
Satellites,
evincing
the Order
of the Coe-
lation.

(12.) But before I leave *Saturn* and *Jupiter*, I cannot but take notice of one admirable Property, for the Knowledge whereof, we are beholden to the *Telescope* ; and that is, the wonderful Agreement which is found in all the several Systems of our Vortex ; as well between the General System of the *Sun*, and Primary Planets with the particular System of *Saturn's* or *Jupiter's* Planets, as between the particular Systems themselves, in this single Property, That the Periodical Times of the Planets Revolutions are in a sesquialtera Ratio of their Distances from the Centre

Centre of the Planet about which they revolve. That is, As the Square of the Period of the first Satellite (for instance) : To the Square of the Period of the second :: So the Cube of the Distance of the first from Jupiter's Centre : To the Cube of the Distance of the second from his Centre. This holds most exquisitely true in Jupiter's Satellites, as is noted by the admirably learned Mr. Newton, in his incomparable Treatise, *Philosophiæ Naturalis Principia Mathematica*, Lib. 3. Hypoth. 5. And the same Law of Motion is strictly observed by the five Primary Planets, and the Earth about the Sun. As he notes, Hypoth. 7, 8. This is also verified by Monsr. Cassini in the five Satellites of Saturn; as appears by his Account of them in the *Philosoph. Transact.* Num. 92. P. 5178. N. 133. P. 831. N. 181. P. 79.

And from hence may we justly fall into the deepest Admiration, that one and the same Law of Motion should be observed in Bodies so vastly distant from each other, and which seem to have no Dependence or Correspondence with each other. This does most evidently demonstrate, that they were all at first put into Motion, by one and the same unerring Hand, even the infinite Power and Wisdom of God, who has fixt this Order amongst them all, and has establish'd a Law, which they cannot transgress. Chance or dull Matter could never produce such an Harmonious Regularity in the Motion of Bodies so vastly distant: This plainly shews a Design and Intention in the first Mover. And with Submission to the Reverend and Learned Divines, I am apt to think, that one Argument drawn from the Order, Beauty and Design of Things, is more forcible against Atheism, than Multitudes of Notional Proofs drawn from Ideas, Apparitions of Specters, Witches, &c. (not that these should lose their due Strength) For besides the Heavens, even the little Globe we inhabit affords us infinite Variety in this kind: And for my own part, I must confess, I can

N n

read

Beauty
Order and
Design of
natural
Bodies
strong
Proofs of
a Deity.

read more Divinity in Mr. *Charlton's* admirable *Museum*, on a Box of beautiful *Shells*, of delicately painted *Plants*, curiously adorned *Insects*, *Serpents*, *Birds*, or *Minerals*; than in large Volumes of Notional Writers. For *Animals*, *Plants*, and *Minerals* do yield us abundant Instances, which visibly shew a *Design* or *End proposed*; which, as it cannot possibly consist with *Chance*, so neither can it be apprehended to have been so *ab aeterno*: For 'tis absolutely unconceivable, that a thing *designed* for some *End* or *Purpose*, should not be so *designed* in *time*, by some *designing Being*. But I beg Pardon for this Digression, in which I am thus engaged before I was aware. To return therefore to our Subject.

(13.) Mars

(13.) *Mars* offers himself next; who, trusting in his own Strength, is attended by no Guards; But the Prying Telescope discovers in his Face *Scars*, *Spots*, and *Ruggedness*; as we may find in the *Philosoph. Transact.* Num. 14. P. 239. &c. By these *Spots* the acute *Cassini* has determin'd, that he turns on his own *Axis* once in 24^h. 40'. tho others assign his Revolution performed in just half that time (a Mistake easie enough) *vid. ibidem*. The fiery Face of this Planet requires a very good Telescope to view him, and a small Aperture on the Object-Glass: or else this Glaring Light makes but a confused Appearance. But however furious his Beams are, he is beholden for them (as all the rest of the Planets) to the great Fountain of Light and Heat the *Sun*. For what the great distance of *Saturn* and *Jupiter*, and their being so much *above* the *Sun* (as I may so speak) hinders us from seeing, *viz.* their *Increase* and *Decrease* in Light like our Moon, is very visible in the Planet *Mars*; who, in his *Quadratures* with the *Sun*, and in his *Perigeon*, may be seen almost *bisected*, but never *corniculated* or *falcated* as the other Inferiours, *Vid. Hevelii Selenograph.* Cap. 4. P. 66.

Increases
and De-
creases-

(14.) The
Sun.
Sun's
Spots.

(14.) The glorious *Sun* does next present; in whose bright Face, we can hardly expect to find dark *Spots*. yet such there are,

are, and frequent too. *Scheinerus* in his *Rosa Ursina* has published a large Book in fol. of nothing else; and *Hevelius*, at the end of his *Selenography*, has many Observations of these *Macule*; as also of some brighter Spots in the *Sun*, called *Facule*. The Way of observing them is taught at large, in the foresaid Authors; and is in short, either by admitting the Light of the *Sun* through a Telescope upon a white Paper, in a dark Room; or by arming the Eye with a small thin Glass smoaked over a Torch, Lamp, or Candle, and with it looking through a Telescope at the *Sun's* Body. The only Discovery that has been made by these *Macule*, is, that the *Sun* revolves round his own Axis, in the Space of about $25\frac{1}{4}$ days. But for a compleat and succinct Account of the Theory of these *Macule* and their Motions, I refer the Reader to the excellent Mathematician *Andrew Tacquet*, *Astronom. Lib. 8. Tract. 3. Num. 7.* 'Tis only to be noted, that for these several years past, the Appearance of these *Macule* has been much more rare, than when *Galileo* (who certainly first discovered them) *Scheinerus*, *Hevelius*, attended their Observations, viz. about 50 or more years ago. About that time, one should seldom see the *Sun's* Face (no more than now our brighter Beauties here below) free from one or more black Patches; but now (as if they were grown out of Fashion) he seldom wears any: One in 5 or 7 years hardly appearing: As if now he put them on, more of necessity, to cover an odd Pimple, that may otherwise disfigure his Countenance, than to adorn his Face. How far the Fair Sex should follow his Example, I dare not venture to determine. But from them we naturally fall to *Venus*. Sun's Rotation.
Sun's Spots rare of late.

(15.) *Venus*, the brightest Planet in the Heavens. She fears not sometimes even at Noon-day to display her Beauty; and in this Armour reposing an entire Confidence, performs her Course alone, and free from all other Attendants. (15.) Venus and Mercury.

Mercury's Wit and Quickness secures him, therefore he has no Train, but generally shelters himself under the Beams of his potent Lord the Sun.

*Increase
and De-
crease.*

*Falsity of
the Ptole-
maick Hy-
pothesis.*

But both these Inferiour Planets are found by the Telescope to *increase* and *decrease*, as our *Moon*. For sometimes they appear *corniculated*, sometimes *falcated*, sometimes *gibbous*, and sometimes *full* even on, or nigh to their Conjunctions with the Sun. By which last Phænomenon, 'tis manifest they move about the Sun, sometimes *farther* from us, sometimes *nigher* to us than he: and consequently the *Ptolemaick Hypothesis* is absolutely *false*. (whatever *Hypothesis* be true.) And for the Demonstrable Detection of this *Error*, we are beholden to the Telescope: And I doubt not but Posterity may, by the same Instrument, discover some *Hypothesis* as *positively true*: For the *Probabilities* of the *Copernican System* are already so strongly confirm'd thereby, that there seems no Room left for any farther Doubt; But Time and Labour will yet discover farther Proofs. How successfully Mr. *Hook* has applyed the Telescope to prove the *Motion* of the *Earth*, I leave the Reader to judge upon Perusal of his *Attempt*.

(16.) *Dis-
coveries
in the
Moon.*

(16.) And thus at last are we arrived at home to contemplate our Neighbour the *Moon*. Her may we properly call *our own*, as making us the Centre of her Periodical Motion. For, as the *Satellites* about *Saturn* or *Jupiter* move round *them*; so moves the *Moon* as a *Satellite* about our *Earth*. *Galileo* with his Telescope first discover'd great *Ruggedness* in the *Moons Face*, after him *Langrenus* the King of *Spain's* Cosmographer, attempted to draw her Picture. But the noble *Hevelius* in his curious and costly Work of *Selenography*, has perfected this Affair, perhaps beyond Amendment. There may we see the *Moons* Countenance distinguished in an admirable difference of Parts, both for Shape and Colour. We may there see greater Parts that resemble our Seas, Lakes, Rivers, Islands, Peninsulas,

Peninsulas and Continents; other lesser Spots (almost infinite in number) that resemble our Mountains, Hills and Vallies. Of the *greater* Parts, those that are something *obscure*, may we reckon *Seas* and *Lakes*; and the brighter may we account *Land*. For just so does our Earth appear, when, from a distant Height we look upon a Mixture of Land and Water enlightened by the Sun. Of the *smaller* Spots, those that are *brightest* and *shine*, are Mountains and Rocks; and the *darker* Parts, which are usually encompassed with these *brighter* Verges, may we esteem *Vallies*.

Now that some Parts of the Moon are much higher than others, is as manifest by the Telescope, as that some Parts of our Earth are higher than others. For if we look upon it about the Quarter-days, we shall plainly see the Edge, towards the dark Part, broken and cragg'd; and many little bright Spots, that are clearly separated from the rest of the enlightned Part. Which is an evident Proof that these are the high Tops of Eminencies, which receive the Suns Light, before the Parts *below* them are enlightned. Moreover the Moons Spots cast their Shadows *opposite* to the Sun, that is, to the *Eastward*, whilst the Moon is *Increasing*, and to the *Westward*, on her *Decrease*. That these Mountains are very *high*, is manifest from the way of Measuring them, delivered in *Riccioli Almagest*. 1. Pag. 208. And *Tacquet Geom. Prac.* Cap. 8. Prob. 2.

For the better distinguishing these Spots, and making them more useful in the Observation of Lunar Eclipses, there are names imposed on them by Authours: *Hevelius* assigns to them the names of Places here on Earth: *Grimaldus* and *Ricciolus* give to them the names of famous Mathematicians and Astronomers.

By means of these Spots, Lunar Eclipses are now much more accurately observed than formerly; to the great Advancement of Geography and Navigation, in setting the Longitudes

Mountains in the Moon.

Names imposed on the Spots.

Eclipses accurately observed by them.

gitudes of Places. For now the Immersions and Emerfions of these Spots from the Shadow of the Earth are most nicely determined.

Moreover, by these Spots the Moon is discovered to have *various librating Motions*, from *East to West*, and from *West to East*, also from *North to South*, and from *South to North*. But hereof we cannot now enlarge, *vid. Bullialdi Astrom. Philolaic. Lib. 3. Cap. 13. Hevelii Selenograph: Riccioli Almagest. 1. Lib. 4. Cap. 9.* Neither is it needful to insist on the Moons *Transits over*, and *Appulses to*, fixt Stars and Planets; which can never be accurately observed but by the Telescope. I cannot tell whether it be worth our while to take notice in this Place, that *Monf. Isaac Vossius* has published a fantastical Conceit of his own, for explicating the Appearance of the Moons Spots; in his *Liber variarum Observationum*.

(17.) *Planners whether inhabited.*

(17.) And now perhaps we may be allowed to sit down, and think awhile, whether all these Celestial Bodies, that thus dance round our Sun, may not be inhabited. But this Disquisition has been already so ingeniously managed by several, (particularly by the Reverend *Dr. Wilkins* Bishop of *Chester*, in his *World in the Moon*; and by *Monf. Fontenel* his *Plurality of Worlds*) that there is little left to be said on the Subject. I shall only add, that there is nothing in *Nature*, *Morality* or *Religion*, that contradicts the *Affirmative* of this Opinion. And 'tis through a narrowness of Thought that some men deny it. They will not think on any other sort of Creatures, than what we see here on Earth; and presently begin to ask, how should Men possibly live in *Saturn's cold Climates*, or in the scorching *Heat* that affects *Mercury*. But shall we thus confine the *Great Creator* to our poor Conceptions? Cannot he that has made a Man, a Whale, an Elephant, a Fly, be able to create indefinite Varieties of Creatures, and all endowed with different Faculties, and various Ways

Ways of Perception? Some adapted to one Planet, others to others? And all these may be engaged in different Ways of Life and Thought; but should all be engaged in Praising and Serving him that gave them all their *Beings*.

(18.) But here I quit these remote Thoughts; and from viewing the admirable Extent, Beauty, Order and Variety of the Creation abroad, betake my self *home* to our own Globe. And here we shall not lay aside our *Telescope* as *useless*: We may imploy it on various Occasions and divers Concerns of Human Life. The Merchant may with this discern afar his rich-laden Vessels, whose Sides and Sails are swell'd, and look big with imported Wealth. The Seaman may discern his Friend or Foe. The wealthy Countryman may survey his distant Herds, Plantations, and Labourers; and Generals may observe their wide-spread Troops. But endless would it be to touch on all; the contriving Head will find it *useful*, and adapt it to his own particular Concerns on many Occasions, that cannot now be thought of.

(19.) And thus much shall suffice in short, concerning the Discoveries made with the *Telescope*. And now I hope it will not be asked, *Cui Bono*? To what *End* are all these Discoveries? What Advantage is there in them? For, if the *Advancement* of *Astronomy* have any good in it; if the furnishing us with a Contemplation from whence we may evince the *Power* and *Wisdom* of an *Almighty Creator* be any Good; If affording an Opportunity of admiring the vast Extent, Order and Beauty of the Creation be any Good; I am sure, hereby we reap all these *Goods* in an ample manner. But supposing that nothing of all these Advantages were at hand just at present; let not the inquisitive Philosopher therefore despond in his Enquiries. The consideration of the *Magnet* (as I have noted before) teaches us what Secret Virtues may lurk in the simplest Things: And what admirable Uses Posterity may raise out

out of them. The *Torricellian Experiment* was long apply'd only to Disquisitions concerning a *Vacuum*, before our incomparable Philosopher the Honorable Mr. *Boyle*, one of the chief Glories of the *English Nation*, discovered its Usefulness in predicting the Weather, by which 'tis become one of the most pleasant Instruments in the World. But this I say, not so much to encourage men in the Prosecution of *useless Enquiries* (for doubtless he is *best* imployed that can propose the *best Advantages* to Mankind from his Studies) But to discourage some men from exclaiming against all Labours as absolutely *useless*, whose *immediate Use* they do not apprehend.

(20.) Of
the Micro-
scope.

(20.) And now that we are arrived at home, let us change our Instrument, and take into our hands a *Microscope*. And indeed with this, our Contemplations may be *endless*; all things affording such admirable Appearances, such curious Texture of Parts, and such delicate vivid Colours; that the Contrivance of the *Almighty Creator* is as visible in the *meanest Insect* or *Plant*, as in the greatest *Leviathan* or strongest *Oak*. To touch upon all the Wonders this Instrument shews us, would be infinite; I shall therefore only refer to those, who have prosecuted Enquiries therewith to great Exactness.

Fontana.

Franciscus Fontana in his *Observationes Caelestium Terrestriumque Rerum* (wherein he challenges to himself the Invention of the *double Microscope*, An. 1618.) is the first (that I can learn) who published *Microscopical Observations* of some few Bodies. After

Borellus.

him *Borellus* at the end of his Tract, *De vero Telescopii Inventore*,

Power.

An. 1650. does the same. Next a learned *Englishman* Dr. *Power*, An. 1664. did the like. But all these went no farther than verbal Description; they want those curious and lively Schemes,

Hook:

which the learned and ingenious Mr. *Hook* presents to the World in his *Micrographia*, An. 1665. a Book full of admirable Discoveries by the *Microscope*, and other curious Enquiries. The learned Dr. *Grew*, and the excellent *Bouonian* Phi-

Grew.

losopher

Philosopher Marcel. Malpighius, have laboured most successfully <sup>Malpighi-
us.</sup> in the *Anatomy of Plants* by the Microscope; and the latter has used it much in his Books, *de Ovo incubato*, *de Bombyce*, *de Viscerum structura*, &c. The Heer Leuwenhoeck of Delft in Holland, <sup>Lewenho-
eck.</sup> has lately apply'd himself with great Diligence to the use of Microscopes: of which Instrument he thinks he has a better kind than was ever yet known. When I visited this Gentleman at Delft, he shew'd me several that indeed were very curious; but nothing more than what I had ordinarily seen before; being compos'd only of one single, very minute Glass-Sphere or Hemisphere, placed between two very thin pierced Lamine, or Plates of Brass, and the Object was brought to its due distance before the Glass by a fine Screw: But for his best fort, he beg'd our Excuse in concealing them. The Observations he has made with his Glasses are Printed in several Letters of his in Dutch; but for the most part, they are to be found dispers'd in the *Philosophical Transactions*. The last Author that has professedly treated of Microscopick Observations, is Johan. Fran. Griendelius in his *Micrographia nova Norimberg.* 1687. where <sup>Griendeli-
us.</sup> in he has taken a great deal of pains in giving the genuine Representations of his Objects as magnify'd.

I have been often delighted with the curious Appearance of many Objects seen through the Microscope. But none ever surpris'd me more, than the visible Circulation <sup>Circulati-
on of the
Blood in
Water-
Newts.</sup> of the Blood in Water-Newts (*Lacerta aquatica*) to be seen as plainly as Water running in a River, and proportionably much more rapid. Of this I have formerly given the Account at large to the Royal Society. And 'tis publish'd in the *Philosophical Transact.* Num. 177. P. 1236.

(21.) I shall conclude all, with two remaining Uses of the Telescope. The first is, To view nigh Objects therewith. <sup>(21. View-
ing nigh
Objects
with the
Telescope.</sup> The most apposite Telescope for our Purpose, is that consisting of a Convex Object-Glass and Convex Eye-Glass. But what is deli-

vered hereof, is easily accommodated to the Telescope with a Concave Eye-Glass; but this latter, taking in but a narrow space of the Object, is not so proper for our Purpose.

Wherefore, suppose we have a Telescope, whose Object-Glass has its *Focal Length* for *very distant Objects* just 3 Foot, (how to find this exactly, I have shewn Chap. IV. Sect. 3.) This we'll call the *Solar Focus*. And suppose the Focus of the Eye-Glass be just 2 Inches; and that we were to view with this Telescope an Object 20 Foot distant from the Object-Glass. 'Tis required to find the *distinct Base* of this Object-Glass exposed now to this *nigh Object* 20 Foot distant, which in the First Part is called the *Respective Focus*.

And this is done by Prop. V. of the First Part: By this Analogy,

As the difference between the Distance of the Object and Glasses Focus :

To the Glasses Focus ::

So the Distance of the Object from the Glass:

To the Distance of the respective Focus, or distinct Base from the Glass.

In the Numbers of our Example thus,

$204.36 :: 240.42.35 + =$ equal to the *distinct Base* of this Object-Glass exposed now to this *nigh Object*. So that the Distance of the Object-Glass from the Eye-Glass, which, for viewing *distant Objects*, was $36 + 2 = 38$ Inches; must now be $42.35 + 2 = 44.35$ Inches; and so much is the Telescope to be lengthened to view this Object distant only 20 Foot.

Viewing
Pictures
in Mini-
ature.

The use of this Theorem is very pretty; and they who draw Pictures in *Miniature*, may practise it to good Purpose. For the small Picture (being *inverted* that it may appear *erect*, and placed in a strong Light, and thus looked at with a Telescope) may

may be made to appear full as *bigg* as the natural Face, or *bigger* if we please. And by this means, the least Errors of the Painter may be easily discovered, which cannot be done so well by a single Convex-Glass as will be manifest to those who shall try and compare both Ways. And this naturally leads me to the last Use of the Telescope I shall here mention; and 'tis as it were the *Converse* of this I have just spoke of. 'Tis,

(22.) To Measure the Distance of an Object at one Station by ^{(22.) Measuring Distances by a Tele-}
a Telescope.

This is the great *Desideratum* in *Practical Geometry*; and always reputed *impossible*. Whatever Attempts have been made towards it have been always found to result at last to a *double Station*, tho some way or other disguised by the Contrivance. This is evidently shewn by Tacquet (*Geom. Pract. Lib. 1. ad Finem Cap. 5.*) concerning the Method proposed by Clavius. And may be shewn of any other, that I have ever yet heard of.

Towards the latter end of the year 1665. (as we may find ^{History of this Affair.} Num. 7. P. 123. *Philosoph. Transact.*) Mons. Auzout did propose to Mr. Hook, to exchange with him a Secret he had in Opticks, for another which Mr. Hook had. Mons. Auzout's was, *Locorum Distantias ex unicâ statione, absque ullo Instrumento Mathematico metiri*. And therein he declares the Instrument, he uses for this Purpose, to be a *great Telescope*, with some necessary Tables. Adding withal, "That tho the Practice do not altogether answer the *Theory* of his *Invention*, because that the "Length of the Telescope admits of some Latitude; yet one "comes near enough, and perhaps as *just*, as by most of the "Ways ordinarily used with Instruments. But tho Mr. Hook discovered the Secret which Mons. Auzout desired from him, (which is that I mention at the End of Chap. IV.) yet Mons. Auzout never (that I know) published any thing farther of this

Invention of his own. And indeed I cannot see, how justly Mons. Azout could say, that his Performance was, *absque ullo Instrumento Mathematico*; and at the same time tell us, that it was done by a great Telescope; which surely is a Mathematick Instrument in the highest sense. Perhaps he means, 'tis none of those Mathematick Instruments commonly used in this Practice.

Mr. Oldenburg, who then publish'd the *Philosoph. Transact.* adds Pag. 125. "That the Secret of measuring the Distance of Places by a Telescope fitted for that Purpose, and for one Station, is a Thing already known (if I am not misinformed) to some Members of the Royal Society; who have been a good while since considering of it, and have contrived Ways for the doing of it. Whether the same with those of Mons. Azout, I know not; nor have I (at the Distance that I am now from them) Opportunity of particular Information.

This is in short the History of this Affair. Wherein (for ought as ever I could learn) nothing more has been done hitherto. So that what I shall propose therein is my own Thought and Contrivance. Whereof I shall first declare the Method; and afterwards shall not conceal the Difficulties thereof: not doubting but more ingenious Heads may have Contrivances of the same kind far exceeding mine; tho' nothing herein has ever yet been publish'd.

The Author's Proposal.

Wherefore let the Distance of the Object from the Object-Glass be called d , the Solar Focus of the Object-Glass, or its Focus at very distant Objects f , the respective Focus or distinct Base at a nigh Object r .

By Prop. V. and the Rule immediately mentioned :

It shall be — — — $d - f . f :: d . r$

And Compounding — $d . f :: d + r . r$

Permute — — — $r . f :: r + d . d$

Divide — — — $r - f . f :: r . d$

Which

Which last Analogy is the Rule I give for finding the Distance of an Object by a Telescope, thus expressed in Words,

As the Difference between the Respective Focus and Solar Focus:

To the Solar Focus ::

So the Respective Focus:

To the Distance of the Object from the Object-Glass.

It now remains to shew how to obtain exactly the three first Terms of this Analogy; That thereby we may find the fourth required. And first for the *Solar Focal Length* of an Object-Glass, 'tis shewn before in Chap. IV. Sec. 3. how to procure it exactly. Secondly for the *Respective Focus*, or distinct Base of any *nigh* Object we look at, I propose the Way to obtain it thus:

As the *Micrometer* is contrived to *open* and *close* in the Focus of the Object-Glass, and the Indices give the exact Measure of this *Opening*: So may we adapt an Instrument, which may *advance* or *withdraw* the curious Point of a slender Needle to or from the Object-Glass. And an *Index* (after the manner of the *Micrometer*) may shew how much the slender Point is withdrawn from the Object-Glass. Then looking through the Telescope at the Object, whose Distance we measure, let us withdraw the Needle, till by moving the Eye before the Eye-Glass, we perceive the Needle, as it were, *fixt* upon the Object: (as is taught Chap. V. Sect. 4.) Then is the Needle in the *Respective Focus*. And by observing the *Index* aforementioned, we have the Measure of this *Respective Focus* from the Object-Glass: And consequently the Difference between it and the *Solar Focus*. With which we are to work according to the Rule, and we obtain what was required, *viz.* the Distance of the Object. Thus suppose, in the foregoing Example, the Glasses Focus be = 36 Inches, and the *Respective*

spective Focus be measured 42.45 : The Analogy will then stand thus,

$42.35 - 36 = 6.35 \cdot 36 :: 42.35 \cdot 240 =$ to the Distance of the Object from the Object-Glass. And thus may we make a Table to any Telescope, that upon the first Sight of the *Respective Focus* shall give the Distance of the Object.

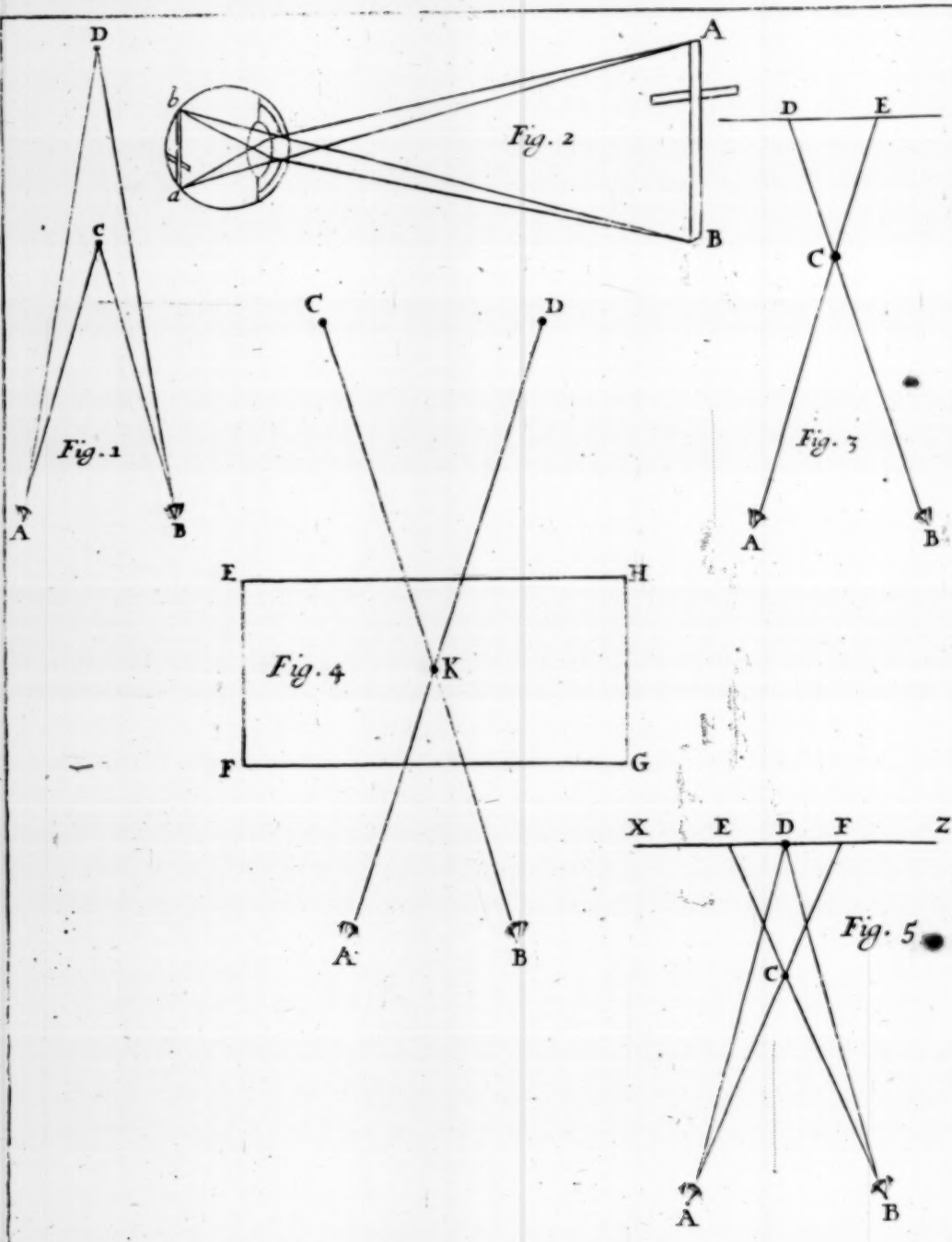
Difficul-
ties and
Uncer-
tainty
thereof.

BUT now I shall not conceal the Difficulties, that attend this Method of observing Distances. And first, The Eye can never be *justly* certain, when the Needle's Point is *exactly* in the *Respective Focus*; for tho the Way I propose of examining it (*viz.* by moving the Eye before the Eye-Glass, and observing whether or not the Needle seems to move on the Object) be the best Expedient, and most certain that I can think of at present: Yet we shall find, that it admits of some Latitude; and that the Needle may be moved a little more *forward* or *backward*, without shewing any visible Motion thereof on the Object; and this too most especially, the longer the Telescopes are that we use, and the wider the Aperture. But for the Remedy hereof, 'tis best to observe where the Needle (being *too nigh* the Object-Glass) *begins first* to move on the Object; as also to observe, where the Needle (being *too far* from the Object-Glass) *begins first* to appear to move on the Object; and to take the *middle* between these two Stations of the Needle for the *true Respective Focus*.

Secondly, where the Distance of the Object is *very great*, and vastly disproportional to the Solar Focal Length of the Object-Glass; the Observation will not be very accurate. And therefore it is that Mon^r. *Auzout* in the forementioned Passage out of the *Philosoph. Transact.* confines his Practice to *great Telescopes*. For unless they be *so*, a considerable Alteration in the Distance of the Object makes no sensible Alteration in the *Respective Focus* of the Object-Glass.

But

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But if we believe the said Monf. *Auzont*, by this Way, (if this Way which I propose be his, as I know not whether it be or not) one comes near enough to *Exactness*, and perhaps as just as by most of the Methods ordinarily used. However, tho the Practice, through some accidental Difficulties, do not so exactly answer the Theory: Yet it cannot be deny'd, that the Theory is true. And perhaps farther Improvements may bring it to Perfection.

CHAP. VII.

An Optick Problem of Double Vision.

IN *Tab. 41. Fig. 1.* AB are the two Eyes, C the nigher *Tab. 41.
Fig. 1.* Object, D the farther Object; If both Eyes open are fixt upon C, the Object D shall seem double, and then shutting the left Eye A, the left Image of D disappears; and shutting the Right Eye B, the right Image of D disappears.

But if both Eyes open are fixed on D, then C shall seem double, and if the left Eye A be shut, the right Image of C vanishes, if the Right Eye B be shut, the left Image of C vanishes.

This is the Declaration of the Phenomenon, which any one may experiment by placing two Candles, D 3 Foot, and C 1 Foot distant from the Eyes, and then standing so that the Nose and two Objects may lie in or near a right Line, he fixing his Eyes on either Object, alternately open and shut them I choose two Candles, and at that Distance and Posture, because the Experiment thereby will be more sensibly evident; tho it hold in any two Objects at whatever Distance, till the Distance be so great, that the Interval between the two
Eyes,

Eyes bears no sensible proportion thereto, that is, till the Angle ACB (and much more ADB) is so very small, that the Lines AC, BC, may be taken to run as it were Parallel.

This being declared, I explicate the Reason of this Appearance, as follows.

In the first Case, both Eyes being fixed on C, if the Right Eye B be shut, the Object D will appear to the left hand of C; then shutting the Left Eye A, and opening the Right Eye B, and looking at C, D will appear to the Right Hand of C; therefore opening both A and B, and looking stedfastly on C, the Object D will appear on both sides C, that is, both to the Right and Left Hand of C, and therefore double. And 'tis manifest, the Right Eye B receives the Right Image of D, and the Left Eye A the Left Image of D; therefore in this Case, to the Right Eye being shut, the Right Image of D disappears, and to the Left Eye being shut, the Left Image of D disappears.

But in the second Case, fixing both Eyes on D, C seems double, for the Left Eye A being shut, C appears to B on the Left Hand of D; then the Right Eye being shut, the Left Eye A sees C on the Right Hand of D; so that in this Case, the Right Eye receives the Left Image of C, and the Left Eye the Right Image of C, and consequently the opening or shutting of either, does in this Case make the contrary Image of C disappear. And why C should seem double, is plain in this Case; for the Right Eye sees it on the Left Hand of D, and the Left Eye on the Right Hand of D; so that both Eyes see it on both Hands of D, and therefore double. But then in the first Case, the Object D, and not the Object C, seems double. And in the second Case, the Object C, and not the Object D, seems double. For in Vision, there is a Difference between *looking* and *seeing*; what-
ever

ever Object I look at with both Eyes appears *single*, and all others more remote or nigher, that I see, appear *double*; for upon the Object, I look at, the Optick Axes do concur, but not so on those I only see. And this is the Reason that at any time I can make all the Objects about me seem confused, only by turning the Optick Axes so, that they may concur in the free Air; as if there were a certain visible Object there before them, tho really there be none.

And that the Concurrence of the *Axes Optici* in a single Point or Object is sufficient to make that Object seem but one besides the Proof of the foregoing Experiments, I shall endeavour to evince, or at least to explicate, by an other known Affection of Vision; the Explanation whereof is allowed by all men as satisfactory, 'tis this, in *Tab. 41. Fig. 2.* T. 41. F. 4. the Image *ab* of the Object *AB* is painted on the *Retina* inverted, and yet the Eye (or rather the Soul by means of the Eye) sees the Object erect and in its natural Posture; Because the Mind takes no notice of what happens to the Rays in the Eye by Refraction or Decussation, but in its direction towards the Object; it follows streight amongst the Rays as they by their Impulse and in their plain Course lead it, and consequently following the Rays *aA*, it is directed strait to the upper part of the Object; and also following the Rays *bB*, it is directed to the lower part of the Object, and so of the rest: for suppose the Ball of the Eye taken out of its Socket, or Cavity in the Skull, and a Man receives in the Socket an impulse by a Stick coming in the posture of *Bb*, and hitting him on the upper part of the Cavity, surely he would never look for the Original of this Blow at *A*, but would be certainly directed to hunt back as it were amongst the Stick *bB* towards the Place from whence the Stroak comes. So the mind does hunt back by means of each Pencil of Rays (which are as it were a Stick giving the *Retina* a certain

P p

Impulse)

Impulse) to the Point from whence it comes, and is thereby directed strait thereto. To apply this to what I intend, I say then, that the Mind or visive Faculty (if I may have leave to use that Word for a thing we all understand and cannot better express) takes no notice that there are two *Axes Optici*, or two Pictures made by those *Axes Optici* on each *Retina*, but following back, and hunting counter alongst these *Axes*, it is directed to, and determined in one single Point, and therefore it sees it as one.

And seeing this Speculation does naturally lead me to the Consideration of an Opinion, first as I think started by the celebrated *Gassendus*, and since embraced by many, viz. that we see but with one Eye at once one and the same Point of an Object, *otiante alio*, (as they term it) whilst the other is idle and does nothing. I shall not think it improper to subjoyn to my former Discourse something relating to that Opinion, confining my self only to what may something illustrate my former Explanations, for to enumerate all the Experiments that prove we see with two Eyes, would swell this far beyond the limits I design it. Therefore to the Matter.

Against our seeing with two Eyes at once one and the same Point of an Object, it is commonly objected, that if it were so, every Object would seem in two places at once, *vide Gassendi Epist. 4. de Magnit. Solis humilis & sublimis, & Taqueti Opt. lib. 1. Prop. 2.* Thus in Tab. 41. Fig. 3. If the Eyes A, B, look at the Object C, and both see it at a time, A would see it on the opposite Wall suppose at E, and B at D.

I am so far from thinking this an Objection, that I assert first, that we do see all Objects in two places, and that this is not taken notice of by us upon these Accounts. First, the common Objects of our sight are large, and the *Axes* of our Eyes directed

directed to but one Physical Point thereof at a time, and then we cannot be expected to see such in two places at once; for when I read on a flat broad Book, where is the opposite Wall for a single small Letter to be seen on at two places, the Letters themselves are fixt to the Surface that determines the Sight. What I Instance in Letters on a Book may be accommodated to most other Objects. Secondly, There are few Objects disposed fitly for shewing us that we see them in two places, for either their Bulk hinders the Experiment from appearing so plainly, or their distance from each other or from the Eyes. Thirdly. The Wall D E and the Parts thereof are seen so confusedly by the Eyes A, B, when fixed on C, that we cannot so clearly observe this Experiment; and this happens to all Objects more remote or nigher to us than C, when the Eyes are fixed on C, still supposing that it self be but in a moderate Distance from us, that is, in such a Distance from us, as may bear some considerable Proportion to the Distance of the two Eyes from each other. Fourthly, the chief Reason why we do not perceive so plainly the Object C to cover both D and E is, because, though D be cover'd by C from the Eye B, yet it is not covered from the Eye A, and therefore we think it not covered at all. Also though E be covered by C from the Eye A, yet it is not covered from the Eye B, and consequently because we see E, tho but with one Eye B, we imagine it not covered at all, so that both the Points D and E being open to both the Eyes A and B, viz. the Point D to the Eye A, and the Point E to the Eye B, we think neither of them covered, whereas really they are both obscur'd, each to its proper Eye, as will be evident by winking. And from hence is manifest the falsity of *Taquet's* Assertion in the forecited Place, viz. that opening both Eyes A and B (*Fig. 3.*) and looking at C, it shall appear only to cover E (he imagining that most Men see generally with their left Eye) whereas I

say it shall appear to obscure neither E nor D, tho to each Eye respectively it cover each. Fifthly, whereas the Bulk of Objects (as I have noted) does hinder the Experiment from being so sensibly evident ; if by any Experiment I can shew how a large Object may be doubled, I think 'twill be sufficiently plain. Therefore in *Tab. 41. Fig. 4.* let A, B, be the two Eyes , before which at a convenient Distance place the two Candles C, D, then take a large piece of Paper EFGHK, in whose middle at K there is a small Hole. This Paper so place between the Eyes and Candles, that the Eye A may through the Hole K see the Candle D ; and at the same time the Eye B may see through the Hole K the Candle C ; the due Distance of the Paper that is requisite for this will be found by Tryals, and winking alternately with the Eyes. When all things are in this Posture, open both Eyes A and B, and direct them to look at either of the Candles, I say the Hole K shall seem double ; and what is proved of the Point K is true of all other Points in the Paper, so the whole Paper appears double ; but no Points thereof are so evidently doubled as the Point K, because they want the Advantage of Lights behind them to render the Experiment more sensible. And upon this Occasion I cannot but hint to all Persons that are desirous to make Experiments about Vision, that they always imploy the most luminous and vigorous Objects they can possibly, for many Experiments will be evident by them, that will not sensibly succeed with others.

But I assert secondly, that unless we saw with both Eyes, the two first mentioned Experiments would not succeed, and they may be reckoned amongst the greatest Arguments that can be produced for it. For suppose in *Fig. 1.* A and B to look stedfastly on C, I say they see C in two Places at once, for they see it both on the Right hand and Left hand
of

of D. For tis the same thing to see C between two D's, as to see it at two Places at once. And this I'll declare more fully *Tab. 41. Fig. 5.* supposing the two Eyes A and B fixed on the Object C, I say C appears on the opposite Wall X Z in two Places, *viz.* at E and F (tho neither E nor F are obscur'd thereby for the foregoing Reason) for let us suppose the Object D of the first Figure to be placed duly on this Wall; shutting A, the Object C shall appear to B on the left hand of D by the Angle D B E: then shutting B, the Object C shall appear to A on the right hand of D, by the Angle D A F; wherefore to both open at a time and looking at it, it shall appear on both sides of D, to A on the right side, to B on the left side, and this happens by doubling of D; so that from hence this Paradoxical Corollary arises, *That an Object may be seen in two Places, and yet not seen double.* T. 41. F. 5.

They that assert that we see but with one Eye, whilst the other is idle and does nothing, assert likewise that all visual Spirits recede from the idle Eye, and only supply the Eye that sees; but I see no Reason why the Eyes A, B, looking at C, either of them should be said destitute of Spirits in respect of C, which they see single, and yet both replenished with Spirits in respect of D, which they see Double. The Images of C and D both are painted on the Retina of A as well as of B, and therefore A and B both being replenished with Spirits to see D, (even by the Concession of our Adversaries) why not to see C also, at which they stedfastly look?

To conclude, I propose it to the learned and ingenious Dr. Briggs, or to any other of the Philosophical Spirits of this Age to explain the foregoing Phenomena by the Doctor's Theory of Vision. And as a Conclusion to the whole shall only add one Experiment that Demonstrates we see with both

both Eyes at once; and 'tis, that which is commonly known and practised in all Tennis-Courts, that the best Player in the World Hoodwinking one Eye shall be beaten by the greatest Bungler that ever handled a Racker; unless he be used to the Trick, and then by Custom he gets an Habit of using one Eye only.

APPENDIX

AS THE
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both Eyes at once; and 'tis, that which is commonly known and practised in all Tennis-Courts, that the best Player in the World Hoodwinking one Eye shall be beaten by the greatest Bungler that ever handled a Racket; unless he be used to the Trick, and then by Custom he gets an Habit of using one Eye only.

APPENDIX

Fig. 1.

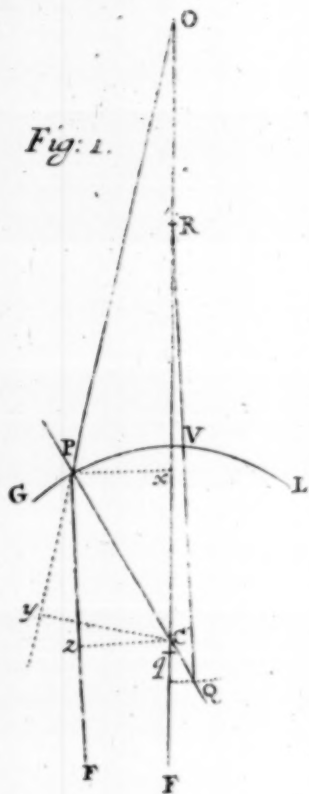


Fig. 2.

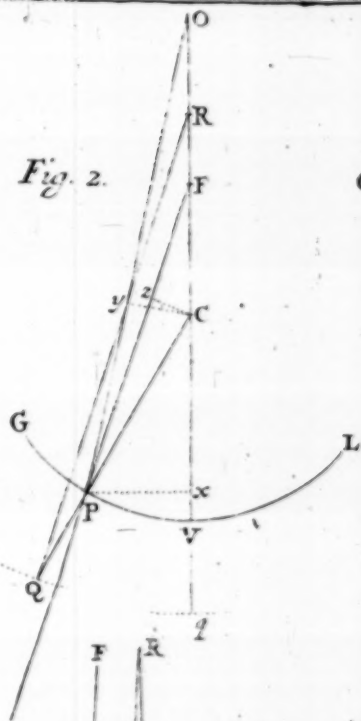


Fig. 3.

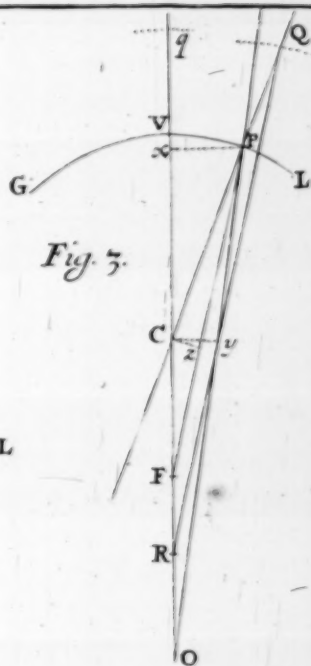


Fig. 6.

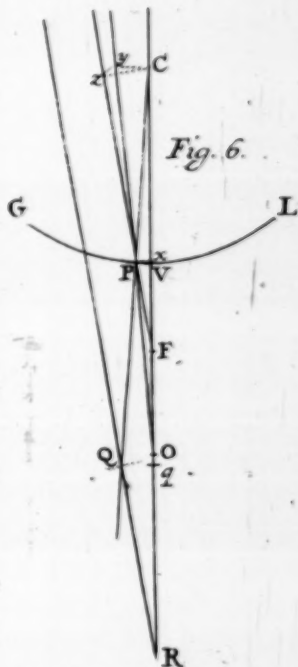


Fig. 5.

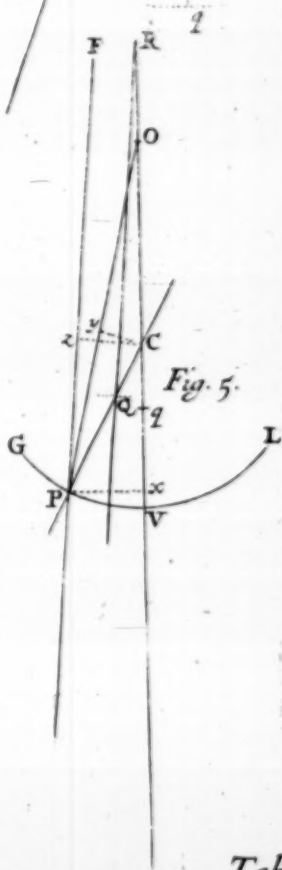
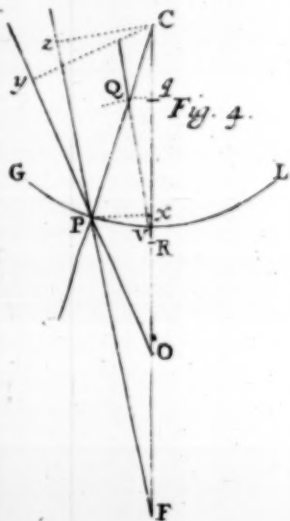


Fig. 4.



APPENDIX.

WHilst this Book was in the Press my Affairs were such, that I could not attend the Perusal and Correction thereof, but therein have made use of my Friend Mr. E. Halley, who was willing to do me that Service: He, after the first Part hereof was finished, sent me a Proposition of his own, which I took to be of that Consequence in Dioptricks, that I importuned him to permit it to be subjoyned by way of Appendix to my Treatise, it being of that Extent as to comprehend the whole Doctrine of the Foci of Spherical Glasses of all Sorts, exposed either to Diverging, Converging, or Parallel Rays. It is as follows.

PROPOSITION.

TO find the Focus of any Parcel of Rays Diverging from, or Converging to a given Point in the Axis of a Spherical Lens, and inclined thereto under the same Angle; the ratio of the Sines in Refraction being known.

Let GL Tab. 42. be the Lens, P any Point in its Surface, V the Pole thereof, C the Center of the Sphere whereof it is a Segment, O the Object or Point in the Axis, to or from which the Rays do proceed, OP a given Ray: and let the ratio of Refraction be as r to s ; make CR to CO as s to r for the immersion of a Ray, or as r to s for the Emerision, (that is, as the Sines of the Angles in the Medium which the Ray enters, to their corresponding Sines in the Medium out of which it comes) and laying CR from C towards O , the Point R shall be the same for all the Rays of the Point O . Then draw the Radius PC if need be continued, and with the Centre R and Distance OP sweep a touch of an Arch intersecting PC in Q ; the Line QR being

being drawn shall be parallel to the refracted Ray, and PF being made parallel thereto shall intersect the Axis in the Point F, which is the Focus sought Or make it as $CQ : CP :: CR : CF$ and CF shall be the Distance of the Focus from the Centre of the Sphere.

Demonstration.

Let fall the Perpendiculars Px on the Axis, Cy on the given Ray and Cz on the refracted Ray. By the Construction PF and QR are parallel, whence the ΔQRC and $PF C$ are similar, and CR to QR as CF to PF, that is, CR to OP as CF to PF. Now $CF : PF :: Cz : Px$ ob *similia Triang.* whence $CR : OP :: Cz : Px$, and $CR : Cz :: OP : Px$. Again CR is to CO as the Sines of Refraction, by construction, that is as s to r or r to s ; and as CR to Cz , so $CO = \frac{r}{s}$ or $\frac{s}{r}$ CR to $\frac{r}{s}$ or $\frac{s}{r}$ Cz , and so PO to Px : But as PO to Px , so CO to Cy . Ergo $Cy = \frac{r}{s}$ or $\frac{s}{r}$ Cz , that is, Cy to Cz is as the Sines of Refraction, but Cy is the Sine of the Angle of Incidence, and Cz of the refracted Angle. Ergo constat *Propositio*.

The several Cases of Rays Diverging or Converging as they enter the curve Surface of a *Convex* or *Concave Lens*, are for the Readers Ease delineated in the first four Figures of *Tab. 42.* as are in *Fig. 5* and *6* thereof, and in *Fig. 1, 2.* of *Tab. 43.* the like Cases of emerging Rays. All which are drawn with the same Letters to their respective Points, only in some the Point F falling far distant, is to be understood in the Intersection of the Line PF with the Axis.

This thus demonstrated in the most difficult Cases, will give all the Rules for the Foci of Rays parallel to the Axis as likewise for the principle Focus, where the Rays nearest the Axis do unite, all which Rules I shall collect in these following Corollaries.

Cor. 1. If OP be equal to CR, then the Points Q and C are coincident, and the Rays OP after Refraction run on parallel to the Axis.

Cor. 2.

Cor. 2. If the Point *Q* fall on the same side the Axis as is the Point *P*, then the Beams after Refraction do tend on, either diverging or converging as before: But if *Q* fall on the other side the Axis, as in *Fig. 1.* the diverging Rays are made to converge by a Convex, or the converging to diverge by a Concave Glafs.

Cor. 3. If *OP* do exceed *CR*, the Focus is in all Cases on the same side of the Glafs as is the Centre of the Sphere *C*. But contrarywise if *OP* be less than *CR*, the Focus falls on the other side of the Glafs beyond the Vertex *V*.

Cor. 4. An Object may be so placed, that the Rays next the Axis of a Convex-Glafs shall have an imaginary Focus, transmitting diverging Rays, when the more remote Parts thereof shall make them converge to a real Focus.

Cor. 5. If *OV* the Distance of the Object from the Pole or Vertex of the Glafs, be taken instead of *OP*, then will *CQ* be the difference of *OV* and *CR*, and as that difference to *CR*, so the Radius *CV* to *CF* the Distance of the principal Focus from the Centre of the Sphere whereof the Glafs is a Segment: or else as *CQ* to *OP* or *RQ*:: so *PC* to *VF* the focal distance from the Pole of the Glafs. Whence follows a general Rule for the Foci of all Glasses, only according to *Corol. 3.* if *OV* do exceed *CR*, the Focus is on the same side of the Glafs as is the Centre of the Sphere. But if *CR* be the greater, then the Focus is on the opposite side of the Glafs, whence it will be determined whether the Focus be real or imaginary.

Cor. 6. What has hitherto been said of one Surface of a *Lens* is easily applicable to the other; taking *F* the *Focus* of the first Surface as an Object, and using it as *O* in the Figures for emerging Rays, whereby the *Focus* of both Surfaces will be determined, as in *Tab. 43. Fig. 3.* where I have given an Example.

Cor. 7. Hitherto we have considered only *oblique Rays* either *Diverging* or *Converging*; it now remains to add something con-

cerning *Rays parallel* to the *Axis*: In this Case the Point *O* must be considered as infinitely distant, and consequently *OP*, *OC*, and *CR* are all infinite; and *OP* and *OC* are in this Case to be accounted as always equal, (since they differ but by a Part of the Radius of the Sphere *GPVL*, which is no part of either of them,) wherefore the *ratio* of *CR* to *OP* will be always the same, viz. as *s* to *r* for *immersing Rays*, and as *r* to *s* for those that *emerge*. And by this Proposition *CF* is to *PF* in the same *ratio*. It remains therefore to shew on the Base *CP*, to find all the Triangles *CPF* wherein *CF* is to *PF* in the *ratio* given by the degree of Refraction. This Problem has been very fully considered by the celebrated Dr. *Wallis* in his late Treatise of Algebra, pag. 258. to which I refer; but I must here repeat the Construction thereof, *Tab. 43. Fig. 4, 5.*

Let *GPVL* be a *Lens*, *VC* or *PC* the Radius of its Sphere, and let it be required to find all the Points *f, f*, such as *Cf*, may be to *Pf* in the given *ratio* of *s* to *r* for *immersing Rays*, or as *r* to *s* for the *emerging*. Divide *CV* in *K*, and continue *CV* to *F*, that *CK* may be to *VK*, and *CF* to *VF* in the proposed *ratio*: Then divide *KF* equally in the Point *a*, and with that Center sweep the Circle *FKF*; this Circle being drawn gives readily all the Foci of the Parallel Rays *OP*, *OP*. For having continued *CP* till it intersect the Circle in *F*, *PF* shall be always equal to *Vf* the Distance of the Focus of each respective Parcel of Rays *OP*, from the Vertex or Pole of the *Lens*.

To demonstrate this, draw the prickt line *VF*, and by what is delivered by Dr. *Wallis* in the aforecited Place, *VF* and *CF*, will be alwayes in the same proposed *ratio*. Again *Vf* being made equal to *PF*, *CF* and *Cf* will be likewise equal, as are *CP*, *VC*; and the Angles *PCf*, *VCf* being *ad verticem* are also equal: Wherefore *Pf* will be equal to *VF*, and consequently *Cf* to *Pf* in the same *ratio* as *CF* to *VF*, whence
and

and by what foregoes, the Points f, f are the several respective Foci of the several Parcels of Rays, OP, OP . Q. E. D.

That CF is to PF in the *ratio* of the Refraction, in the case of parallel Rays, will be yet more evident, if it be consider'd, that the Angle at C is equal to the Angle of Incidence, and the Angle at P to the refracted Angle; wherefore PF the side opposite to the Angle at C , is as the Sine of the Angle of Incidence, and CF opposite to the Angle at P , is as the Sine of the respective refracted Angle; whence in all Cases of parallel Rays, CF is to PF in the same constant *ratio* of Refraction.

If it shall be desired to effect in Numbers what we have here done by Lines, it will be most easie to adapt a Calculus to the foregoing Geometrical Construction. For if in the Triangle POC there be given the Radius CP equal to *Unity*, CO the distance of the Object from the Centre of the Sphere, and the Perpendicular Px equal to the Sine of the Angle PCO , the side $PO = QR$ will be equal to $\sqrt{CO^2 + CP^2} \pm 2CO$ in $\sqrt{CP^2 - Px^2}$. Then as QR or PO to Px :: so CR to the Sine of the Angle CQR , and the Complement to 180° of the sum of the Angles CPO and CQR is the Angle $CRQ = CFP$; and as Px to PO so the Sine of the Angle CRQ to CQ ; and as CQ to CP so CR to CF , which is the distance of the respective Focus of all the Rays PO from the Centre of the Sphere C .

But the Foci of Rays parallel to the *Axis* may be more readily computed, following the Footsteps of the Construction delivered in *Coroll. 7. (Tab. 43. Fig. 4, 5.)* for thereby it will appear, that the Radius of the Circle KF , *viz.* aF , is equal to $\frac{rs}{rr - ss} CP$, and $Ca = \frac{rr}{rr - ss} CP$, for emerging Rays, as in *Fig. 4*; but for immerging Rays, as in *Fig. 5*, Ca will be found to be $\frac{ss}{rr - ss} CP$: and supposing the distance of the Ray from the

Q q 2

Axis

Axis = Px , in the Case of parallel Rays Emerging, the distance of the Focus will be found, $PF = \frac{rr}{rr-ss} \sqrt{CP^2 - Px^2} + \sqrt{\frac{rs}{rr-ss} CP^2 - \frac{rr}{rr-ss} Px^2} - CP$: that is, r to s being as 3 to 2, $PF = \frac{9}{5} \sqrt{CP^2 - Px^2} + \sqrt{\frac{6}{5} CP^2 - \frac{9}{5} Px^2} - CP$. And for immerging Rays, the Focal distance is found by a like Rule, $PF = \frac{ss}{rr-ss} \sqrt{CP^2 - Px^2} + \sqrt{\frac{rs}{rr-ss} CP^2 - \frac{ss}{rr-ss} Px^2} + CP$: that is r and s being as 3 to 2 as before, PF is equal to $\frac{4}{5} \sqrt{CP^2 - Px^2} + \sqrt{\frac{6}{5} CP^2 - \frac{4}{5} Px^2} + CP$. These Canons are so easily deduced from the Constructions, that I shall not need to trouble the Reader with their Demonstration; only I shall add two Tables which I computed from them, with little more work than a continual Addition; which may by way of Example, serve to instruct and exercise the young Student in this part of Mathematicks.

Suppose CP the Radius of the Sphere of Glas 2 Inches, and the *ratio* of Refraction as 3 to 2; at each tenth of an Inch distance from the Axis, the *Foci* are as follows.

For Emerging Rays.

Px	PF
0	$\sqrt{12,9600} + \sqrt{3,7600} - 2$
1	$\sqrt{12,9276} + \sqrt{3,7276} - 2$
2	$\sqrt{12,8304} + \sqrt{3,6304} - 2$
3	$\sqrt{12,6684} + \sqrt{3,4684} - 2$
4	$\sqrt{12,4416} + \sqrt{3,2416} - 2$
5	$\sqrt{12,1500} + \sqrt{2,9500} - 2$
6	$\sqrt{11,7936} + \sqrt{2,5936} - 2$
7	$\sqrt{11,3724} + \sqrt{2,1724} - 2$
8	$\sqrt{10,8864} + \sqrt{1,6864} - 2$
9	$\sqrt{10,3356} + \sqrt{1,1356} - 2$
10	$\sqrt{9,7200} + \sqrt{0,5200} - 2$

For Immerging Rays.

Px	PF
0	$\sqrt{2,5600} + \sqrt{5,7600} + 2$
1	$\sqrt{2,5536} + \sqrt{5,7536} + 2$
2	$\sqrt{2,5344} + \sqrt{5,7344} + 2$
3	$\sqrt{2,5024} + \sqrt{5,7024} + 2$
4	$\sqrt{2,4576} + \sqrt{5,6576} + 2$
5	$\sqrt{2,4000} + \sqrt{5,6000} + 2$
6	$\sqrt{2,3296} + \sqrt{5,5296} + 2$
7	$\sqrt{2,2464} + \sqrt{5,4464} + 2$
8	$\sqrt{2,1504} + \sqrt{5,3504} + 2$
9	$\sqrt{2,0416} + \sqrt{5,2416} + 2$
10	$\sqrt{1,9200} + \sqrt{5,1200} + 2$

But

But it is to be Noted, that these *Foci* for Immerging Rays, must not be taken for the Foci of a Plano-Convex, when the Convex Side is towards the Object, for the plane Side by its Refraction, does contract the Focal length by about a Semidiameter of the Sphere; These suppose the Body of Glas continued, as in the First Proposition of this Treatise.

F I N I S.

ERRATA.

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220	1	shall be	as shall be
264	22	his	this

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